Pricing Weather Derivative: an Equilibrium Approach*

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Abstract

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1. Introduction

It is estimated that about $ 1 trillion of the $ 7 trillion U.S. economy is weather-sensitive (Challis 1999 and Hanley 1999). Weather conditions directly affect the outputs of agricultural products and the demand for energy products, and indirectly affect retail businesses. For example, the inventory of winter coats at a department store depends on weather forecast for the coming winter and eventual sales depend on the actual weather condition (Agins and Kranhold, 1999). Likewise, earnings in the power industry depend on the retail prices and the sales quantities of electricity. Although weather conditions may not play a key role in determining the electricity retail price, it is certainly one of the most important factors affecting the electricity demand. Until 1997, earnings stabilization for utility firms was primarily achieved through price hedging mechanisms while volumetric risks were largely left unhedged. The increasing competition caused by ongoing deregulation has created incentives for utility firms to stabilize revenues. It has become necessary for companies to hedge against the volumetric risks caused by unexpected weather conditions. Such needs have created a new family of over-the-counter weather derivatives. Meantime, the Chicago Mercantile Exchange has introduced futures contracts on the temperature of several U.S. cities.

The underlying variables for weather contracts include temperatures, rainfall, snowfall and precipitations.[1] However, the most commonly used variables are Heating-Degree-Days (HDD) and Cooling-Degree-Days (CDD) defined on daily average temperatures. The daily average temperature, in turn, is the arithmetic average of the maximum and minimum temperature recorded on a

[1] For a complete survery, see Hanley (1999).

midnight-to-midnight basis. The precise expressions of HDD and CDD are defined below:

Daily HDD = max(65° degree Fahrenheit - daily average temperature, 0);

Daily CDD = max(daily average temperature - 65° degree Fahrenheit, 0).

To simplify the language in the text, we refer to the daily average temperature as daily temperature. Intuitively, HDD measures the coldness of the day compared to a benchmark of 65 Fahrenheit for the winter season while CDD measures the extent to which a summer day is hot. An HDD season includes winter months from November to March and the CDD season (or summer season) from May to September. April and October are commonly referred to as the shoulder months.

Because of the high correlation between the electricity consumption and HDD/CDD, most contracts are written on the accumulation of HDD or CDD over a certain period (e.g., a calendar month or a season) so that one contract can be used to hedge a particular period. The popular transactions in the OTC market include HDD / CDD swaps and options for large cities like Atlanta, Chicago, Dallas, New York and Philadelphia. [2] The swap contracts or forward contracts are similar to the exchange-traded futures contracts. There are four basic elements in these contracts: (i) the underlying variable: HDD or CDD; (ii) the accumulation period: a season or a calendar month; (iii) a specific weather station reporting daily temperatures for a particular city; and (iv) the tick size: the dollar amount attached to each HDD or CDD. Table 1 presents the typical transactions of an HDD swap (or forward) on New York and a CDD option on Chicago. In the New York HDD swap, the tick size is set at \$5,000 per HDD. XYZ Co. agrees to pay ABC Co. a fixed rate of 1,000 HDD and in return for a floating rate which is the actual accumulated HDD during January, 1999. The realized HDD for January, 1999 is 956. Then the payoff for XYZ Co. at maturity is $\$5000 \times (956 - 1000) = -\$220,000$. Similarly, the Chicago CDD option has a tick size of \$5,000

[2] See Smithson and Choe (1999) for a brief survey of the market.

and a strike level of 190 CDD. The actual accumulated CDD in June, 1999 is 196, which is higher than the strike level. Thus, xyz Co. would exercise the call option at maturity and receive a payoff of $\$5,000 \times (196 - 190) = \$30,000$.

Table 1: Examples of HDD- and CDD-based Swap and Option

	HDD Swap (or Forward)	CDD Call Option
Location	La Guardia Airport, New York	O'Hare Airport, Chicago
Buyer	YYY Co. (paying fixed rate)	xyz Co. (paying call premium)
Seller	AAA Co. (paying floating rate)	abc Co.
Accumulation Period	January 1 - 31, 1999	June 1 - 30, 1999
Tick Size	$5,000 per HDD	$5,000 per CDD
Fixed Rate	1,000 HDD	
Strike Level		190 CDD
Floating Rate	the actual HDD for January, 1999 = 956 HDD	
Settlement Price		the actual CDD for June, 1999 = 196 CDD
Payoffs at Maturity for the Buyer	$(956 - 1000) \times 5000 = -\$220,000$	$(196 - 190) \times 5000 = \$30,000$

Despite the rapid growth of weather derivatives in the OTC market, the bid / ask spread is still very large and there is no effective pricing method accepted by all industry participants. In this paper we propose an equilibrium model to price weather derivatives. Our objectives are three-fold. First, we will present a realistic model for the dynamics of the daily average temperature, which is very different from that of a security price. For example, temperatures are seasonal and cyclical, can be predicted accurately for the very near future, and will vary within a well-defined range in the long run. The traditional stochastic diffusion process used for modelling security prices will not be suitable for temperature dynamics. Second, we would like to establish, in a rudimental way, whether the market price of risk associated with the temperature variable significantly affects the valuation of weather derivatives. Third, we will develop a pricing framework for derivatives based on the accumulated HDD or CDD. Challenges arise because the underlying is the accumulation of the daily HDD/CDD which are non-linear in daily temperatures.

The key contributions of this paper lie in the accomplishments of the aforementioned objectives. First, we propose an auto-regressive, mean-reverting dynamic system for the daily temperature. This system is capable of capturing the seasonality and the global warming trend and can incorporate weather forecast information. Second, the market price of risk associated with the temperature variable is analyzed in an equilibrium framework. Specifically, we extend Lucas' (1978) equilibrium asset-pricing model where fundamental uncertainties in the economy are generated by aggregate dividend and a state variable representing the daily temperature. It is shown that the equilibrium prices of weather derivatives depend on the agent's risk preference and the correlation between the temperature variable and the aggregate dividend. Lastly, we use Maximum Likelihood method to estimate the temperature dynamics for Atlanta, Chicago, Dallas, New York and Philadelphia. Based on the estimated parameters, we simulate forward and option prices for seasonal and monthly HDD / CDD contracts and examine the importance of the market price of risk thereof.

The paper is organized as follows. Section 2 presents the underlying equilibrium model, discusses the dynamics of the fundamental variables: the aggregate dividend and daily temperatures. Section 3 discusses the equilibrium valuation of weather derivatives. Section 4 presents temperature estimation results for five cities. Section 5 analyzes the simulated weather derivative prices. Section 6 concludes the paper. Proofs and exhibitions are collected in appendices.

2. The Structure of the Economy

In a discrete-setting, consider an extension of the Lucas (1978) pure exchange economy with two state variables: aggregate dividends (δ) and weather conditions (Y) which generate the fundamental uncertainties in the economy. Aggregate dividends can be viewed as aggregate outputs or dividend on the market portfolio. The weather variable can refer to temperatures, rainfall, snowfall or precipitations. The dynamics governing the aggregate dividend and the temperature variable are assumed to be exogenous processes on a given probability space (- , $\mathcal{F}$, $\mathcal{P}$). There is a representative investor whose information structure is given by the filtration $\mathcal{F}_t \equiv \sigma(\delta_\tau, Y_\tau; \tau \in (0, 1, 2, ..., t))$. The agent has an infinite lifetime horizon.

In the financial market, the representative agent can trade a single risky stock, pure discount bonds and a finite number of other contingent claims at any time. The risky stock can be viewed as the market portfolio. Therefore, its dividend stream $\{\delta_t\}$ is understood as aggregate dividends in the economy. The total supply is normalized to one share and the contingent claims are written on the risky stock, the pure discount bond and the weather variable. The net supply of all contingent claims and the riskless bond is zero.

In the following subsections, we specify the fundamentals in the economy: the agent's preference, the dynamics of the aggregate dividend and the weather variable.

2.1. Agent's Preference and Aggregate Dividend Behavior

The agent's preference is described by a smooth time-additive expected utility function

$$V(c) = E_0 \left(\sum_{t=0}^{\infty} U(c_t, t) \right),$$

where $U : \mathcal{R}_+ \times (0,\infty) \rightarrow \mathcal{R}$ is smooth on $(0,\infty) \times (0,\infty)$ and, for each $t \in (0,1,2,...,\infty)$, $U(\cdot,t) : \mathcal{R}_+ \rightarrow \mathcal{R}$ is increasing, strictly concave, and has a continuous derivative $U_c(\cdot,t)$ on $(0,\infty)$. For analytical tractability, the agent's period t utility is assumed to be the Constant-Relative-Risk-Aversion (CRRA) type of preference:

Assumption 1. *The representative agent's period utility is described by*

$$U(c_t, t) = e^{-\rho t} \frac{c_t^{\gamma+1}}{\gamma+1},$$

with the rate of time preference, $\rho > 0$ and the risk parameter $\gamma \in [-1, 0]$.

For the aggregate dividend process, we appeal to Marsh and Merton (1987), whose estimation results suggest mean-reversion in dividend rate changes. This feature is incorporated in the following assumption:[3]

Assumption 2. *The aggregate dividend, δ, evolves according to the following Markov process:*

$$\ln \delta_t = \alpha + \mu \ln \delta_{t-1} + \sigma_\delta \, \epsilon_t, \quad \forall \quad \mu \leqslant 1$$

where ϵ_t is an independently and identically distributed standard normal random variable.

2.2. Dynamics of the Temperature Variable

We will focus on temperature-related weather derivatives since they are the most traded products. The equilibrium model proposed here can be easily adapted to weather derivatives on other variables such as rainfall, snowfall or precipitations.

[3]Lintner (1956) and Fama and Babiak (1968) study the dividend behavior for individual stocks. They use the accounting earnings variable instead of the changes in stock prices.

2.2.1. Summary of Temperature Behavior

To ensure accurate modeling of the temperature variable, we first study its behavior. We examine 20-year historical daily temperature data for Atlanta, Chicago, Dallas, New York and Philadelphia obtained from the National Climate Data Center (NCDC), a subsidiary of the National Oceanic Atmospheric Administration (NOAA). The sample period is from 1979 to 1998. Exhibits 1 and 2 summarize sample statistics and Exhibit 3 depicts the warming trend in Atlanta, which is typical of all cities studied.[4] The following remarks are in order.

Remark 1. *The sample means of the two Southern cities (Atlanta and Dallas) are higher than those of the three Northern counterparts. The highest and lowest sample means are 66 and 50 of Dallas and Chicago, respectively.*

Remark 2. *Northern cities generally have larger standard deviations. Chicago has the highest sample standard deviation (20 degrees), indicating large temperature swings. Atlanta has the lowest sample standard deviation (15 degrees).*

Remark 3. *Correlations among the five cities are very high and are above 0.84. New York and Philadelphia, the two nearest cities, present the highest correlation, 0.9853.*

Remark 4. *Daily temperatures exhibit strong auto-correlations.*

Remark 5. *Standard deviations of monthly CDD's for the two Southern cities are higher than those of their Northern counterparts. The reserve is true for HDD standard deviations (Exhibit 2).*

[4] To simplify the analysis, we have omitted the observations for February 29 from the sample. Therefore, each year consists of 365 days and the sample size for 20 years is 7300.

To facilitate further discussions, let us index the years in the sample period by yr, thus $yr = 1$ for 1979, $yr = 2$ for 1980, ..., $yr = 20$ for 1998. Also, index January 1 as $t = 1$, January 2 as $t = 2$, and so on for 365 days in a year. Denote $Y_{yr,t}$ as the temperature on date t in year yr. Below, we define the mean ($\overline{Y}_t$) and the standard deviation (ψ_t) for date t as

$$\overline{Y}_t = \frac{1}{20}\sum_{yr=1}^{20} Y_{yr,t} \qquad \& \qquad \psi_t = \sqrt{\frac{1}{20}\sum_{yr=1}^{20}\left(Y_{yr,t} - \overline{Y}_t\right)^2}; \qquad \forall \quad t = 1, 2, ..., 365.$$

We plot in Exhibit 4 the daily standard deviations for Atlanta and Chicago, which show a clear seasonal pattern: the temperature variation in the HDD season is larger than that in the CDD season. This is common for all cities in consideration.

