A categorical approach to Weyl modules

V. Chari¹ G. Fourier² T.Khandai³

¹UC Riverside

²Universität zu Köln

³HRI, Allahabad

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Notations Current and loop modules

Simple complex Lie algebra

• g is a simple, complex Lie algebra

• *R*, *R*⁺ set of roots, *Q*, *Q*⁺, *P*, *P*⁺ (positive) root and weight lattice

•
$$x_{\alpha}^{\pm}$$
 and $h_{\alpha} = [x_{\alpha}^{+}, x_{\alpha}^{-}], \alpha_{i}, \omega_{i}$

• $\mathfrak{g} = \mathfrak{n}^+ \oplus \mathfrak{h} \oplus \mathfrak{n}^-$

Let A be a commutative associative algebra with unit over \mathbb{C} . We define a Lie bracket on $\mathfrak{g} \otimes A$ by

$$[x \otimes a, y \otimes b] := [x, y] \otimes ab$$

for $x, y \in \mathfrak{g}$ and $a, b \in A$.

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Weyl modules of current and loop algebras

• Relation to char p

- Chari-Pressley defined global and local Weyl modules for g ⊗ C[t^{±1}] (resp. g ⊗ C[t]) by generators and relations
- Motivated by representations of quantum affine algebras
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Projectives

Let $\mathcal{I}_{\mathcal{A}}$ be the category of $\mathfrak{g} \otimes A$ modules which are integrable as \mathfrak{g} modules. The morphism are $\mathfrak{g} \otimes A$ module homomorphism. Let V be a left \mathfrak{g} -module, define a left $\mathfrak{g} \otimes A$ module

$$P(V) := \mathbf{U}(\mathfrak{g} \otimes A) \otimes_{\mathbf{U}(\mathfrak{g})} V.$$

Proposition

Let V be an integrable g module, then P(V) is a projective module in $\mathcal{I}_{\mathcal{A}}$. If $\lambda \in P^+$, then $P(V(\lambda))$ is generated by $p_{\lambda} = 1 \otimes v_{\lambda}$ with relations

$$\mathfrak{n}^+\otimes \mathtt{1}=\mathtt{0}\ ,\ (h-\lambda(h))=\mathtt{0}\ ,\ (x^-_{lpha_i}\otimes \mathtt{1})^{\lambda(h_{lpha_i})+\mathtt{1}}=\mathtt{0}.$$

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For $\nu \in P^+$ and $V \in Ob \mathcal{I}_A$, let $V^{\nu} \in Ob \mathcal{I}_A$ be the unique maximal $\mathfrak{g} \otimes A$ -quotient of V satisfying

$$\operatorname{wt}(V^{\nu})\subset
u-Q^+.$$

We define $\mathcal{I}^{\nu}_{\mathcal{A}}$ to be full subcategory of $\mathcal{I}_{\mathcal{A}}$ of objects V s.t.

$$V = V^{\nu}$$
.

We define for $\lambda \in P^+$

$$W_A(\lambda) := P(V(\lambda))^{\lambda}$$

the "global Weyl module".

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Category of locally finite modules The Weyl functor Functorial in A

Original definition

There is another definition of the global Weyl module by generator and relations, which is the "original" definition by Chari-Pressley in the case $A = \mathbb{C}[t^{\pm 1}]$.

Proposition

For $\lambda \in P^+$, the module $W_A(\lambda)$ is generated by $w_\lambda \neq 0$ with relations:

$$(\mathfrak{n}^+\otimes A)w_\lambda=0, \quad hw_\lambda=\lambda(h)w_\lambda, \quad (x^-_{lpha_i}\otimes 1)^{\lambda(h_{lpha_i})+1}w_\lambda=0.$$

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Annihilator algebra

Set

$$\operatorname{Ann}_{\mathfrak{h}\otimes A}(w_{\lambda})=\{u\in \mathbf{U}(\mathfrak{h}\otimes A): uw_{\lambda}=0\},\$$

and define

$$\mathbf{A}_{\lambda} := U(\mathfrak{h} \otimes A) / \operatorname{Ann}_{\mathfrak{h} \otimes A}(w_{\lambda}).$$

Define a right $\mathfrak{h} \otimes A$ -module structure on $W_A(\lambda)$ by

$$zw_{\lambda}.(h\otimes a):=z(h\otimes a)w_{\lambda}$$

for $z \in \mathbf{U}(\mathfrak{g} \otimes A)$, $h \otimes a \in \mathfrak{h} \otimes A$. $W_A(\lambda)$ is a bi-module for $(\mathfrak{g} \otimes A, \mathfrak{h} \otimes A)$, in fact for $(\mathfrak{g} \otimes A, \mathbf{A}_{\lambda})$.

