# Mathematics Education for the 21<sup>st</sup> Century

A Fields-Nortel White Paper





## **Executive Summary**

In July 1997, The Fields Institute for Research in Mathematical Sciences (Fields) and Northern Telecom (Nortel) hosted a workshop to provide a forum for discussion of the global and technological challenges facing secondary mathematics education for the twenty-first century.

This Fields-Nortel Workshop brought together a highly committed group of mathematics educators, mathematicians and industry representatives — from Ontario and abroad — to explore the stated themes. The thirty-six participants brought expertise from universities, colleges, schools, industry and government, and included members of eight national and provincial professional mathematics organizations.

The outcome of the workshop is this White Paper, which presents a model of mathematics education for the twenty-first century based on a profile of the mathematics learner as a creative, thinking, learning citizen with the knowledge and skills to compete and succeed in a global economy. The underlying philosophy of this White Paper is revealed in our profile of the learner, and it is this profile which drives our recommendations to the Ontario Ministry of Education and Training for curriculum, delivery, assessment and resources.

In this model of mathematics education, concepts are introduced within the context of solving problems, whenever possible. Students learn that mathematics is a dynamic discipline to be valued for its own sake. Lessons are organized in a coherent fashion so they build toward an understanding of major mathematical concepts. Students use appropriate technology to investigate, find models, organize, prepare and present the results of their investigations. Authentic assessment ensures accountability to this profile of the mathematics learner.

The recommendations presented here result from a sharing of ideas, research, visions, exciting achievements, concerns and challenges expressed in presentations from the industry, business and education sectors and in working group discussions. These recommendations represent a model of mathematics education that is intended to help Ontario secondary school mathematics students develop into successful graduates who match our profile of the learner and are equipped to meet the challenges of life in the twenty-first century. This model is not a list of options: each recommendation is essential to the whole. The ultimate goal of this White Paper is to help bring about changes that will make mathematics education in Ontario the best in the world.

## Introduction

We live in an era of unprecedented change in knowledge-based industries and communication technologies that are transforming the workplace, schools and everyday life. As Ontario embarks upon a major reform of the secondary school curriculum, it is important not only to look critically at what has been taught in the past, but also to develop a vision of the future needs of our high school graduates who will live and work in the twenty-first century. The Ontario Ministry of Education and Training (MET), through its Business Plan of 1997-98, has identified the need for a quality education and training system, which is a "passport to opportunity for students living in a rapidly changing world". This new information and knowledge-based economy puts increased value on learning, particularly in mathematics. A strong foundation in mathematics and numeracy is essential for our students' success in a competitive global economy. These issues were explored in depth at the Fields-Nortel Workshop on Mathematics Education for the 21<sup>st</sup> Century, held July 15-19, 1997 at the Fields Institute in Toronto.

In many ways, education in Ontario is the envy of the world. However, keynote speakers from industry drew attention to the growing need for mathematical skills in the workforce. As Tony Marsh, CEO of Canadian Microelectronics Corp. and Chair of the Education Council of the Conference Board of Canada, stated in his Opening Address to the Workshop, "A shortage of quantitative skills affects our competitive advantage in the global economy." He felt that we should engage more students in mathematics, because Canada's living standards depend increasingly on our brainpower. Marsh suggested that our challenge is to help students experience and creatively embrace mathematics and science. Ron Dembo, President of Algorithmics Inc., supported this view. He stressed that he has difficulty finding employees with adequate mathematical skills. He believes that more exciting and challenging careers would be available to graduates with stronger mathematical skills and knowledge. Robert Long, manager of Nortel's Education Interaction Department, who presented statistics on job opportunities for our knowledge-based economy, further illustrated this message. He stressed the need for good mathematical skills, which are the foundation for employment in this growth sector.

This White Paper begins with a summary of the deliberations at the Fields-Nortel Workshop, then presents an integrated model for mathematics education centered on a profile of the mathematics learner. This model and the  A strong foundation in mathematics and numeracy is essential for our students' success in a global economy

 Better math skills open up possibilities to more exciting and challenging careers profile of the learner lead to a set of recommendations for the transformation of mathematics education in Ontario, to better prepare students for life in the twentyfirst century.