2.2.2. Modelling Daily Temperature Behavior

In light of the properties identified in the previous section, a model for the daily temperature must possess the following features. First, it must capture the seasonal cyclical patterns; second, the daily variations in temperature must be around some average "normal" temperature, to be elaborated on later; third, it should allow forecasts to play a key role in projecting temperature paths in the future; fourth, it should incorporate the autoregressive property in temperature changes (i.e., a warmer day is most likely to be followed by another warmer day, and vice versa); fifth, the extent of variation must be bigger in the winter and smaller in the summer; sixth, a projected temperature path into the future should never wander outside of the normal range of the temperature for each projected point in time; and seventh, the model must reflect the global warming trend.

A diffusion process would fail most of the above requirements, especially the first and the sixth. Even with the most plausible candidate, the mean-reverting process, it is always possible (and this has been verified by our empirical estimation and subsequent simulations) that a particular path of temperatures does not resemble a temperature evolution at all. For this reason, we resort to a discrete, autoregressive model. To this end, define the de-meaned and de-trended residual of the daily temperature as $U_{yr,t}$,

$$U_{yr,t} = Y_{yr,t} - \widehat{\overline{Y}}_{yr,t}, \quad \forall \quad yr = 1, 2, ..., 20 \; \& \; t = 1, 2, ..., 365. \tag{2.1}$$

Assumption 3. *The daily temperature residual, $U_{yr,t}$, is described by a k-lag autocorrelation system:* [5]

$$\begin{aligned} &U_{yr,t} = \textstyle\sum_{i=1}^{k} \rho_i U_{yr,t-i} + \sigma_{yr,t} * \xi_{yr,t} \\ &\sigma_{yr,t} = \sigma - \sigma_1 \mid \sin(\pi t/365 + \phi) \mid, \\ &\xi_{yr,t} \sim i.i.d. \; N(0,1), \\ &\forall \quad yr = 1, 2, ..., 20; \quad \& \quad t = 1, 2, ..., 365. \end{aligned} \tag{2.2}$$

The correlation between ξ and ϵ (the white noise in aggregate dividend) is assumed to be:

$$corr(\xi_{yr,t} \; , \; \epsilon_{yr',t'}) = \begin{cases} \varphi & if \; yr = yr' \; and \; t = t'; \\ 0 & otherwise. \end{cases}$$

In the above, $\widehat{\overline{Y}}_t$ serves the purpose of de-mean and de-trend. It is tempting to use the historical daily average, $\overline{Y}_t$ as an input for $\widehat{\overline{Y}}_t$. But this will be a poor choice, because for a very cold or very warm year, all the realized temperatures could be below or above the historical averages. To illustrate, let us consider a colder-than-normal winter in New York from November 1980 to March 1981. Panel A of Exhibit 5 compares date t's average temperature with the realized temperature. (The plots are extended at both ends to have a complete view.) Although $\overline{Y}_t$ was more or less in the center of realized temperatures before the winter months, it was much higher for those extremely cold days. To correct this problem, we make certain adjustments in the following steps. 1) For

[5] We confess a slight abuse of notation here. Notice that at the beginning of year yr, we must use the data from the end of the previous year, $(yr - 1)$ to calculate the auto-regressive terms. It is understood that the index yr will automatically take appropriate values when required.

each month of the year, we average the daily means, $\overline{Y}_t$; 2) for the same year, we calculate the realized, average temperature of each month; 3) for each month, we find the difference between the averages from Step 2 and Step 1; and 4) we adjust the simple mean for each day of the month by the quantity calculated in Step 3, and this adjusted mean is referred to as $\widehat{\overline{Y}}_t$. The period of one month is chosen as a trade-off. Too long a period will not solve the non-centering problem and too short a period will unnecessarily exaggerate the short term fluctuations and diminish the meaning of "average" or "mean". Needless to say, one could be more sophisticated in making the adjustments. For instance, rather than following the calendar months, one could always center the day in question in a 30 day (e.g.) period and make the above adjustments on a rolling basis. However, as shown in Panel B of Exhibit 5, our scheme already works very well.

The volatility specification in (2.2) using the sine wave reflects the fifth requirement and the feature in Exhibit 4. The autocorrelation setup reflects the fourth feature. The other features (including the global warming trend) are captured by the specification of $\widehat{\overline{Y}}_t$. The correlation specification indicates that the current temperature is correlated with the current aggregate dividend.[6]

2.3. Agent's Optimization Problem

Initially, the agent is endowed with one share of the risky stock. Denote his portfolio holdings at time t as $\theta_t = (\theta_t^s, \theta_t^B, \theta_t^{x\prime})$, where θ_t^s, θ_t^B and $\theta_t^{x\prime}$ represent the number of shares invested in the risky stock, the discount bond and other contingent claims, respectively. Also denote the security prices at time t by a vector X_t and the corresponding vector of dividends by D_t. The agent's consumption over time is financed by a trading strategy $\{\theta_t, t \geq 0\}$. His decision problem is to

[6] A more general specification would allow the current aggregate dividend to be affected by the lagged temperatures. This type of setup will make the estimation and valuation much more complicated, and we will leave it to future research.

choose an optimal trading strategy so as to maximize his expected lifetime utility. The first order conditions yield the standard Euler equation:

$$X_t = E_t \left(\sum_{\tau=t+1}^{\infty} \frac{U_c(c_\tau, \tau)}{U_c(c_t, t)} D_\tau \right).$$

Thus, the price of any security equals the expected discounted sum of its dividends, with the marginal rate of substitution being the stochastic state price deflator. [7]

In equilibrium, both the financial market and the goods market clear so that aggregate consumption equals the dividends generated from the risky stock. Therefore, for a riskless bond paying 1 unit of consumption goods at T and 0 at all other times, its equilibrium price at time t, denoted by $B(t,T)$, is

$$B(t,T) = \frac{1}{U_c(\delta_t, t)} E_t \left(U_c(\delta_T, T) \right), \quad \forall \quad t \in (0,T). \tag{2.3}$$

For any contingent claim with a payoff q_T at maturity T, its price at time t, denoted by $F_t(t,T)$, is

$$F(t,T) = \frac{1}{U_c(\delta_t, t)} E_t \left(U_c(\delta_T, T) q_T \right), \quad \forall \quad t \in (0,T). \tag{2.4}$$

For example, if K denotes the delivery level of a forward contract written on the temperature variable, Y, and the tick size is assumed to be \$1 per degree days, then $q_T = Y_T - K$ is the payoff of the contract at maturity.

3. Pricing HDD / CDD Derivatives

3.1. Term Structure of the Interest Rate under the CRRA Preference

Proposition 3.1. *Under the CRRA utility and Assumption 2, the pure discount price at time t is*

[7] All expectations in this paper are taken with respect to the filtration specified in Assumptions 2 & 3.

$$B(t,T) = \delta_t^{\gamma(\mu^{T-t}-1)} \exp\left(-\rho(T-t) + \sum_{i=t+1}^{T} [\alpha\gamma\mu^{T-i} + \frac{1}{2}\gamma^2\sigma_\delta^2\mu^{2(T-i)}]\right),$$

and the yield-to-maturity defined through $e^{-R(t,T)(T-t)} = B(t,T)$ *is*

$$R(t,T) = \rho - \frac{\sum_{i=t+1}^{T}[\alpha\gamma\mu^{T-i} + \frac{1}{2}\gamma^2\sigma_\delta^2\mu^{2(T-i)}]}{T-t} - \frac{\gamma(\mu^{T-t}-1)\ln\delta_t}{T-t}.$$

Proof: see Appendix A.

A special case is $\mu = 1$ where changes in the dividend growth rate follow a random walk. In this case, the pure discount bond price reduces to

$$B(t,T) = \exp\left[\left(-\rho + \alpha\gamma + \frac{1}{2}\gamma^2\sigma_\delta^2\right)(T-t)\right],$$

which yields a flat term structure $R(t,T) = \rho - \alpha\gamma - \frac{1}{2}\gamma^2\sigma_\delta^2$. In order to ensure a stable system, the yield-to-maturity has to be positive, i.e., $\rho - \alpha\gamma - \frac{1}{2}\gamma^2\sigma_\delta^2 > 0$.

3.2. HDD / CDD Forward Prices and Market Price of Risk

Consider an HDD forward contract with a tick size of \$1 and delivery price, K. The accumulation period starts at T_1 and ends at maturity $T_2 > T_1$. Denote $HDD(T_1, T_2) = \sum_{\tau=T_1}^{T_2} \max(65 - Y_\tau, 0)$. The value of the HDD forward contract at time t, $f_{HDD}(t, T_1, T_2, K)$, can be determined by equation (2.4):

$$\begin{aligned} f_{HDD}(t, T_1, T_2, K) &= E_t\left(\tfrac{U_c(\delta_{T_2}, T_2)}{U_c(\delta_t, t)}[HDD(T_1, T_2) - K]\right) \\ &= e^{-\rho(T_2 - t)} E_t\left(\frac{\delta_{T_2}^\gamma}{\delta_t^\gamma}[HDD(T_1, T_2) - K]\right). \end{aligned} \tag{3.1}$$

When a forward contract is initiated, the delivery price, K is chosen so that the value of the contract is zero. Therefore, the forward price at time t, $F_{HDD}(t, T_1, T_2)$, is the value of K which makes $f = 0$ in (3.1). That is,

$$F_{HDD}(t, T_1, T_2) = \frac{E_t\left(\delta_{T_2}^{\gamma} HDD(T_1, T_2)\right)}{E_t(\delta_{T_2}^{\gamma})} = \frac{e^{-\rho(T_2-t)}\delta_t^{-\gamma} E_t\left(\delta_{T_2}^{\gamma} HDD(T_1, T_2)\right)}{B(t, T_2)}.$$

Similar expressions can be written for CDD contracts:

$$f_{CDD}(t, T_1, T_2, K) = e^{-\rho(T_2-t)} E_t\left(\frac{\delta_{T_2}^{\gamma}}{\delta_t^{\gamma}}[CDD(T_1, T_2) - K]\right),$$

$$F_{CDD}(t, T_1, T_2) = \frac{e^{-\rho(T_2-t)}\delta_t^{-\gamma} E_t\left(\delta_{T_2}^{\gamma} CDD(T_1, T_2)\right)}{B(t, T_2)}.$$

It is easier to analyze the market price of risk or the risk premium with closed-form formulas. To this end, we set the autocorrelation parameters to zero and obtain the following proposition.