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Weyl functor

Let mod \textbf{A}_{λ} be the category of left $\textbf{A}_{\lambda}\text{-modules}.$ Let

$$\textbf{W}^{\lambda}_{\textbf{\textit{A}}}: \textbf{mod} ~ \textbf{A}_{\lambda} \rightarrow \mathcal{I}^{\lambda}_{\mathcal{A}}$$

be given by

$$\mathbf{W}^{\lambda}_{\mathcal{A}}M = W_{\mathcal{A}}(\lambda) \otimes_{\mathbf{A}_{\lambda}} M, \qquad \mathbf{W}^{\lambda}_{\mathcal{A}}f = \mathbf{1} \otimes f,$$

where $M, M' \in \text{mod } \mathbf{A}_{\lambda}$ and $f \in \text{Hom}_{\mathbf{A}_{\lambda}}(M, M')$. We have

- $\mathbf{W}^{\lambda}_{A}M \in \operatorname{Ob} \mathcal{I}^{\lambda}_{A}$.
- \mathbf{W}_{A}^{λ} is right exact.
- $W^{\lambda}_{A} \mathbf{A}_{\lambda} \cong_{\mathfrak{g} \otimes A} W_{A}(\lambda).$

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Restriction functor

- For $\lambda \in P^+, V \in \mathsf{Ob}\,\mathcal{I}^\lambda_\mathcal{A}$, we have $V_\lambda \in \mathsf{mod}\,\mathsf{A}_\lambda$
- Define $\mathbf{R}^{\lambda}_{\mathcal{A}} : \mathcal{I}^{\lambda}_{\mathcal{A}} \to \mathsf{mod} \, \mathbf{A}_{\lambda} \text{ by } \mathbf{R}^{\lambda}_{\mathcal{A}} V = V_{\lambda}$
- \mathbf{R}^{λ}_{A} is an exact functor
- $\mathsf{id}_{\mathbf{A}_{\lambda}} \cong \mathbf{R}^{\lambda}_{A}\mathbf{W}^{\lambda}_{A}$
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We have a categorical definition of $\mathbf{W}_{A}^{\lambda}M$, which is maybe the most important improvement in this paper.

Theorem

Let $V \in \mathcal{I}_{\mathcal{A}}^{\lambda}$. Then $V \cong W_{\mathcal{A}}^{\lambda} R_{\mathcal{A}}^{\lambda} V$ iff for all $U \in \mathcal{I}_{\mathcal{A}}^{\lambda}$ with $U_{\lambda} = 0$, we have $\operatorname{Hom}_{\mathcal{I}_{\lambda}^{\lambda}}(V, U) = 0$, $\operatorname{Ext}_{\mathcal{I}_{\lambda}^{\lambda}}^{1}(V, U) = 0$.

We can deduce from this

Corollary

The functor \mathbf{W}^{λ}_{A} is exact iff for all $U \in \mathcal{I}^{\lambda}_{A}$ with $U_{\lambda} = 0$, we have

$$\operatorname{Ext}_{\mathcal{I}_{\lambda}^{\lambda}}^{2}(\mathbf{W}_{A}^{\lambda}M,U)=0, \ \forall \ M\in \operatorname{mod} \mathbf{A}_{\lambda}.$$

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Induced maps

For $f : A \to B$, denote also by f the morphism $f : \mathfrak{g} \otimes A \to \mathfrak{g} \otimes B$. For a *B*-module (resp. $\mathfrak{g} \otimes B$ -module) M, denote by f^*M the *A* (resp. $\mathfrak{g} \otimes A$)-module. For $\lambda \in P^+$ we have

$$f_{\lambda}: \mathbf{A}_{\lambda} \to \mathbf{B}_{\lambda}$$

and bi-module map

$$f^*_{\lambda}: W_{\mathcal{A}}(\lambda) \to f^*(W_{\mathcal{B}}(\lambda)).$$

For $M \in \text{mod } \mathbf{B}_{\lambda}$ we have

$$\mathbf{W}_{A}^{\lambda} f_{\lambda}^{*} M \to f^{*} \mathbf{W}_{B}^{\lambda} M$$
 as $\mathfrak{g} \otimes A$ – modules