## The Fields-Nortel Workshop

The Fields-Nortel Workshop on Mathematics Education for the 21<sup>st</sup> Century focused on the mathematical needs of secondary school students and explored the effects of agents of change, which are external to the classroom. The two main themes were the *Global Challenge* and the *Technological Challenge*, which are changing priorities in both "what" and "how" we should teach our students. The expert presentations and intensive discussions led the participants to a vision of mathematics education for Ontario in the twenty-first century.

The global ▶ challenge In considering the *Global Challenge*, the workshop addressed such questions as: Will the emerging global economy and knowledge-based industries demand new and different mathematical skills of our graduates? How can we motivate more students (including females) to enroll in the challenging high school mathematics courses which lead to rewarding high-tech careers? Is the present mathematics curriculum relevant to "real-world" problem solving; for example, does it prepare students for the interdisciplinary problems increasingly faced in industry? What are other school systems doing in mathematics education? What are the skills that will enable students to continue learning long after graduation? What essential mathematics skills will be required by all future citizens, whether or not they go on to post-secondary education?

Discussions of the *Technological Challenge* raised questions including: How can educators best use the new communication technologies, such as the World Wide Web, which provide virtually unlimited access to information by students, and in-

service professional development opportunities for teachers? Computers have dramatically changed how mathematics is used in the workplace: how should this be reflected in the classroom? How will Computer-Guided Learning modules provide new approaches and new resources, ranging from remedial learning to challenging enrichment for gifted students? Do computational tools such as graphing calculators, CAD, Maple, Geometer's Sketchpad and spreadsheets make obsolete the need for traditional manipulative skills? Are there inherent dangers in these seductive new technologies?

In examining these broad issues, workshop participants came to the conclusion that fundamental changes in mathematics education are urgently needed in Ontario schools. Representatives from advanced technology industries told the workshop that too many students drop mathematics early, and then are unqualified for post-secondary programs leading to rewarding careers where skilled applicants are in short supply. In the words of American engineer Robert M. White, *"Mathematics ... must become a pump instead of a filter in the pipeline of education"*<sup>11</sup>. Ontario's economic health in te twenty-first century depends on increasing the flow of highly skilled graduates. Experience shows that each new appointment at the cutting edge of technology typically leads to at least ten other new jobs throughout the industry. Mathematics is a resource that fuels the growth of the knowledge-based industries which create new jobs for Canadians.

Another impetus for reform of mathematics education is the changing nature of mathematics itself. Researchers in applied mathematics and statistics in the twentieth century have developed whole new branches of mathematics and invented powerful new tools for problem solving. We give just a few examples: Game theory and optimization methods are now used to improve both service and profitability of airlines and express couriers; mathematical modeling has become indispensable to designers of automobiles and aircraft; advances in differential equations and dynamical systems theory have improved robotic controllers and weather forecasting; probability theory is essential to the design of telephone networks and the testing of new drugs. The revolutionary developments in mathematical sciences that have occurred in this century remain unappreciated by many citizens, and are rarely taught in the classroom. There is no other major subject of study in high school for which the present curriculum

The technological challenge

The current curriculum reflects little of the modern development and practice of mathematics

<sup>&</sup>lt;sup>1</sup> Everybody Counts, page 6, National Academy of Sciences, USA (1989).

reflects so little of its modern development and practice. A priority of mathematics education reform must be to bring the curriculum in line with current and future use of mathematics in the real world.

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Canadian leadership in new information technologies provides exciting opportunities for enhancement of education in Ontario. For example, Ethel Thayer showed the workshop how Canada's Schoolnet delivers up-to-the-minute information into the classroom, and enables far-flung communities to share ideas and work together via the Internet. This Canadian initiative has become a model for web-based learning communities around the world. Another Canadian initiative, Waterloo Maple Inc., produces software for performing advanced mathematical operations of algebra, calculus and geometry on a computer far faster than ever possible by hand, thus rendering feasible calculations which were unthinkable a generation ago. Maple software is very highly regarded by scientists around the world, but its full potential for mathematics education has yet to be exploited. Contrary to popular perception, computers have not rendered mathematics obsolete: computers have opened the door to ways of doing mathematics more powerfully than ever before. As we enter the new millennium, we expect a flood of new computer-based learning tools which will go far beyond even those demonstrated at this workshop. They will provide opportunities for more individualized learning and explorations, which can make mathematics more stimulating and relevant for students.