Proposition 3.2. *When $\rho_i = 0$ ($i = 1, 2, .., k$), the equilibrium forward prices at time $t < T_1$ before the accumulation period are*

$$\begin{aligned} F_{HDD}(t, T_1, T_2) &= \textstyle\sum_{\tau=T_1}^{T_2} \left([65 - \mu'_Y(\tau)] \cdot N(\frac{65-\mu'_Y(\tau)}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}} \exp\left[-\frac{(65-\mu'_Y(\tau))^2}{2\sigma_{yr,\tau}^2}\right]\right); \\ F_{CDD}(t, T_1, T_2) &= \textstyle\sum_{\tau=T_1}^{T_2} \left([\mu'_Y(\tau) - 65] \cdot N(\frac{\mu'_Y(\tau)-65}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}} \exp\left[-\frac{(65-\mu'_Y(\tau))^2}{2\sigma_{yr,\tau}^2}\right]\right); \\ \textit{with} \quad \mu'_Y(\tau) &= \widehat{\overline{Y}}_\tau + \gamma\varphi\mu^{\tau-t}\sigma_\delta\sigma_{yr,\tau} \ . \end{aligned}$$

The equilibrium forward prices at time $t \in (T_1, T_2)$ during the accumulation period are

$$\begin{aligned} F_{HDD}(t, T_1, T_2) &= HDD(T_1, t) + \textstyle\sum_{\tau=t+1}^{T_2} \left([65 - \mu'_Y(\tau)] \cdot N(\frac{65-\mu'_Y(\tau)}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}} \exp\left[-\frac{(65-\mu'_Y(\tau))^2}{2\sigma_{yr,\tau}^2}\right]\right); \\ F_{CDD}(t, T_1, T_2) &= CDD(T_1, t) + \textstyle\sum_{\tau=t+1}^{T_2} \left([\mu'_Y(\tau) - 65] \cdot N(\frac{\mu'_Y(\tau)-65}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}} \exp\left[-\frac{(65-\mu'_Y(\tau))^2}{2\sigma_{yr,\tau}^2}\right]\right). \end{aligned}$$

The equilibrium values of the forward contracts at time t are

$$\begin{aligned} f_{HDD}(t, T_1, T_2, K) &= B(t, T_2)(F_{HDD}(t, T_1, T_2) - K) \qquad \textit{and} \\ f_{CDD}(t, T_1, T_2, K) &= B(t, T_2)(F_{CDD}(t, T_1, T_2) - K). \end{aligned}$$

Proof: (see Appendix B).

Remark 6. *The equilibrium value of an HDD/CDD forward contract in Proposition 3.2 is the present value of the difference between the forward price and the delivery price discounted at the riskfree rate, which is consistent with results for stock or currency forward contracts when interest rates are non-stochastic.*

Remark 7. *The equilibrium forward prices converge to the spot prices at maturity T_2:*

$$F_{HDD}(T_2, T_1, T_2) = HDD(T_1, T_2) \qquad and \qquad F_{CDD}(T_2, T_1, T_2) = CDD(T_1, T_2).$$

A special case of Proposition 3.2 is when $\varphi = 0$, i.e., when the temperature variable Y is not correlated with the aggregate dividend. In this case, the forward prices at time $t < T_1$ collapse to

$$F^*_{HDD}(t, T_1, T_2) = \sum_{\tau=T_1}^{T_2} \left([65 - \mu_Y(\tau)] \cdot N(\tfrac{65-\mu_Y(\tau)}{\sigma_{yr,\tau}}) + \tfrac{\sigma_{yr,\tau}}{\sqrt{2\pi}} \exp\left[-\tfrac{(65-\mu_Y(\tau))^2}{2\sigma^2_{yr,\tau}} \right] \right)$$

$$F^*_{CDD}(t, T_1, T_2) = \sum_{\tau=T_1}^{T_2} \left([\mu_Y(\tau) - 65] \cdot N(\tfrac{\mu_Y(\tau)-65}{\sigma_{yr,\tau}}) + \tfrac{\sigma_{yr,\tau}}{\sqrt{2\pi}} \exp\left[-\tfrac{(65-\mu_Y(\tau))^2}{2\sigma^2_{yr,\tau}} \right] \right)$$

$$with \quad \mu_Y(\tau) = \widehat{\widehat{Y}}_\tau .$$

It can be verified that the forward prices, F^*_{HDD} and F^*_{CDD}, are simply the expected values of HDD and CDD, respectively, conditional on today's information on temperature. That is,

$$F^*_{HDD}(t, T_1, T_2) = E_t(HDD(T_1, T_2)) \quad and \quad F^*_{CDD}(t, T_1 T_2) = E_t(CDD(T_1, T_2)).$$

No risk premium is required for the temperature variable in this case.

Given the parameter restrictions $-1 < \gamma < 0$, $0 < \mu \leqslant 1$, $\sigma_\delta > 0$ and $\sigma_{yr,t} > 0$, parameter φ plays a key role in determining the relationship between the forward prices and the expected HDD/CDD values. To gain more insights, we compute the following comparative statics:

$$\frac{\partial F_{HDD}}{\partial \varphi} = -\sum_{\tau=T_1}^{T_2} \left(N(\frac{65 - \mu'_Y(\tau)}{\sigma_{yr,\tau}}) \gamma \mu^{\tau-t} \sigma_\delta \sigma_{yr,\tau} \right) > 0;$$

$$\frac{\partial F_{CDD}}{\partial \varphi} = \sum_{\tau=T_1}^{T_2} \left(N(\frac{\mu'_Y(\tau) - 65}{\sigma_{yr,\tau}})\gamma\mu^{\tau-t}\sigma_\delta\sigma_{yr,\tau} \right) < 0.$$

Since $\varphi = 0$ corresponds to the case where the forward prices are equal to the expect HDD/CDD, we obtain the following relations:

Table 2: Comparative Statics

$\varphi < 0$	$\varphi > 0$
$F_{HDD} < E_t\,(HDD(T_1, T_2))$	$F_{HDD} > E_t\,(HDD(T_1, T_2))$
$F_{CDD} > E_t\,(CDD(T_1, T_2))$	$F_{CDD} < E_t\,(CDD(T_1, T_2))$

Note: $-1 < \gamma < 0$, $0 < \mu \leqslant 1$, $\sigma_\delta > 0$ and $\sigma_{yr,t} > 0$.

When the temperature is negatively correlated with the aggregate dividend, i.e., $\varphi < 0$, HDD forward prices are lower than the expected HDD, but CDD forward prices are higher than the expected CDD. The reverse is true when $\varphi > 0$. Simulation results in subsequent sections show that the results in Table 2 still obtain when the auto-correlations are not zero.

3.3. Valuation of HDD and CDD Options

HDD and CDD options can be priced in a similar way as the corresponding forward contracts. Consider a European option written on $HDD(T_1, T_2)$ with maturity T_2 and a strike price X. Denote the call and put prices at time t as $C_{HDD}(t, T_1, T_2, X)$ and $P_{HDD}(t, T_1, T_2, X)$, respectively. According to (2.4), the call and put values can be determined as

$$C_{HDD}(t, T_1, T_2, X) = e^{-\rho(T_2-t)}\delta_t^{-\gamma} E_t \left(\delta_{T_2}^{\gamma} \max(HDD(T_1, T_2) - X, 0) \right),$$

$$P_{HDD}(t, T_1, T_2, X) = e^{-\rho(T_2-t)}\delta_t^{-\gamma} E_t \left(\delta_{T_2}^{\gamma} \max(X - HDD(T_1, T_2), 0) \right).$$

Similarly, call and put options written on $CDD(T_1, T_2)$ can be priced as

$$
\begin{aligned}
C_{CDD}(t, T_1, T_2, X) &= e^{-\rho(T_2-t)}\delta_t^{-\gamma} E_t\left(\delta_{T_2}^{\gamma} \max(CDD(T_1, T_2) - X, 0)\right), \\
P_{CDD}(t, T_1, T_2, X) &= e^{-\rho(T_2-t)}\delta_t^{-\gamma} E_t\left(\delta_{T_2}^{\gamma} \max(X - CDD(T_1, T_2), 0)\right).
\end{aligned}
$$

It is important to note that, when $\varphi \neq 0$, the above option pricing equations also incorporate market price of risks for the temperature variable. When $\varphi = 0$, similar to forward prices, the option prices can be determined by discounting the future payoffs at the riskfree rate, i.e.,

$$
\begin{aligned}
C_{HDD}(t, T_1, T_2, X) &= e^{-\rho(T_2-t)}\delta_t^{-\gamma} E_t\left(\delta_{T_2}^{\gamma} \max(HDD(T_1, T_2) - X, 0)\right), \\
&= e^{-\rho(T_2-t)}\delta_t^{-\gamma} E_t\left(\delta_{T_2}^{\gamma}\right) E_t\left(\max(HDD(T_1, T_2) - X, 0)\right) \qquad (\text{when } \varphi = 0) \\
&= B(t, T_2) E_t\left(\max(X - HDD(T_1, T_2), 0)\right) \qquad (\text{by Appendix A.1}) \\
&= e^{-R(t,T_2)(T_2-t)} E_t\left(\max(X - HDD(T_1, T_2), 0)\right). \qquad (\text{by definition})
\end{aligned}
$$

Since the above pricing equations do not admit closed-form formulas, we will perform the analysis of market price of risk in Section 5 through simulations.

4. Estimation of Temperature Dynamics

Our setup calls for the estimation of the joint distribution of the aggregate dividend and the daily temperature. The aggregate dividend can be approximated by several variables. To start with, one could use the aggregate output, although the lower frequency of such data will make the joint estimation difficult since the temperature data are daily. One may also use the dividend yield of a stock market index. Since our primary task in this paper is to lay out a pricing framework, we will not perform a joint estimation. Instead, we only estimate the parameters for the temperature behavior. For simulations, parameters for the utility function (ρ, γ) and the aggregate dividend (α, μ, σ_δ) will be calibrated via the results in Proposition 3.1.

Let Θ be the vector containing all the parameters (ρ_1, ρ_2, ρ_3, ..., ρ_k, σ, σ_1, ϕ), then the log-likelihood function is

$$l(\Theta; Y) = -\frac{1}{2}\sum_{yr=1}^{20}\sum_{t=1}^{T}\left(\frac{[Y_{yr,t} - E_{yr,t-1}(Y_{yr,t})]^2}{\sigma_{yr,t}^2} + \log\left(2\pi\sigma_{yr,t}^2\right)\right)$$

with

$$\begin{aligned} E_{yr,t-1}(Y_{yr,t}) &= \widehat{\overline{Y}}_{yr,t} + \sum_{i=1}^{k}\rho_i U_{yr,t-i}; \qquad \sigma_{yr,t} = \sigma - \sigma_1 \mid \sin(\pi t/365 + \phi) \mid, \\ \forall \quad yr &= 1, 2, ..., 20; \quad \& \quad t = 1, 2, ..., 365 \end{aligned}$$

Given the large sample size, the estimation variances can be computed based on the asymptotic distribution of $\widehat{\Theta}$:

$$\sqrt{T}(\widehat{\Theta} - \Theta) \;\tilde{}\; N[0, \quad \Im(\Theta)^{-1}] \quad with \quad \Im(\Theta) \equiv \lim_{T\to\infty} -E\left(\frac{1}{T}\frac{\partial^2 l(\theta; Y)}{\partial\theta\partial\theta'}\right).$$

In order to determine k, the number of lags, we estimate the system separately for $k = 1, 2, 3, ...$, and perform sequential maximum likelihood ratio tests (i.e. χ^2 tests). We stop when the maximum likelihood value ceases to improve. It turns out that three lags describe the data the best. For brevity, we only report the estimation and testing results for $k = 3$, 4 in Exhibit 6.

Several observations are in order. First, for $k = 3$, almost all parameters are estimated with very low standard errors, implying the proper specification of the estimation system. This is by no means a fluke since we have extensively explored and eliminated many other systems. For instance, we have estimated a system which utilizes the simple mean $\overline{Y}_t$ together with a trending parameter. The likelihood value of such a system is much smaller. Second, standard errors of the parameter

σ_1 is very small, implying the appropriateness of using the sine wave to fit the overall volatility structure. (When the system is estimated by specifying a constant volatility throughout the year (i.e. $\sigma_1 = 0$), the likelihood value is much smaller.) Third, the first order auto-regressive behavior tends to be stronger for Southern cities, and ρ_1 has the highest value for Atlanta. Roughly, a stronger auto-correlation means less dramatic changes in temperature, and vice versa. As shown in Exhibit 1, Atlanta does have the lowest overall standard deviation in the sample period.

5. Numerical Analysis

5.1. Simulation Design

Broadly speaking, the simulation procedure consists of 1) generating bivariate paths for the dividend process in Assumption 2 (by assuming $\mu = 1$) and the daily temperature process in Assumption 3 (by using the parameters in Exhibit 6 for the case of $k = 3$); 2) tracking realized HDD/CDD values of each path; 3) calculating the payoff of the derivative security in question according to (2.4) by using CRRA utility; and 4) repeating steps 1 through 3 a large number of times and averaging the payoffs to obtain the desired derivative security value. To accomplish the above we must first make some specifications and adopt certain simulation strategies.