Category of locally finite modules The Weyl functor Functorial in A

Induced maps

The comultiplication Δ of $\mathbf{U}(\mathfrak{h} \otimes A)$ induces

$$\Delta: \mathbf{A}_{\lambda+\mu} \to \mathbf{A}_{\lambda} \otimes \mathbf{A}_{\mu}.$$

The assignment $w_{\lambda+\mu} \mapsto w_{\lambda} \otimes w_{\mu}$ induces a bi-module map

$$au: W_A(\lambda + \mu) \to W_A(\lambda) \otimes W_A(\mu).$$

For $M \in \operatorname{mod} \mathbf{A}_{\lambda}, N \in \operatorname{mod} \mathbf{A}_{\mu}$ we have

 $\tau: \mathbf{W}^{\lambda+\mu}_{\mathcal{A}} \Delta^*(\mathcal{M} \otimes \mathcal{N}) \to \mathbf{W}^{\lambda}_{\mathcal{A}} \mathcal{M} \otimes \mathbf{W}^{\mu}_{\mathcal{A}} \mathcal{N} \text{ as } \mathfrak{g} \otimes \mathcal{A} - \text{modules}$

What is this algebra? Irreducible modules Finitely generated Weyl modules Tensor product phenomen

Recall

$\mathbf{A}_{\lambda} = \mathbf{U}(\mathfrak{h} \otimes \mathbf{A}) / \operatorname{Ann}_{\mathbf{U}(\mathfrak{h} \otimes \mathbf{A})}(\mathbf{w}_{\lambda})$

What is \mathbf{A}_{λ} ?

$$egin{aligned} \lambda &= \sum r_i \omega_i \;,\; r_\lambda = \sum r_i \;,\; S_\lambda := S_{r_1} imes \ldots imes S_{r_n} \subset S_{r_\lambda} \ & (A^{\otimes r_\lambda})^{S_\lambda} := \bigotimes_i (A^{\otimes r_i})^{S_{r_i}} \end{aligned}$$

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Theorem

For $\lambda \in P^+$, we have

$$\mathbf{A}_{\lambda}\cong (\mathbf{A}^{\otimes r_{\lambda}})^{\mathcal{S}_{\lambda}}$$

as algebras. If A is finitely generated, than \mathbf{A}_{λ} is finitely generated.

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What is this algebra? Irreducible modules Finitely generated Weyl modules Tensor product phenomen

From now on, we will suppose that A is finitely generated!

$$\max(\mathbf{A}_{\lambda}) = \max(\bigotimes_{i} (A^{\otimes r_{i}})^{S_{r_{i}}})$$

which is

$$\max(A)^{\times r_1}/S_{r_1}\times\ldots\times\max(A)^{\times r_n}/S_{r_n}$$

 $\max(\mathbf{A}_{\lambda})$ are orbits of the S_{λ} action on $\max(\mathbf{A}^{\otimes r_{\lambda}})$.

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Irreducibles

Lemma

Let λ ∈ P⁺ and assume that V ∈ I^λ_A is irreducible. There exists μ ∈ P⁺ ∩ (λ − Q⁺) such that

wt
$$V \subset \mu - Q^+$$
, dim $V_{\mu} = 1$.

- *V* is the unique irreducible quotient of $\mathbf{W}^{\mu}_{A}\mathbf{R}^{\mu}_{A}V_{\mu}$.
- If V' ∈ Ob I_A, then V ≅ V' as g ⊗ A–modules iff V_µ ≅ V'_µ as A_µ-modules.
- For M ∈ irr A_λ, we denote the unique irreducible quotient of W^λ_AM by V^λ_AM.

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Finitely supported functions

- Define $\Xi := \{\xi : \max(A) \to P^+ \mid \xi \text{ finitely supported} \}$
- supp $\xi := \{ S \in \max(A) \mid \xi(S) \neq 0 \}$
- wt $\xi := \sum_{S \in \max(A)} \xi(S)$

•
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What is this algebra? Irreducible modules Finitely generated Weyl modules Tensor product phenomen

Finitely supported functions

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A is still finitely generated.

Theorem

- For $\lambda \in P^+$, $W_A(\lambda)$ is a finitely generated right \mathbf{A}_{λ} -module.
- If M ∈ mod A_λ is finitely generated then W^λ_AM is a finitely generated left g ⊗ A–module. Same for finite–dimensional modules.
- In particular for $M \in \operatorname{irr} \mathbf{A}_{\lambda}$, we have $\dim \mathbf{V}_{A}^{\lambda}M < \infty$.