Our society accepts low performance standards in mathematics Research on learning dispels the myth that skill in mathematics is an innate ability possessed only by a few. Mathematics can be learned by everyone. Compared with other countries, our society accepts standards of performance in mathematics that are too low. Mathematics educators would like to eradicate the common cocktail party comment "I was never any good at math". This attitude is not justified. Students *can* succeed in mathematics. If more is expected, more will be achieved.

While fully recognizing the importance of mathematics education for the growth of our knowledge-based industries, the Workshop participants had even greater concern for the mathematics education of the majority of high school students who are not headed for research or high-tech careers. Mathematical literacy (or "numeracy") has become every bit as essential as language skills for survival in our society. As we pass from the industrial age to the information age, many traditional jobs are disappearing, and the workplace is changing to a digital world requiring higher levels of numeracy and problem-solving skills. Nineteenth century "shopkeeper arithmetic" no longer suffices in this world of barcodes and digital cash. Ontario must raise the general level of numeracy in the population. We cannot afford the continued waste, in both human and economic terms, of tragically high youth unemployment rates. Numeracy involves more than familiarity with numbers; tomorrow's citizens must deal confidently with many mathematical concepts, for example: chance, graphs, logic and the dynamics of change. A citizen lacking in numeracy can hardly make valid judgements on issues ranging from interpretation of opinion surveys to predictions of global climate change. Recognizing the overriding importance of mathematics education for a successful and informed citizenry, the Workshop participants chose not to focus on the needs of the technological elite. The model of high school mathematics education presented in this White Paper is intended for *everyone*. If we can raise the quality of mathematics education for all students, then more will be enabled to advance to higher levels.

The outcome of all the presentations and intensive working groups at the Workshop is this Fields-Nortel White Paper, respectfully submitted to the Ontario Ministry of Education and Training and offered for wide circulation and discussion. The Workshop participants sincerely hope that it will be useful in the process of reform of the secondary school mathematics curriculum, and will contribute to giving our students what they deserve: the very best in mathematics education, to prepare them for life in the twenty-first century.

## Background

This Workshop built on the strong base of experience in previous studies of mathematics education brought to the table by the participants. It marks the first collaboration involving the Fields Institute Mathematics Education Forum with the Nortel Education Interaction Department. Participants were knowledgeable about the recent contributions and recommendations of the Conference Board

 Numeracy is as essential as language skills for all students of Canada *Employability Skills Profile* and *Science Literacy for the World of Work*, Ontario Association for Mathematics Education (OAME) *Submission to Expert Panel*, Ontario Mathematics Coordinators Association (OMCA) *Submission to Expert Panel*, and the Ontario MET *Background Paper on Mathematics Education* (G. Roulet, 1997). The group endorses the process so ably begun by the Conference Board and the Mathematics Associations, the pedagogical framework of the MET Background Paper, and the 1985 Ministry document *Mathematics: Intermediate and Senior Divisions*. The Workshop also considered important work on mathematics reform outside Ontario, such as the provincial mathematics consortia in Western Canada and Atlantic Canada, the recent reforms in England, and the NCTM Standards in the USA. Throughout its deliberations, the Workshop recognized the primary importance of the individual classroom teacher in the delivery of quality education.

## An Integrated Model for Mathematics Education

The Workshop participants agreed that the first step in charting a course for education reform is to develop a *profile of the mathematics learner*. The ideal mathematics learner will graduate as a creative, thinking, learning citizen with the knowledge and skills to compete and succeed in a global economy. This profile of the mathematics learner drives our recommendations for curriculum content, delivery, assessment and resource requirements. To achieve the profile of the mathematics learner, it will be necessary to change fundamentally the way we implement mathematics education. The traditional transmission model, with students as passive learners, prevents students from acquiring the attitudes, knowledge and skills necessary for success in the twentyfirst century. To be successful students must be engaged in the learning process, use technology when appropriate, and learn how to learn. They must acquire critical and creative thinking skills, abstraction and modeling capabilities earlier in their education. Mathematical concepts should be introduced within a context of inquiry and problem solving and should be organized around major concepts and themes. To measure the success and drive the changes, assessment must reflect the profile of the mathematics learner and the methods of instruction.