To begin with, the riskfree rate does not explicitly enter the valuation equations in Sections 3.2 and 3.3. Instead, it is endogenously determined by ρ, α, γ, and σ_δ, as shown in Proposition 3.1. The rate of time preference, ρ is typically set equal to, or estimated as the real riskfree interest rate. We set it at 0.03. The volatility of the dividend growth rate, σ_δ is set at 0.2, mimicking the average volatility of a stock market index. The risk-aversion parameter, γ will serve as a comparative static parameter in our analysis, ranging from -1 to 0. Given a value of γ, we will assign a value to α so that the riskfree rate is maintained at 0.06. The only other free standing parameter in the bi-variate system is the correlation coefficient, which we will also use as a comparative static parameter.

Next, we must decide on the inputs for the adjusted mean temperature, $\widehat{\overline{Y}}_{yr,t}$. In the estimation context, $\widehat{\overline{Y}}_{yr,t}$ serves as the "anchoring point" and the temperature dynamics are estimated around these anchoring points. In the valuation context which by necessity is forward looking, $\widehat{\overline{Y}}_{yr,t}$ can naturally be considered as daily temperature forecasts. If the forecasts were of 100% accuracy, then derivatives valuation becomes a simple arithmetic calculation using these forecasts. As a matter of fact, derivative securities won't even exist in this case, because perfect planning is achievable. In reality, it is precisely the uncertainty in the forecasts that drives the value of weather derivatives. The random term in the temperature dynamic captures this uncertainty. (Indeed, this is one of the key advantages of our model, since it allows forecasts to be used as inputs and is capable of accommodating deviations from forecasts commensurate with history, i.e., data.) As for the choice of forecasts, there are no restrictions. In our simulations, we will use two sets of inputs for $\widehat{\overline{Y}}_{yr,t}$, one being the historical daily average temperatures and the other being last season's adjusted daily average temperatures.

To reduce simulation errors, we employ the antithetic variable technique. Each value estimate is based on 10,000 runs.

Finally, we employ a procedure which is equivalent to the control variate technique. Notice that the fundamental variable in our framework is the daily temperature, and the underlying variable for most weather derivatives is HDD or CDD, which is a nonlinear function of daily temperatures. While our model will produce almost "unbiased" temperature forecasts in that the average temperature for a future point will be almost equal to the input forecast, it can not guarantee an unbiased forecast for the HDD / CDD. To further illustrate this point, we compare some forward prices in Exhibit 7. The first column reports the average seasonal CDD and HDD for each city averaged across the twenty years. (It is actually nineteen years for the HDD.) The third column contains the CDD and HDD calculated from the average daily temperatures. (The difference between the two columns is due to the different sequence of averaging.) The fifth column is the counterpart of Column three except that the inputs are the adjusted daily average temperatures of the last season. Columns two and four report the simulated forward CDD's and HDD's using the corresponding forecast inputs when the correlation between the dividend process and the temperature is set to zero. Let us compare Column two with Column three, and Column four with Column five. Notice that the difference in some cases is very small, almost entirely attributable to simulation errors; but in some other cases, the difference is too big to be explained by simulation errors. The reason for the sizable differences lies in the aforementioned fact that the simulated quantities are nonlinear functions of the underlying variable. To ensure correct pricing, we perform a two-stage simulation for each value estimate. In the first stage, we simulate forward prices by setting the correlation parameter to zero ($\varphi = 0$), and record the difference between the model price and the implied forward price from the forecasts (e.g., in Exhibit 7, when average temperatures are used as forecasts, this is the difference between Column two and Column three). Then, in the second stage, we simulate the derivative security's price by setting the correlation parameter back to its actual value. Here, before calculating the derivative's payoff for each path, we first adjustment the realized CDD or HDD by the difference found in the first stage. In a nutshell, the above procedure amounts to ensuring unbiased paths of the CDD and HDD which are underlying variables for weather derivatives. It should be pointed out that when the temperature dynamic, especially the volatility structure, is modelled and estimated perfectly, this procedure will not be necessary.

5.2. HDD / CDD Derivative Prices and Market Price of Risk

For all subsequent simulations, we will examine two risk-aversion cases ($\gamma = -0.5$ corresponding to a power utility and $\gamma = -1.0$ corresponding to a log utility) and two correlation cases ($\varphi = -0.2$ and $\varphi = 0.2$). For each parameter combination, we perform two sets of simulations, each based on a different set of forecasts. We will examine both CDD and HDD contracts. Throughout the discussions we will attempt to assess the importance of market price of risk. (Recall that, when the interest rate is maintained at a constant level, $\varphi = 0.0$ amounts to a zero market price of risk for the temperature variable, irrespective of the risk-aversion level.)

5.2.1. CDD Forward Prices and Option Prices

Exhibit 8 reports CDD forward prices. Several observations are in order. First, the level of forward price depends heavily on the forecast inputs, as reflected by the differences between the two panels. Second, since the forecast forward prices correspond to $\varphi = 0$, comparing the forecast forward prices with those under $\varphi = -0.2$ and $\varphi = 0.2$ reveal that the predictions in Table 2 also hold for the general case where the auto-correlations are not zero. Third, depending on the sign of the correlation, a higher risk-aversion may lead to a higher or lower forward price. Specifically, with a negative correlation φ, a higher risk-aversion ($\varphi = -1.0$) leads to a higher forward price, and vice versa. The intuitive reason lies in the valuation equations in Section 3.2. Notice that the future payoff is "discounted" back at a rate which is a function of certain fixed parameters and the stochastic dividend ratio raised to the power of γ. Since $\gamma < 0$ and since the average dividend growth rate is positive (i.e. $\alpha > 0$), a higher dividend ratio leads to a smaller discount factor (i.e. a lower present value), and vice versa. Now, with a negative correlation, a higher CDD ending value is most likely accompanied by a lower dividend ratio, and a lower dividend ratio (meaning close to one) will lead to a higher present value. The closer γ is to -1, the more manifest the above effect, and hence the pattern. (To see the last point, realize that in general, let $f = 1/A^a$ and $g = 1/A^b$ and $1 > a > b > 0$, then $\mid \frac{\partial f}{\partial A} \mid > \mid \frac{\partial g}{\partial A} \mid$.) Of course, this also explains why, under a particular risk-aversion parameter, the relationship between the forward price and the correlation is negative. Lastly, irrespective of the above patterns, the overall impact of the market price of temperature risk does not seem to be significant. Going from a zero to a non-zero market price of risk, the forward price only changes by less than one percent, which is true across different parameter combinations.

Exhibit 9 contains CDD option prices. We first explain the so-called historical simulation prices. Some authors (e.g. Hunter, 1999) have discussed the use of historical simulation in pricing weather derivatives. The idea is similar to the historical simulation used in some of the VaR calculations. Specifically, a derivative contract's payoff is calculated using realized, historical underlying variable values, and the average payoff over a sample period (say 10 years) is taken as the estimate of the derivative's value. In our case, it boils down to evaluating the payoff for each of the twenty years and then averaging the payoffs to arrive at a value. (The discounting is done using the 6% constant rate.) The strike price is set equal to the historical average CDD for each city. It is seen that the call and put option values are equal for each city. This is expected since the exercise price is set at the (realized) forward price level.

In Exhibit 9, we also report simulated option values under different parameter combinations. For Panel A, the strike price is set equal to the forecast forward price level (from the first column of Exhibit 8). For Panel B, two sets of strike prices are used. Option prices in Column 4 are based on the strike prices in Column 3. Other option prices are based on the strike prices in Column 1. We use different strike prices to examine the behavior of away-from-the-money options.

The first observation is the significant difference between the historical simulation prices and the regular simulated prices, especially in Panel A where the historical simulation prices are much higher. To understand why, realize that in the regular simulations, the overall level of the temperature one year into the future is more or less contained by the forecasts, and variations are around this overall level. But with historical simulations, we are implicitly assuming that next year's temperatures can have extremely large variations, even in terms of the general level; the very cool and the very hot summers in the past 20 years command the same probability in realization. In reality however, meteorologists are at least able to forecast the general level of temperatures (i.e. cool, normal or hot) for the next summer with reasonable accuracy. Historical simulations tend to exaggerate the impact of extreme realizations when the sample size is not large, and in our case, we have only 20 observations for each season. This points out a serious drawback of historical simulations used in pricing weather derivatives.

Another observation has to do with the impacts of parameter values. First of all, we observe exactly the same patterns in CDD call option prices as in CDD forward prices: the higher the risk-aversion, the lower the option's price, when the correlation is positive, and so on. Similarly, a positive correlation is associated with lower option prices for a given level of risk-aversion. The explanations for the CDD forward prices also apply to CDD call options. It is seen that the patterns for put options are reversed. This is easy to understand, since the value of a put option is inversely related to the level of CDD. Second, the option prices do not seem to change significantly across different parameter combinations, although percentage-wise, the change is bigger than that for the forward prices. The higher the risk-aversion, the bigger the impact of the market price of risk. However all percentages are less than 3.5% in absolute values. If an investor is willing to tolerate an error in that magnitude, the market price of risk can be safely ignored.

Finally, it is observed in Panel B that different strike prices can lead to quite different option values, as expected. What is interesting is the non-linear relationship between the option price and the strike price. For example, for Dallas, when the strike price changes from 3153.45 to 2424.55, representing a decrease of 728.9, the call value increases from 35.01 to 697.57, representing an increase of 662.56. (We ignore the minor impact of the correlation.) The option value's increase is smaller than the decrease in the strike price. It is tempting to calculate one option value based on a set of forecasts and simply adjust linearly for another set of forecasts to get a new option value. Exhibit 9 shows that this is not valid.

5.2.2. HDD Forward Prices and Option Prices

Exhibits 10 and 11 are HDD counterparts of Exhibits 8 and 9. All the conclusions in the previous section apply here. Of course, it is seen that all the patterns are the reverse of their CDD counterparts. This is because HDD as an underlying contract variable is negatively related to the temperature level, whereas CDD is positively related to the temperature.

5.2.3. Monthly HDD / CDD Options

Finally, in Exhibit 12, we report option values for individual months of a CDD or HDD season. The setup is exactly the same as those in Exhibits 9 and 11, except that every quantity is for a particular month. For brevity, we only report results for two cities under two parameter combinations. It is seen that all the qualitative conclusions drawn before also apply here. However, we would like to draw the reader's attention to an important result implied in the exhibit. Take the CDD contracts as an example, and let's restrict our attention to New York under $\gamma = -0.5$ and $\varphi = -0.2$. It can be verified that the sum of the strike prices across the five months is equal to the CDD seasonal strike reported in Exhibit 9: 1101.80 (allowing for a small rounding error). The sum of the call option prices is 75.22 and that of the puts is 74.26. But the corresponding seasonal contract values are 34.84 and 33.98 respectively (from Exhibit 9), much smaller than the sums. This is the well known result that the option on a portfolio is worth less than the sum of options written on the individual components of the portfolio.

6. Summary and Conclusion

In this paper, we propose and implement an equilibrium valuation framework for weather derivatives. We specialize the framework to HDD/CDD contracts. The framework is the generalized Lucas's model of 1978. The underlying economic variable is the aggregate dividend and the underlying variable for weather derivatives is the daily temperature, and the two are correlated. We study the temperature behavior in the past twenty years for five major cities in the U.S., and derive key properties of the temperature dynamics. We develop a model system that not only allows easy estimation, but also incorporates key features of the daily temperature dynamics such as seasonal cycles and uneven variations throughout the year. The system is estimated using the maximum likelihood method, and HDD and CDD contracts are priced accordingly.