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A generalization of the tensor product phenomenon

Theorem

Suppose that A and B are finite–dimensional commutative, associative algebras and let $\lambda, \mu \in P^+$. For $M \in \text{mod } \mathbf{A}_{\lambda}$, $N \in \text{mod } \mathbf{B}_{\mu}$, finite–dimensional, we have,

$$\mathbf{W}_{A\oplus B}^{\lambda+\mu}(M\otimes N)\cong \mathbf{W}_{A}^{\lambda}M\otimes \mathbf{W}_{B}^{\mu}N,$$

as $\mathfrak{g} \otimes (A \oplus B)$ -modules.

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What is this algebra? Irreducible modules Finitely generated Weyl modules Tensor product phenomen

Theorem

Let $\lambda, \mu \in P^+$, and $M \in \text{mod } \mathbf{A}_{\lambda}, N \in \text{mod } \mathbf{A}_{\mu}$, finite–dimensional with supp $M \cap \text{supp } N = \emptyset$. Then we have

• $\mathbf{W}_{A}^{\lambda+\mu}(M\otimes N)\cong_{\mathfrak{g}\otimes A}\mathbf{W}_{A}^{\lambda}M\otimes \mathbf{W}_{A}^{\mu}N.$

• If M, N are irreducible, then $\mathbf{V}_{\mathcal{A}}^{\lambda+\mu}(M \otimes N) \cong_{\mathfrak{g} \otimes \mathcal{A}} \mathbf{V}_{\mathcal{A}}^{\lambda}M \otimes \mathbf{V}_{\mathcal{A}}^{\mu}N.$

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What is this algebra? Irreducible modules Finitely generated Weyl modules Tensor product phenomen

Theorem

Let λ, μ ∈ P⁺, and M ∈ mod A_λ, N ∈ mod A_μ,
finite-dimensional with supp M ∩ supp N = Ø. Then we have
W^{λ+μ}_A(M ⊗ N) ≅_{g⊗A} W^λ_AM ⊗ W^μ_AN.
If M, N are irreducible, then

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We want to analyze $\mathbf{W}^{\lambda}_{A}M_{\xi}$, $M_{\xi} \in \operatorname{irr} \operatorname{mod} \mathbf{A}_{\lambda}$. The tensor product theorem gives

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We define

$$\Xi_{\lambda}^{\mathrm{ns}} = \{ \xi \in \Xi_{\lambda} : \xi(S) \in \{0, \omega_1, \dots, \omega_n\}, \ \forall S \in \max A \}.$$

Then $\Xi^{\rm ns}_\lambda$ is an open subset and

 $\Xi_{\lambda}^{ns} \leftrightarrow \{ \text{orbits of non-singular points of the } S_{r_{\lambda}} - \text{action on } \max(A^{\otimes r_{\lambda}}) \}$

Want to analyze the Weyl modules of Ξ_{λ}^{ns} . It is enough to analyze $\mathbf{W}_{A}^{\omega_{i}}M_{S}, i \in I$.

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Open subsets Fundamental Weyl modules

Notations

Let $J_0 \subset I$ be defined as follows:

$$J_0 = \begin{cases} I, \ \mathfrak{g} \text{ of type } A_n, \ C_n, \\ \{n\}, \ \mathfrak{g} \text{ of type } B_n, \\ \{n-1, n\}, \ \mathfrak{g} \text{ of type } D_n. \end{cases}$$

Given $m \in \mathbb{Z}_+$, let $\mathbf{c}(m, k)$ be the dimension of the space of polynomials of degree *m* in *k*-variables, i.e

$$\mathbf{c}(m,k)=\#\{\mathbf{s}=(s_1,\cdots,s_k):\in\mathbb{Z}_+^k:s_1+\cdots+s_k=m\}.$$

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Open subsets Fundamental Weyl modules

Theorem

Let \mathfrak{g} be of classical type. Let $S \in \max A$ be such that $\dim S/S^2 = k$ and for $i \in I$, let $M_S \in \operatorname{irr} \operatorname{mod} \mathbf{A}_{\omega_i}$.

- If $i \in J_0$, then $\mathbf{W}_A^{\omega_i} M_S \cong_{\mathfrak{g}} V(\omega_i)$.
- If $i \notin J_0$, then

$$\mathbf{W}_{A}^{\omega_{i}}M_{S}\cong_{\mathfrak{g}}\bigoplus_{j}V(\omega_{i-2j})^{\oplus\mathbf{c}(j,k)}$$

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