These fundamental components come together in an *integrated model for mathematics education*, which is represented as a wheel with the learner at the hub (see Figure 1). The profile of the student relies on the integration of six themes: curriculum, implementa

A new model of mathematics education tion plan, integration of technology, professional development, assessment and resources. These six themes are like the spokes of a wheel with the learner at the center: each theme is essential to the whole. For successful reform, all six themes must be addressed. The rim of the wheel represents the interface between education and society; here issues of accountability and responsibilities to our society are addressed. This integrated wheel model provides the necessary framework for reform of mathematics education.

## **Profile of the Mathematics Learner**

The first step in mathematics education reform is a consensus on the profile of the mathematics learner. These characteristics and behaviors provide us with a vision of a secondary school mathematics student and enable us to make statements about a progressive mathematics curriculum. We propose that the community recognize a new image of the mathematics student and we are providing a suggested profile.

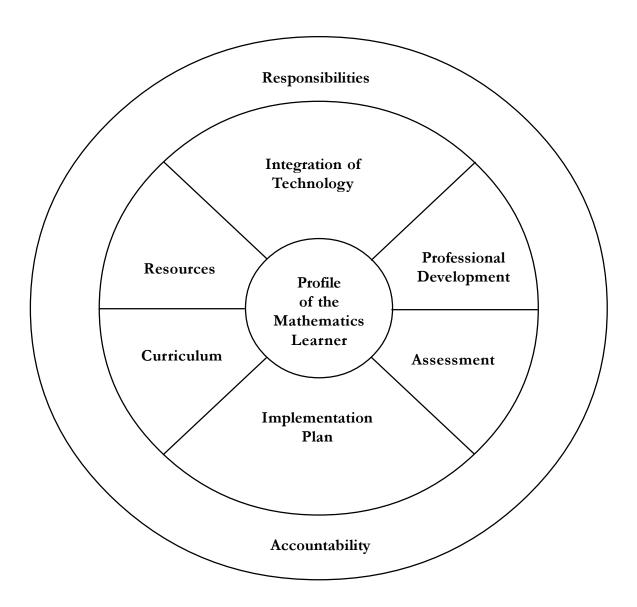
In this profile, we do not distinguish between students bound for post-secondary education and those entering the workforce directly from high school. Recent studies have shown that Ontario has the highest proportion in the world of young adults in post-secondary education; however, many of these are mature students who do not go directly from high school. In a changing workplace, all graduates will need the skills to adapt to change and become lifelong learners. All graduates should attain this basic profile of the mathematics learner.

This profile of the successful mathematics learner is defined in the following table, with the essential attributes grouped under four headings: **Attitudes and Behavior, Skills, Critical Thinking** and **Creative Thinking**. While there is some overlap among these groups, all are seen as essential components of our profile of the mathematics learner for the twenty-first century.

For successful reform, all six themes must be addressed

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 Recognizing the changing image of the mathematics student Figure 1. Integrated Model for Mathematics Education



## **Profile of the Mathematics Learner**

Attitudes and Behavior	Skills
<ul> <li>confident in ability to learn mathematics</li> <li>able to adapt to changing situations</li> <li>shows initiative</li> <li>values and embraces mathematics</li> <li>comfortable with technology and uses it as part of the learning process</li> <li>broad interdisciplinary understanding</li> <li>flexible, resourceful thinker</li> <li>appreciates the beauty of mathematics</li> <li>appreciates the value of mathematics in society</li> <li>aware of career opportunities for mathematically-educated students</li> <li>accepts responsibility for being a shareholder in the learning process</li> <li>works in a team with a variety of members</li> <li>understands mathematics as an evolving, modern science</li> <li>appreciates the historical development of mathematical ideas</li> </ul>	<ul> <li>uses information, communication and computation technologies effectively</li> <li>capable of algorithmic thinking</li> <li>recognizes contexts where the use of mathematics is appropriate</li> <li>has sense of number, ability to estimate magnitude and know when it matters</li> <li>represents mathematical ideas in various ways (algebraic, graphic, symbolic)</li> <li>verifies predictions by using different approaches, including mathematical proof</li> <li>communicates ideas through appropriate mathematical language</li> <li>able to recognize phenomena that can be modeled, and to construct models</li> <li>performs computations within contexts and appraises reasonableness of results</li> <li>formulates hypotheses through extrapolation and interpretation</li> </ul>
Critical Thinking	Creativity
<ul> <li>develops and follows logical arguments</li> <li>searches for answers, reflects on actions, reflects on thoughts of others</li> <li>uses mathematical thinking to provide evidence and justification for a point of view</li> <li>critically analyses and compares information from a variety of sources</li> <li>capable of both abstraction and synthesis</li> <li>listens critically to others</li> <li>understands and employs mathematical proofs</li> <li>poses questions and finds solutions through analysis</li> <li>presents solutions clearly and convincingly</li> <li>uses logical discourse, supported argument and effective communication</li> <li>sees the big picture</li> </ul>	<ul> <li>formulates problems</li> <li>generalizes from local to global problems</li> <li>creates own knowledge and tools</li> <li>observes patterns and evaluates pattern relationships by reference to known situations</li> <li>capable of divergent and lateral thinking</li> <li>explores and investigates with mathematical tools</li> <li>introduces new ideas and innovative ways of solving problems</li> <li>uses mathematical intuition</li> <li>makes conjectures based on given data and information</li> <li>transfers skills to new, real-life situations</li> <li>reformulates an unfamiliar problem in a more familiar form</li> </ul>