Our valuation framework has many advantages. It allows the use of weather forecasts in modelling the future temperature behavior. In addition, since our starting point is the daily temperature, the framework is capable of handling temperature contracts of any maturity, for any season, and it requires only a one-time estimation. In contrast, if one starts by modelling CDD or HDD directly, then by nature of the temperature behavior, the CDD or HDD will necessarily be season and maturity specific, which implies that each contract will require a separate estimation procedure. This will not only create potential inconsistency in pricing, but also render the whole idea impractical if many different contracts are dealt with or if the valuation is to be done on an ongoing basis. Last but not least, our equilibrium framework allows us to answer a very important question: Can one use the riskfree rate in deriving weather derivative values without incurring too big an error?

Several conclusions can be drawn from the study. First and foremost, the market price of risk associated with the temperature variable does not seem to affect the derivatives' value in a significant way. This is true for the range of plausible correlations between the aggregate dividend and the temperature variable, and under different utility functions within the CRRA family. This conclusion is significant in that the common practice in the industry is to use the riskfree rate as the discount rate. Our study represents the first attempt to inquire into the validity of such a practice. Notwithstanding, we are obliged to admit that the conclusion is not universal, because we use a special class of utility function and we only allow contemporaneous correlation between the aggregate dividend and the temperature.

The second conclusion has to do with another common practice in the industry, which is to use the historical simulation approach to estimate weather derivative values. We show that in most cases, this is not valid. Weather contracts typically cover a period to come and do not extend very far into the future. However, historical simulations implicitly assume that the next season's temperature can resemble any of the past seasons in the sample, including extreme seasons (very cold or very warm). As a result, in most cases, the historical simulation method tends to over estimate option prices.

As for future research directions, one obvious avenue is to generalize the framework to incorporate intertemporal correlations between the aggregate dividend and the temperature variable and investigate the impact of market price of risk from within. This will be useful in that, intuitively, an abnormal weather season may affect the future output more than the current. Our efforts in this paper represent the first step in such a general pursuit.

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Appendices

A. Proof of Proposition 3.1

A.1. Equilibrium Discount Bond Price

Since the marginal distribution of $\ln\delta_T$ conditional on $\ln\delta_t$ is

$$f(\ln\delta_T \mid \ln\delta_t) = \frac{1}{\sqrt{2\pi\Sigma_\delta(t,T)}} \exp\left(-\frac{(\ln\delta_T - \mu_\delta(t,T))^2}{\Sigma_\delta(t,T)}\right)$$

with

$$\mu_\delta(t,T) = E_t(\ln\delta_T) = E_t\left(\mu^{T-t}\ln\delta_t + \sum_{i=t+1}^{T}\mu^{T-i}(\alpha + \sigma_\delta\epsilon_i)\right)$$
$$= \mu^{T-t}\ln\delta_t + \alpha\sum_{i=t+1}^{T}\mu^{T-i};$$

$$\Sigma_\delta(t,T) = var_t(\ln\delta_T) = \sigma_\delta^2\sum_{i=t+1}^{T}\mu^{2(T-i)},$$

the price of a pure discount bond at time t with maturity T is

$$B(t,T) = E_t\left(\frac{U_c(c_T,T)}{U_c(c_t,t)}\cdot 1\right) = e^{-\rho(T-t)}E_t\left(\frac{\delta_T^\gamma}{\delta_t^\gamma}\right)$$

$$= e^{-\rho(T-t)}E_t\left(\frac{\delta_t^{\gamma\mu^{T-t}}e^{\gamma\sum_{i=t+1}^{T}\mu^{T-i}(\alpha+\sigma_\delta\epsilon_{1i})}}{\delta_t^\gamma}\right)$$

$$= \delta_t^{\gamma(\mu^{T-t}-1)}\exp\left(-\rho(T-t) + \sum_{i=t+1}^{T}[\alpha\gamma\mu^{T-i} + \tfrac{1}{2}\gamma^2\sigma_\delta^2\mu^{2(T-i)}]\right)$$

$$= \delta_t^{\gamma(\mu^{T-t}-1)}\exp\left(-\rho(T-t) + \sum_{i=t+1}^{T}[\alpha\gamma\mu^{T-i} + \tfrac{1}{2}\gamma^2\sigma_\delta^2\mu^{2(T-i)}]\right).$$

Then, the yield-to-maturity, $R(t,T)$, is

$$R(t,T) = -\frac{\ln B(t,T)}{T-t} = \rho - \frac{\sum_{i=t+1}^{T}[\alpha\gamma\mu^{T-i} + \frac{1}{2}\gamma^2\sigma_\delta^2\mu^{2(T-i)}]}{T-t} - \frac{\gamma(\mu^{T-t}-1)\ln\delta_t}{T-t}.$$

B. Proof of Proposition 3.2: Equilibrium forward Prices

When there is no autocorrelation in daily temperatures, i.e., $\rho_i = 0$ (for $i = 1, 2, ..., k$), the joint distribution of $(\ln\delta_T,\, Y_T)$ conditional on $(\ln\delta_t,\, Y_t)$ is a bi-variate normal distribution with correlation coefficient φ :

$$f(\ln\delta_T, Y_{yr,T}|\ln\delta_t, Y_{yr,t}) = N[\ln\delta_T, Y_{yr,T}; \mu_\delta(t,T), \Sigma_\delta(t,T), \mu_Y(t,T), \Sigma_Y(t,T), \varphi],$$

where

$$\mu_Y(T) = E_t(Y_{yr,T}) = \widehat{\overline{Y}}_{yr,T} \qquad and \qquad \Sigma_Y(T) = var_t(Y_{yr,T}) = \sigma_{yr,T}^2.$$

B.1. Forward Prices before the Accumulation Period

For an HDD forward contract with the accumulation period from T_1 to maturity T_2, its forward price at time $t < T_1$ (before the accumulation period) is

$$F_{HDD}(t,T_1,T_2) = \frac{1}{B(t,T_2)} E_t\left(\frac{U_c(c_{T_2},T_2)}{U_c(c_t,t)} \cdot HDD(T_1,T_2)\right)$$

$$= \frac{e^{-\rho(T_2-t)}\delta_t^{-\gamma}}{B(t,T_2)} E_t\left(\delta_{T_2}^{\gamma} \sum_{\tau=T_1}^{T_2} \max(65 - Y_{yr,\tau}, 0)\right)$$

$$= \frac{e^{-\rho(T_2-t)}\delta_t^{-\gamma}}{B(t,T_2)} \delta_t^{\gamma\mu^{T_2-t}} E_t\left(e^{\sum_{i=t+1}^{T_2}\gamma\mu^{(T_2-i)}[\alpha+\sigma_\delta\varepsilon_i]} \sum_{\tau=T_1}^{T_2} \max(65 - Y_{yr,\tau}, 0)\right).$$

We can rewrite the above as

$$= \frac{e^{-\rho(T_2-t)}\delta_t^{\gamma(\mu^{T_2-t}-1)}}{B(t,T_2)} E_t\left(\sum_{\tau=T_1}^{T_2} e^{\sum_{i=t+1, i\neq\tau}^{T_2}\gamma\mu^{(T_2-i)}[\alpha+\sigma_\delta\varepsilon_i]} e^{\gamma\mu^{T_2-\tau}(\alpha+\sigma_\delta\varepsilon_\tau)} \max(65 - Y_{yr,\tau}, 0)\right)$$

$$= \frac{e^{-\rho(T_2-t)}\delta_t^{\gamma(\mu^{T_2-t}-1)}}{B(t,T_2)} \sum_{\tau=T_1}^{T_2} E_t\left[e^{\sum_{i=t+1, i\neq\tau}^{T_2}\gamma\mu^{(T_2-i)}[\alpha+\sigma_\delta\varepsilon_i]}\right] E_t\left[e^{\gamma\mu^{T_2-\tau}(\alpha+\sigma_\delta\varepsilon_\tau)} \max(65 - Y_{yr,\tau}, 0)\right]$$

$$= \frac{e^{-\rho(T_2-t)}\delta_t^{\gamma(\mu^{T_2-t}-1)}}{B(t,T_2)} \sum_{\tau=T_1}^{T_2} \left[e^{\sum_{i=t+1, i\neq\tau}^{T_2}\gamma\mu^{(T_2-i)}[\alpha+\frac{1}{2}\gamma\sigma_\delta^2\mu^{(T_2-i)}]}\right] E_t\left[e^{\gamma\mu^{T_2-\tau}(\alpha+\sigma_\delta\varepsilon_\tau)} \max(65 - Y_{yr,\tau}, 0)\right]$$

Tedious exercise shows that

$$E_t\left[e^{\gamma\mu^{T_2-\tau}(\alpha+\sigma_\delta\varepsilon_\tau)} \max(65 - Y_{yr,\tau}, 0)\right]$$

$$= e^{\gamma\mu^{T_2-\tau}(\alpha+\frac{1}{2}\gamma\sigma_\delta^2\mu^{(T_2-\tau)})}\left([65-\mu_Y'(\tau)]\cdot N(\frac{65-\mu_Y'(\tau)}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}}\exp\left[-\frac{(65-\mu_Y'(\tau))^2}{2\sigma_{yr,\tau}^2}\right]\right)$$

$$\text{with} \qquad \mu_Y'(\tau) = \widehat{\widehat{Y}}_{yr,\tau} + \gamma\varphi\mu^{\tau-t}\sigma_\delta\sigma_{yr,\tau}\,.$$

Therefore,

$$F_{HDD}(t,T_1,T_2) =$$

$$\frac{e^{-\rho(T_2-t)}\delta_t^{\gamma(\mu^{T_2-t}-1)}}{B(t,T_2)} \sum_{\tau=T_1}^{T_2}\left(\begin{array}{c} \left[e^{\sum_{i=t+1, i\neq\tau}^{T_2}\gamma\mu^{(T_2-i)}[\alpha+\frac{1}{2}\gamma\sigma_\delta^2\mu^{(T_2-i)}]}\right] e^{\gamma\mu^{T_2-\tau}(\alpha+\frac{1}{2}\gamma\sigma_\delta^2\mu^{(T_2-\tau)})} \\ \times\left[[65-\mu_Y'(\tau)]\cdot N(\frac{65-\mu_Y'(\tau)}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}}\exp\left(-\frac{(65-\mu_Y'(\tau))^2}{2\sigma_{yr,\tau}^2}\right)\right] \end{array}\right)$$

$$= \sum_{\tau=T_1}^{T_2}\left(\left[[65-\mu_Y'(\tau)]\cdot N(\frac{65-\mu_Y'(\tau)}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}}\exp\left(-\frac{(65-\mu_Y'(\tau))^2}{2\sigma_{yr,\tau}^2}\right)\right]\right).$$

Similarly, the corresponding forward prices on CDD is

$$F_{CDD}(t,T_1,T_2) = \frac{1}{B(t,T_2)} E_t\left(\frac{U_c(c_{T_2},T_2)}{U_c(c_t,t)} \cdot CDD(T_1,T_2)\right)$$

$$= \sum_{\tau=T_1}^{T_2}\left([\mu'_Y(\tau)-65]\cdot N(\frac{\mu'_Y(\tau)-65}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}}\exp\left[-\frac{(65-\mu'_Y(\tau))^2}{2\sigma^2_{yr,\tau}}\right]\right).$$

B.2. Forward Prices during the Accumulation Period

For HDD forward price at time $t \in (T_1,T_2)$ during the accumulation period, we have

$$F_{HDD}(t,T_1,T_2) = \frac{1}{B(t,T_2)} E_t\left(\frac{U_c(c_{T_2},T_2)}{U_c(c_t,t)} \cdot HDD(T_1,T_2)\right)$$

$$= \frac{1}{B(t,T_2)} E_t\left(\frac{U_c(c_{T_2},T_2)}{U_c(c_t,t)} \cdot HDD(T_1,t)\right) + \frac{1}{B(t,T_2)} E_t\left(\frac{U_c(c_{T_2},T_2)}{U_c(c_t,t)} \cdot HDD(t+1,T_2)\right)$$

$$= HDD(T_1,t)\frac{1}{B(t,T_2)} E_t\left(\frac{U_c(c_{T_2},T_2)}{U_c(c_t,t)}\right) + \frac{e^{-\rho(T_2-t)}\delta_t^{-\gamma}}{B(t,T_2)} E_t\left(\delta_{T_2}^{\gamma} HDD(t+1,T_2)\right)$$

$$= HDD(T_1,t) + \sum_{\tau=t+1}^{T_2}\left([65-\mu'_Y(\tau)]\cdot N(\frac{65-\mu'_Y(\tau)}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}}\exp\left[-\frac{(65-\mu'_Y(\tau))^2}{2\sigma^2_{yr,\tau}}\right]\right).$$

Also, the CDD forward price at time $t \in (T_1,T_2)$ is

$$F_{CDD}(t,T_1,T_2)$$

$$= CDD(T_1,t) + \sum_{\tau=t+1}^{T_2}\left([\mu'_Y(\tau)-65]\cdot N(\frac{\mu'_Y(\tau)-65}{\sigma_{yr,\tau}}) + \frac{\sigma_{yr,\tau}}{\sqrt{2\pi}}\exp\left[-\frac{(65-\mu'_Y(\tau))^2}{2\sigma^2_{yr,\tau}}\right]\right).$$

Exhibit 1: Summary Statistics

	Atlanta	Chicago	Dallas	New York	Philadelphia
Mean	63	50	66	56	56
Median	64	50	67	56	56
Mode	79	70	86	72	75
Standard Devia	15	20	16	17	18
Minimum	5	-17	9	3	1
Maximum	92	93	97	93	92
Sample Size	7,300	7,300	7,300	7,300	7,300
			Correlation		
Atlanta	1.0000				
Chicago	0.8847	1.0000			
Dallas	0.8777	0.9038	1.0000		
New York	0.8966	0.8964	0.8443	1.0000	
Philadelphia	0.9125	0.8970	0.8455	0.9853	1.0000
			Auto Correlation		
k-lags					
1	0.9402	0.9421	0.9354	0.9448	0.9462
2	0.8690	0.8809	0.8680	0.8896	0.8926
3	0.8281	0.8494	0.8318	0.8654	0.8678
4	0.8069	0.8304	0.8132	0.8533	0.8550
5	0.7952	0.8181	0.8005	0.8470	0.8486
6	0.7867	0.8091	0.7918	0.8431	0.8437
7	0.7804	0.8022	0.7855	0.8394	0.8380
8	0.7764	0.7973	0.7813	0.8346	0.8330
9	0.7728	0.7925	0.7773	0.8297	0.8283
10	0.7687	0.7894	0.7731	0.8246	0.8228
11	0.7665	0.7870	0.7718	0.8197	0.8175
12	0.7652	0.7857	0.7720	0.8164	0.8142
13	0.7614	0.7835	0.7683	0.8124	0.8098
14	0.7562	0.7793	0.7608	0.8099	0.8054
15	0.7534	0.7759	0.7558	0.8070	0.8017

Exhibit 2: Summary Statistics of Monthly HDD and CDD (1979 - 1998)

		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
							Atlanta						
HDD	Average	679	493	328	143	21	1	0	1	11	117	336	586
	Std. Dev.	125	90	85	63	18	4	0	2	10	43	97	126
	Maximum	882	657	465	261	63	18	0	11	32	188	514	797
	Minimum	462	297	156	36	2	0	0	0	0	22	131	346
CDD	Average	0	2	14	59	198	385	502	452	275	67	8	2
	Std. Dev.	1	3	10	39	66	68	72	63	59	38	13	3
	Maximum	3	8	36	141	322	494	639	589	440	178	49	13
	Minimum	0	0	2	4	62	221	372	349	192	12	0	0
							Chicago						
HDD	Average	1308	1065	857	516	230	51	6	10	112	407	766	1132
	Std. Dev.	189	165	101	80	81	31	6	12	46	79	106	184
	Maximum	1627	1359	1095	643	364	118	19	37	189	583	958	1562
	Minimum	956	733	733	393	87	6	0	0	34	288	598	891
CDD	Average	0	0	2	9	48	161	283	240	93	9	0	0
	Std. Dev.	0	0	3	15	40	62	73	92	36	10	0	0
	Maximum	0	0	13	53	167	254	398	445	158	44	1	0
	Minimum	0	0	0	0	4	38	152	106	19	0	0	0
							Dallas						
HDD	Average	627	442	269	91	10	0	0	0	8	69	305	557
	Std. Dev.	124	101	63	40	9	0	0	0	10	32	66	128
	Maximum	911	635	384	186	29	2	0	0	38	145	414	933
	Minimum	401	299	164	26	0	0	0	0	0	14	190	389
CDD	Average	1	4	28	87	270	491	642	622	399	145	22	4
	Std. Dev.	2	9	20	35	78	74	77	73	64	35	16	5
	Maximum	6	37	77	158	464	668	844	737	563	220	52	13
	Minimum	0	0	4	24	171	382	551	480	302	78	3	0
							New York						
HDD	Average	988	838	702	374	120	13	0	1	35	234	513	824
	Std. Dev.	145	125	80	46	40	12	1	3	16	73	69	139
	Maximum	1241	1173	913	446	187	49	3	9	74	376	651	1198
	Minimum	735	671	613	286	51	1	0	0	11	101	389	644
CDD	Average	0	0	1	4	63	224	392	350	153	22	1	0
	Std. Dev.	0	0	4	8	42	55	56	56	37	23	2	0
	Maximum	0	0	18	34	184	325	490	444	222	95	7	2
	Minimum	0	0	0	0	4	112	271	239	100	0	0	0
							Philadelphia						
HDD	Average	1002	835	676	349	106	10	0	2	39	261	538	852
	Std. Dev.	154	124	95	60	46	11	1	5	19	75	86	136
	Maximum	1243	1170	911	440	191	42	4	21	92	408	704	1219
	Minimum	738	644	542	193	38	0	0	0	14	138	392	701
CDD	Average	0	0	2	9	79	245	409	352	155	22	1	0
	Std. Dev.	0	0	5	11	47	66	71	65	42	21	2	0
	Maximum	0	0	20	34	230	401	540	470	244	83	6	0
	Minimum	0	0	0	0	9	133	283	268	90	0	0	0

Exhibit 3: Global Warming Trend

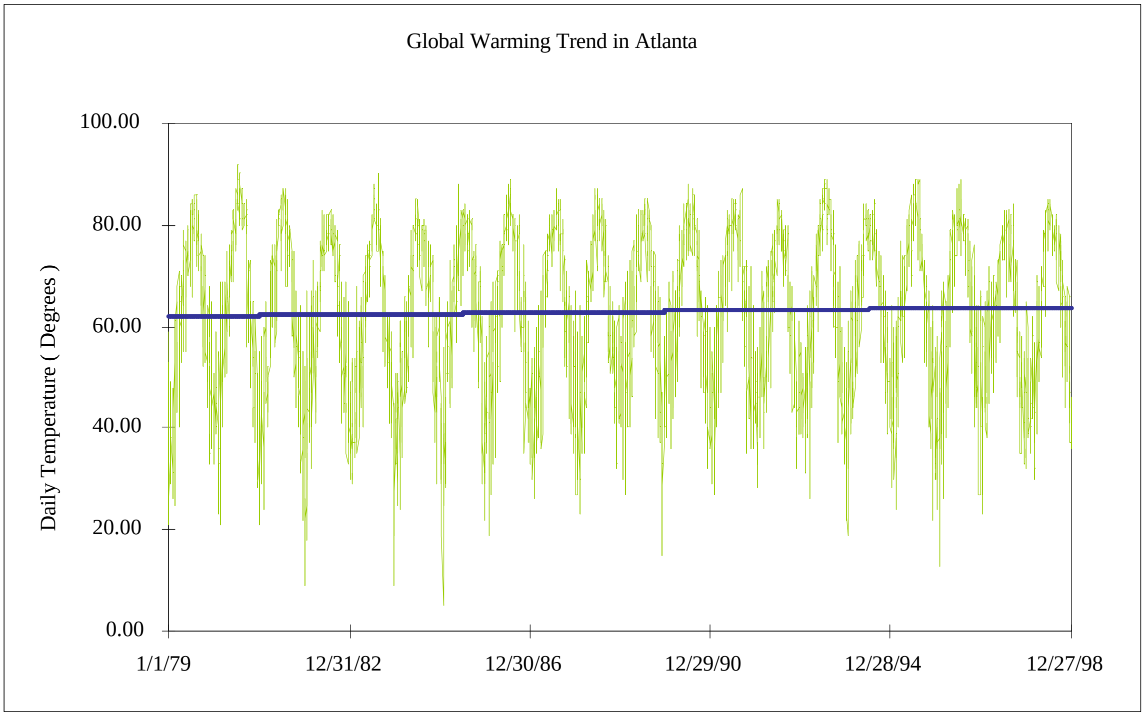


Exhibit 4: Standard Deviation of Date t's Temperature (ψ_t)

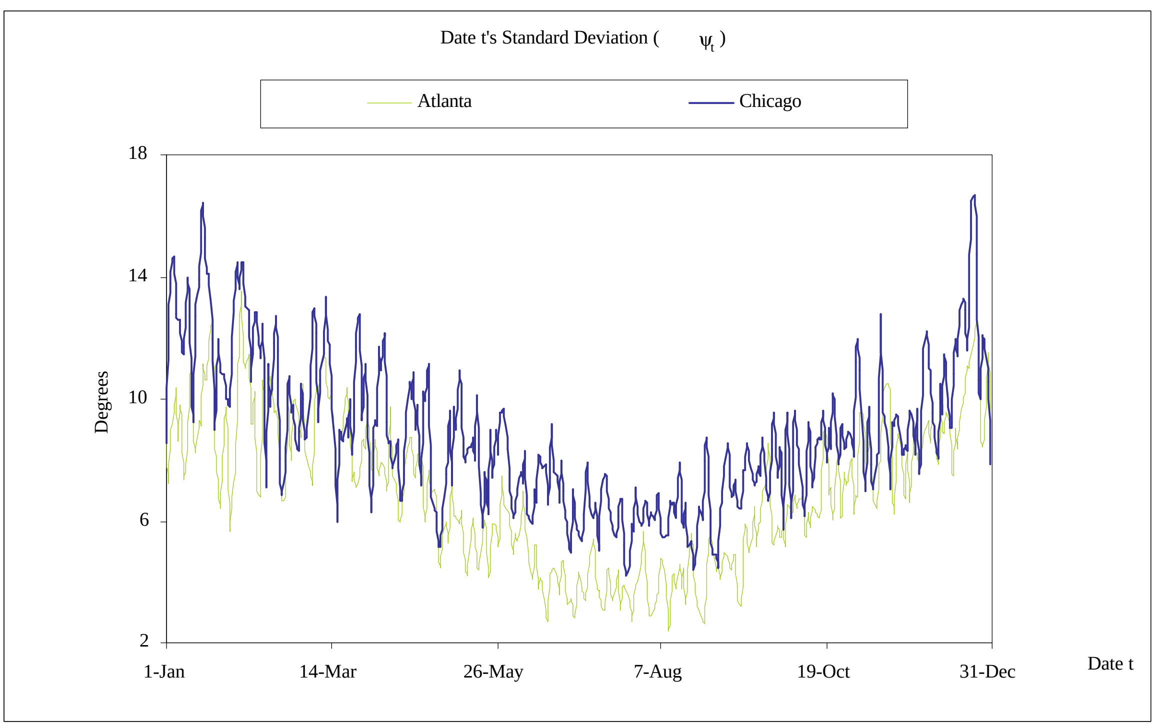
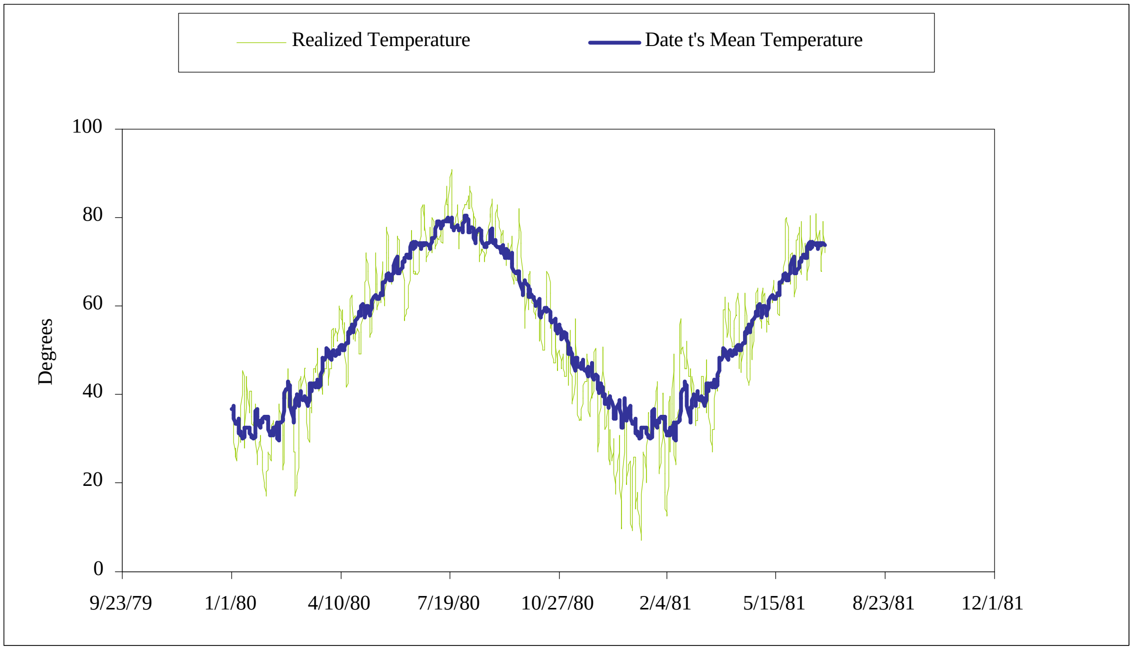


Exhibit 5: Realized Temperatures in New York
For a Colder-Than-Normal Winter (November 1980 - March 1981)

Panel A: Date t's Mean Temperature ($\overline{Y}_t$) vs Realized Temperatures



Panel B: Date t's Adjusted Mean Temperature ($\widehat{\overline{Y}}_t$) vs Realized Temperatures

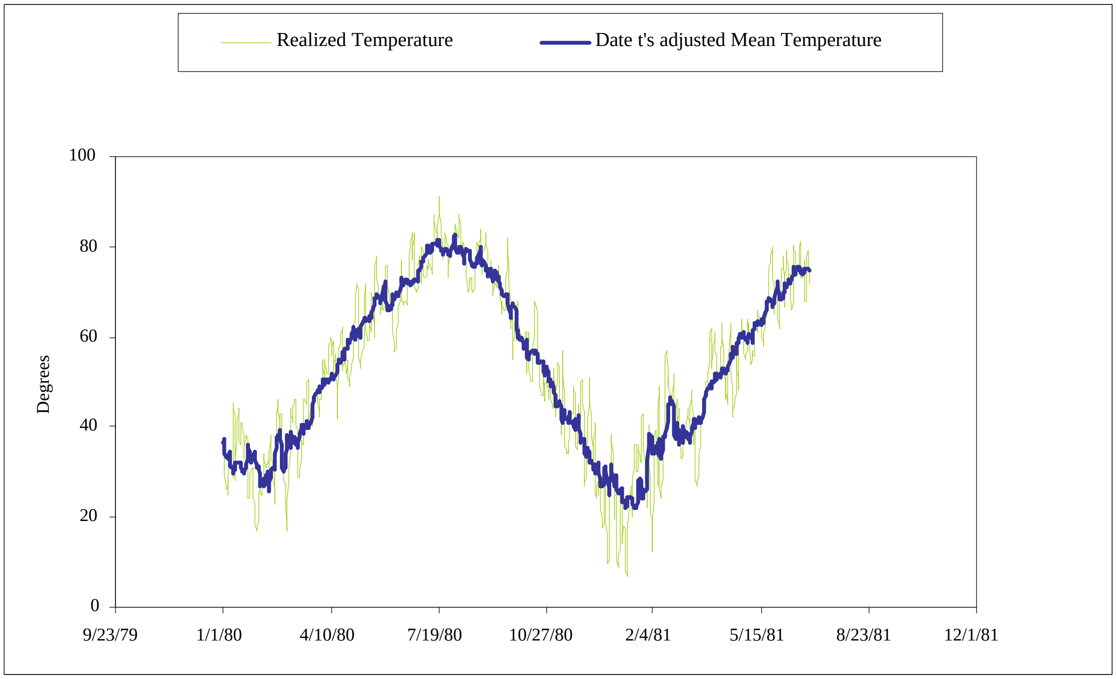


Exhibit 6: Maximum Likelihood Estimation Results

ρ_1	ρ_2	ρ_3	ρ_4	σ	σ_1	ϕ	Log - Likelihood		LR = $2\ln(L_1/L_2)$
					Atlanta				
0.8833	-0.3035	0.0322		7.5980	5.0912	-0.1881	$\ln(L_1)$ =	-20,626	250 *
(0.01170)	(0.01520)	(0.01169)		(0.12086)	(0.14603)	(0.01067)			
0.9482	-0.3158	0.0760	0.0083	7.6565	5.0781	-0.1940	$\ln(L_2)$ =	-20,751	
(0.01166)	(0.01567)	(0.01166)	(0.01288)	(0.12208)	(0.14776)	(0.01088)			
					Chicago				
0.7989	-0.2570	0.0428		7.8289	3.1294	-0.2014	$\ln(L_1)$ =	-23,130	268 *
(0.01170)	(0.01467)	(0.01170)		(0.13922)	(0.18181)	(0.02316)			
0.8619	-0.2697	0.1029	-0.0188	7.9315	3.1211	-0.1998	$\ln(L_2)$ =	-23,264	
(0.01166)	(0.01509)	(0.01165)	(0.01905)	(0.14057)	(0.18135)	(0.02369)			
					Dallas				
0.8158	-0.2436	0.0201		8.9378	6.3349	-0.1418	$\ln(L_1)$ =	-21,381	258 *
(0.01170)	(0.01483)	(0.01170)		(0.14060)	(0.16800)	(0.00953)			
0.8711	-0.2482	0.0567	0.0162	9.0257	6.3488	-0.1497	$\ln(L_2)$ =	-21,510	
(0.01168)	(0.01522)	(0.01167)	(0.01267)	(0.14175)	(0.16936)	(0.00970)			
					New York				
0.7558	-0.2631	0.0463		6.5372	2.7035	-0.2432	$\ln(L_1)$ =	-21,719	298 *
(0.01169)	(0.01433)	(0.01169)		(0.11241)	(0.14520)	(0.02238)			
0.8117	-0.2612	0.1007	-0.0071	6.7129	2.8152	-0.2420	$\ln(L_2)$ =	-21,868	
(0.01166)	(0.01472)	(0.01166)	(0.01374)	(0.11510)	(0.14831)	(0.02201)			
					Philadelphia				
0.7726	-0.2595	0.0473		6.9034	3.1654	-0.2015	$\ln(L_1)$ =	-21,792	306 *
(0.01169)	(0.01446)	(0.01169)		(0.11957)	(0.15360)	(0.01932)			
0.8290	-0.2559	0.0973	0.0015	7.0545	3.2360	-0.2024	$\ln(L_2)$ =	-21,945	
(0.01166)	(0.01486)	(0.01166)	(0.01459)	(0.12207)	(0.15675)	(0.01932)			

Note: 1. The Estimated Systems:

$$U_{yr,t} = \rho_1 U_{yr,t-1} + \rho_2 U_{yr,t-2} + \rho_3 U_{yr,t-3} + \sigma_{yr,t} * \xi_{yr,t} \quad (1)$$

$$U_{yr,t} = \rho_1 U_{yr,t-1} + \rho_2 U_{yr,t-2} + \rho_3 U_{yr,t-3} + \rho_4 U_{yr,t-4} + \sigma_{yr,t} * \xi_{yr,t} \quad (2)$$

with $U_{yr,t} = Y_{yr,t} - \widehat{\overline{Y}}_{yr,t}$, $\quad \sigma_{yr,t} = \sigma - \sigma_1 \mid \sin(\pi t/365 + \phi) \mid$,

$$\xi_{yr,t} \sim i.i.d.\ N(0,1), \qquad \forall \quad yr = 1,2,...,20 \quad \& \quad t = 1,2,...,365.$$

2. The numbers in the parentheses are standard errors.
3. The null hypothesis (H_0) is $\rho_4 = 0$. The likelihood ratio (LR) test is computed as $LR = 2\ln L_1 - 2\ln L_2$ which is asymptotically distributed as $\chi^2(1)$ under H_0.
4. The 1 percent critical level for χ^2 with 1 degree of freedom is 6.6 and * indicates that the test statistic is significant.

Exhibit 7: Comparison of Forward Prices

	Sample Average	Theoretical Model Price	Price Based on Average Temperature	Theoretical Model Price	Price Based on Adjusted Avg. Temperature of 1998
	(1)	(2)	(3)	(4)	(5)
			CDD Season (May - September)		
Atlanta	1812.00	1797.22	1777.95	1902.17	1893.41
Chicago	823.60	799.26	674.80	1002.25	858.24
Dallas	2424.55	2414.13	2405.65	3154.69	3153.45
New York	1181.80	1169.98	1101.80	1296.11	1226.85
Philadelphia	1239.75	1220.75	1149.35	1351.34	1286.41
			HDD Season (November - March)		
Atlanta	2419.47	2423.70	2396.95	2729.68	2715.65
Chicago	5114.37	5127.72	5126.15	4508.18	4506.00
Dallas	2179.21	2202.51	2141.05	2246.26	2192.98
New York	3859.63	3864.00	3862.35	3419.25	3417.25
Philadelphia	3901.00	3901.62	3899.75	3406.81	3404.08

Note: 1. Theoretical model prices are calculated based on $\gamma = -0.5$, $\phi = 0.0$.
2. When forecasts are the adjusted average temperatures, we use November, and December of 1997 and the first three months of 1998 for HDD calculation.

Exhibit 8: Forward Prices for a CDD Season

	Forecast Forward	$\gamma = -0.5$				$\gamma = -1.0$			
		$\phi = -0.2$		$\phi = 0.2$		$\phi = -0.2$		$\phi = 0.2$	
	(1)	(2)	(3) = (2)/(1)-1	(4)	(5) = (4)/(1)-1	(6)	(7) = (6)/(1)-1	(8)	(9) = (8)/(1)-1
Panel A: Forecast is Historical Average									
Atlanta	1777.95	1779.02	0.06%	1776.86	-0.06%	1780.06	0.12%	1775.86	-0.12%
Chicago	674.80	675.93	0.17%	673.63	-0.17%	676.98	0.32%	672.41	-0.35%
Dallas	2405.65	2406.75	0.05%	2404.52	-0.05%	2407.86	0.09%	2403.57	-0.09%
New York	1101.80	1102.85	0.10%	1100.72	-0.10%	1103.82	0.18%	1099.65	-0.19%
Philadelphia	1149.35	1150.45	0.10%	1148.22	-0.10%	1151.48	0.19%	1147.09	-0.20%
Panel B: Forecast is Adjusted Historical Average of 1998									
Atlanta	1893.41	1894.51	0.06%	1892.29	-0.06%	1895.58	0.11%	1891.28	-0.11%
Chicago	858.24	859.58	0.16%	856.85	-0.16%	860.81	0.30%	855.41	-0.33%
Dallas	3153.45	3154.56	0.04%	3152.31	-0.04%	3155.72	0.07%	3151.45	-0.06%
New York	1226.85	1227.95	0.09%	1225.72	-0.09%	1228.98	0.17%	1224.62	-0.18%
Philadelphia	1286.41	1287.59	0.09%	1285.20	-0.09%	1288.69	0.18%	1284.02	-0.19%

Note: 1. "Forecast Forward" prices are the CDD values implied in the forecasts.

2. Forward prices under different parameter combinations are calculated by making an adjustment to each path of the simulated CDD so that when the correlation is zero, the model always gives the forecast forward price. See the text for details.

Exhibit 9: Option Prices for a CDD Season

Historical Simulation			γ = - 0.5	γ = - 0.5				γ = - 1.0			
			ϕ = 0.0	ϕ = - 0.2		ϕ = 0.2		ϕ = - 0.2		ϕ = 0.2	
Strike Price (1)	Option Value (2)	Strike Price (3)	Option Value (4)	Option Value (5)	% (6) = (5)/(4) - 1	Option Value (7)	% (8) = (7)/(4) - 1	Option Value (9)	% (10) = (9)/(4) - 1	Option Value (11)	% (12) = (11)/(4) - 1
			Panel A: Forecast is Historical Average								
1812.00	87.62	1777.95	33.16	33.70	1.62%	32.64	-1.59%	34.17	3.04%	32.05	-3.36%
	87.62		33.44	32.91	-1.59%	33.97	1.59%	32.46	-2.93%	34.59	3.45%
823.60	84.88	674.80	39.25	39.82	1.46%	38.68	-1.44%	40.32	2.73%	38.05	-3.05%
	84.88		39.37	38.83	-1.38%	39.92	1.39%	38.36	-2.58%	40.54	2.96%
2424.55	86.02	2405.65	34.35	34.91	1.62%	33.81	-1.59%	35.40	3.04%	33.20	-3.36%
	86.02		34.72	34.17	-1.59%	35.28	1.60%	33.70	-2.96%	35.92	3.44%
1181.80	57.72	1101.80	34.31	34.84	1.53%	33.79	-1.51%	35.29	2.87%	33.21	-3.20%
	57.72		34.49	33.98	-1.48%	35.00	1.49%	33.55	-2.74%	35.60	3.22%
1239.75	80.29	1149.35	35.70	36.26	1.55%	35.16	-1.52%	36.74	2.90%	34.55	-3.22%
	80.29		35.89	35.35	-1.50%	36.43	1.50%	34.90	-2.77%	37.06	3.25%
			Panel B: Forecast is Adjusted Historical Average of 1998								
1812.00	87.62	1893.41	33.95	86.88		85.10		87.66		84.10	
	87.62		34.24	8.24		8.64		8.06		8.87	
823.60	84.88	858.24	43.61	62.81		61.15		63.53		60.22	
	84.88		43.76	28.52		29.50		28.09		30.07	
2424.55	86.02	3153.45	35.01	697.57		695.34		698.59		694.17	
	86.02		35.50	0.00		0.00		0.00		0.00	
1181.80	57.72	1226.85	35.23	61.47		59.97		62.12		59.14	
	57.72		35.43	17.51		18.18		17.22		18.57	
1239.75	80.29	1286.41	37.17	64.31		62.72		65.00		61.83	
	80.29		37.38	18.74		19.47		18.43		19.89	

Note: Under "Historical Simulation", we assume that the future will mimic history exactly according to the sample data. The strike prices are the historical average CDD's. For all other simulations, in Panel A, the strike prices are reported under the heading "$\gamma = -0.5$, $\phi = 0.0$". They are the CDD's implied in the forecasts. In Panel B, the option prices under "$\gamma = -0.5$, $\phi = 0.0$" are based on the strike prices reported under the same heading; but for other option prices, the stirke prices are those reported under "Historical Simulation".

<u>Exhibit 10: Forward Prices for an HDD Season</u>

	Forecast Forward	γ = - 0.5				γ = - 1.0			
		ϕ = – 0.2		ϕ = 0.2		ϕ = – 0.2		ϕ = 0.2	
	(1)	(2)	(3) = (2)/(1)-1	(4)	(5) = (4)/(1)-1	(6)	(7) = (6)/(1)-1	(8)	(9) = (8)/(1)-1
				Panel A: Forecast is Historical Average					
Atlanta	2396.95	2394.44	-0.10%	2399.43	0.10%	2391.60	-0.22%	2402.25	0.22%
Chicago	5126.15	5123.18	-0.06%	5129.09	0.06%	5118.98	-0.14%	5132.22	0.12%
Dallas	2141.05	2138.41	-0.12%	2143.64	0.12%	2135.59	-0.26%	2146.63	0.26%
New York	3862.35	3860.13	-0.06%	3864.55	0.06%	3856.99	-0.14%	3866.88	0.12%
Philadelphia	3899.75	3897.35	-0.06%	3902.13	0.06%	3894.05	-0.15%	3904.67	0.13%
				Panel B: Forecast is Adjusted Historical Average of 1998					
Atlanta	2715.65	2713.07	-0.10%	2718.20	0.09%	2710.07	-0.21%	2721.09	0.20%
Chicago	4506.00	4503.07	-0.06%	4508.90	0.06%	4499.15	-0.15%	4512.03	0.13%
Dallas	2192.98	2190.32	-0.12%	2195.60	0.12%	2187.45	-0.25%	2198.61	0.26%
New York	3417.25	3415.06	-0.06%	3419.41	0.06%	3412.11	-0.15%	3421.75	0.13%
Philadelphia	3404.08	3401.72	-0.07%	3406.42	0.07%	3398.63	-0.16%	3408.97	0.14%

Note: 1. "Forecast Forward" prices are the HDD values implied in the forecasts.
2. When forecasts are the adjusted average temperatures, we use November and December of 1997 and the first three months of 1998.
3. Forward prices under different parameter combinations are calculated by making an adjustment to each path of the simulated HDD so that when the correlation is zero, the model always gives the forecast forward price. See the text for details.

Exhibit 11: Option Prices for an HDD Season

		Historical Simulation			γ = - 0.5	γ = - 0.5				γ = - 1.0			
					φ = 0.0	φ = - 0.2		φ = 0.2		φ = - 0.2		φ = 0.2	
		Strike Price	Option Value	Strike Price	Option Value	Option Value	% (6) =	Option Value	% (8) =	Option Value	% (10) =	Option Value	% (12) =
		(1)	(2)	(3)	(4)	(5)	(5)/(4) - 1	(7)	(7)/(4) - 1	(9)	(9)/(4) - 1	(11)	(11)/(4) - 1
Panel A: Forecast is Historical Average													
Atlanta	Call	2419.47	108.78	2396.95	68.44	67.35	-1.59%	69.53	1.59%	66.48	-2.87%	70.86	3.54%
	Put		108.78		68.74	69.82	1.58%	67.67	-1.55%	70.63	2.75%	66.35	-3.47%
Chicago	Call	5114.37	144.22	5126.15	74.96	73.76	-1.61%	76.17	1.61%	72.79	-2.89%	77.65	3.59%
	Put		144.23		75.59	76.81	1.61%	74.40	-1.58%	77.71	2.80%	72.91	-3.55%
Dallas	Call	2179.21	60.40	2141.05	73.17	72.01	-1.58%	74.33	1.59%	71.08	-2.85%	75.74	3.51%
	Put		60.40		73.44	74.58	1.55%	72.32	-1.52%	75.42	2.70%	70.93	-3.42%
New York	Call	3859.63	105.53	3862.35	55.97	55.07	-1.61%	56.87	1.61%	54.35	-2.89%	57.97	3.58%
	Put		105.53		56.44	57.35	1.61%	55.55	-1.58%	58.02	2.79%	54.44	-3.54%
Philadelphia	Call	3901.00	109.67	3899.75	61.14	60.16	-1.61%	62.12	1.61%	59.37	-2.89%	63.33	3.58%
	Put		109.67		61.62	62.61	1.61%	60.65	-1.58%	63.34	2.80%	59.44	-3.54%
Panel B: Forecast is Adjusted Historical Average of 1997 / 1998													
Atlanta	Call	2419.47	108.78	2715.65	69.32	276.48		280.69		274.75		283.24	
	Put		108.78		69.65	4.24		3.99		4.33		3.84	
Chicago	Call	5114.37	144.22	4506.00	74.88	0.08		0.09		0.08		0.09	
	Put		144.23		75.43	567.50		562.76		569.12		559.80	
Dallas	Call	2179.21	60.40	2192.98	73.29	78.55		81.01		77.56		82.51	
	Put		60.40		73.57	68.37		66.22		69.17		64.89	
New York	Call	3859.63	105.53	3417.25	55.93	0.07		0.08		0.07		0.09	
	Put		105.53		56.35	412.74		409.21		413.95		407.00	
Philadelphia	Call	3901.00	109.67	3404.08	61.03	0.06		0.07		0.06		0.07	
	Put		109.67		61.45	463.50		459.64		464.81		457.23	

Note: 1. Under "Historical Simulation", we assume that the future will mimic history exactly according to the sample data. The strike prices are the historical average HDD's. For all other simulations, in Panel A, the strike prices are reported under the heading "$\gamma = -0.5$, $\phi = 0.0$". They are the HDD's implied in the forecasts. In Panel B, the option prices under "$\gamma = -0.5$, $\phi = 0.0$" are based on the strike prices reported under the same heading; but for other option prices, the stirke prices are those reported under "Historical Simulation".

2. When forecasts are adjusted average temperature, we use November and December of 1997 and the first three months of 1998.

Exhibit 12: Option Prices for Monthly Contracts

		ATLANTA					NEW YORK				
					γ = -0.5					γ = -0.5	
		Historical Simulation			ϕ = - 0.2	ϕ = 0.2	Historical Simulation			ϕ = - 0.2	ϕ = 0.2
		Strike Price	Option Value	Strike Price	Option Value	Option Value	Strike Price	Option Value	Strike Price	Option Value	Option Value
Panel A: Months of a CDD Season											
May	Call	197.60	25.48	176.30	16.16	15.92	62.70	14.63	24.50	9.78	9.65
	Put		25.48		15.97	16.20		14.63		9.68	9.78
June	Call	384.90	25.72	383.70	15.23	15.01	223.90	20.58	210.80	16.71	16.47
	Put		25.67		15.12	15.34		20.53		16.54	16.77
July	Call	502.10	26.33	502.20	13.54	13.34	391.90	21.39	391.60	17.27	17.02
	Put		26.28		13.44	13.64		21.34		17.10	17.35
August	Call	452.10	24.70	451.50	14.19	13.99	350.10	21.24	348.90	17.41	17.16
	Put		24.75		14.03	14.23		21.19		17.20	17.45
September	Call	275.20	20.44	264.30	15.17	14.96	153.10	14.01	126.10	14.05	13.85
	Put		20.39		15.00	15.21		14.06		13.88	14.06
Panel B: Months of a HDD Season											
January	Call	679.10	53.33	678.90	39.25	39.83	987.90	56.82	987.90	29.13	29.56
	Put		53.33		39.76	39.19		56.82		29.46	29.04
February	Call	493.00	33.07	491.50	33.84	34.33	837.60	46.20	837.60	27.37	27.76
	Put		33.07		34.48	34.00		46.15		28.03	27.63
March	Call	327.70	35.37	313.40	27.06	27.46	702.50	31.77	701.40	25.31	25.68
	Put		35.37		27.53	27.14		31.82		25.85	25.47
November	Call	336.40	36.35	328.30	23.45	23.79	512.50	26.95	511.40	21.31	21.62
	Put		36.30		23.84	23.50		26.95		21.70	21.39
December	Call	586.40	48.97	584.90	31.08	31.52	824.10	49.49	824.00	24.56	24.90
	Put		48.97		31.56	31.13		49.49		24.96	24.62

Note: 1. Under "Historical Simulation", we assume that the future will mimic history exactly according to the sample data. The strike prices are the historical average CDD's or HDD's. The stirke prices for all other simulations are reported under the heading "$\gamma = -0.5$". They are the CDD's or HDD's implied in the forecasts.

2. In this table, the forecasts are historical average temperature.