## **Recommendations**

Building from this profile of the mathematics learner, the Workshop participants developed recommendations for mathematics education reform along six ( Achieving themes, radiating outward from the learner at the center. Achievement of this profile relies on the integration of all six themes: Curriculum, Implementation Plan, Integration of Technology, Professional Development, Assessment and Resources.

the profile

#### Curriculum 1.

As well as being one of the great human intellectual achievements in its own right, mathematics is of fundamental importance in business and industry, and is an essential life skill in society. The curriculum must reflect this multi-faceted nature of mathematics. Mathematics education is indispensable in the information age, even for those not going on to high-tech careers. Therefore, for all students, we advocate a curriculum in which:

- Mathematics is studied in every grade from 9 to 12, and a minimum of four mathematics credits is required for graduation.
- The mathematics curriculum is broadened to a *mathematical sciences* curriculum, incorporating twentieth century advances in mathematical sciences, in areas such as statistics, modeling, optimization, data analysis and computing.
- Mathematics is learned in a context-rich, problems-based environment, showing the relevance of mathematics to other disciplines.
- Transferable skills such as abstract thinking and logical reasoning are developed early in the curriculum.
- The study of mathematics includes mathematical modeling in a variety of fields, such as business finances, engineering, health, environment and statistics.
- Mathematical curiosity and creativity are encouraged at all levels.
- Students have opportunities to pose their own problems and solve them through investigations and project-based learning.
- Skills to promote life long learning are incorporated.
- Mathematical intuition is both valued and developed early.
- The mathematics curriculum is stimulating, relevant and rewarding, to encourage students to stay in mathematics.
- Opportunities are available for Co-op experience.

### 2. Implementation Plan

The best-designed curriculum on paper is worthless without an effective implementation plan. Successful implementation will require a major commitment on the part of all parties involved: the Ministry, school boards and officials, teachers, parents, students and the private sector. This investment will pay off in a better future for Ontario citizens. An implementation plan is required that:

- Makes a long-term commitment to see these changes through to completion.
- Recognizes the need for continuous review and improvement in the delivery of mathematics education, as is natural and necessary for quality to be maintained.
- Is accountable to the community by demonstrating success through assessment.
- Addresses all components of the system, including students' needs, ongoing professional development for teachers, pre-service training and public accountability.
- Includes partnerships with business and the community to create curriculum units, to build relationships between students and business people and give a sense of how mathematics is used in society today.
- Insures the intended curriculum is valued, respected, and followed in that light.
- Includes co-operative work experience and career guidance for students.
- Provides a realistic timetable with milestones for curriculum adoption and provision of resources and training.

### 3. Integration of Technology

Today's rapidly evolving information technologies are transforming the workplace, but their influence is only beginning to be felt in many schools. In the classrooms of tomorrow, computers must become as commonplace as blackboards and paper are today. The Workshop viewed the integration of information technologies into the curriculum as a priority, not only to prepare students for work, but more importantly for their potential to enhance the learning process. Computer-based technologies in the classroom must be enabling, not intimidating. Computing technologies enable three distinct modes of learning activity which may be identified as communication, information and calculation. These modes are easily illustrated by three contemporary examples of uses of computers: *communication* via e-mail or discussion groups, *information* 

from databases or web-browsers, and *calculation* with graphing calculators or Maple. The first two will affect every subject area, while the third has special relevance to the mathematical sciences. All three modes are developing so rapidly at this time that specific recommendations of particular products or standards would be inappropriate. The examples below are for illustration only and will change with time. However, it is clear that students are enabled to reach a deeper level of understanding in a broader range of mathematics through the appropriate use of these technologies. In the classroom of the future, texts, software, videos, computer networks and databases will blend into a hybrid educational resource. This new curriculum will be the first to fully integrate the new information technologies in the classroom. To this end, we recommend that policies be adopted which will:

- Require the use of all three enabling modes of computing technology (communication, information and calculation) in mathematics courses at all levels.
- Ensure that every student has access to up-to-date technologies at all times (i.e. on demand, not just by appointment).
- Ensure that every classroom has access to information technology resources on the Internet (such as Canada's Schoolnet).
- Ensure that every student confidently employs computing technologies that are common in the workplace (for example spreadsheets, web browsers and graphing tools).
- Ensure that every student is able to explore mathematical concepts through computational hardware and software (for example Maple, Geometer's Sketchpad, modeling tools and graphing calculators).
- Guarantee access to a library of the best available computer based learning software packages, just as textbooks and reference books are available to students today.

### 4. Professional Development

Given the pivotal role of the classroom teacher in the delivery of quality education, implementation of mathematics education reform is impossible without substantial investment in professional skill development for all teachers. Many of those now teaching mathematics were not trained as mathematics teachers, and of those who were, much of their training does not relate to the new curriculum. Demographic studies show that more than half of those now teaching in Ontario schools will retire or leave the profession before the end of the first decade of the twenty-first century. Therefore, our faculties of education will be called upon to train increasing numbers of new teachers. This presents an opportunity to inject change into the system. Teachers must remain competent and comfortable with rapidly changing technologies, new advances in mathematics and new ways of using mathematics, throughout their careers. These needs extend also to school boards, administrators, ministry officials, faculty of education professors and the community at large. Therefore, we recommend the following:

- Pre-service teacher training programs must include more study of mathematics, with more emphasis on the use of technology and on research and learning in mathematics.
- Accessible in-service training must be an integral part of all curriculum changes.
- Ontario MET will adopt a requirement that teachers of grades 7 to 12 mathematics be subject specialists, and provide a timetable for implementation of this requirement.
- Teachers must have direct access to up-to-date technology at all times.
- There must be support and direction for all new initiatives, from the ministry, board, and school administrative levels.
- Teacher-initiated innovations should be encouraged and rewarded.
- Curriculum changes will be integrated with teacher accreditation and evaluation by the Ontario College of Teachers.
- Action will be taken to recruit a higher proportion of mathematics majors from universities into faculty of education programs and the teaching profession.
- University-level mathematics courses will be available that are more relevant to public school teaching; for example, showing the historical and cultural as well as technological contexts of mathematical ideas.
- There must be recognition that curriculum change is an ongoing process and that professional development will be available to all teachers throughout their careers.

### 5. Assessment

It is recognized that assessment not only provides feedback and motivation to the learner, it also helps to define and measure the effectiveness of the curriculum. Thus assessment is a multifaceted tool, and must be used appropriately. For example, assessment that only tested rote mechanical skills would be an impediment to the implementation of a high quality mathematics curriculum. When used appropriately, assessment instruments provide the feedback that is essential to maintain progress. The assessment process itself must be carefully monitored, to ensure that:

- It provides a complete portrait of the student through a wide variety of assessments.
- Assessment reflects the goals of the curriculum, the profile of the mathematics learner, and the methods of instruction.
- Assessment is linked to a continuous improvement model of learning and the school system through a feedback cycle.
- Individual student assessment results are made available to and are understood by the student, and by the parents where appropriate.

### 6. Resources

We recognize mathematics education as a resource which fuels economic growth and contributes to a better quality of life for all citizens in the information age. The changes required for this new vision of mathematics education in Ontario represent a significant investment of time and money by all parties concerned. However, the cost of this reform is insignificant compared to the consequences of inaction. To implement successfully this model of mathematics education for the twenty-first century, targeted funding must be provided to ensure that:

• Enabling computing technologies (for communication, information and calculation) are available to every student and teacher at all times.

- Materials to implement the curriculum successfully, including books, periodicals, teacher manuals, manipulatives and software are available.
- Appropriate human resources for training and implementation of these initiatives are allocated, both to university faculties of education for preservice training and to boards and teachers for inservice development.
- Educators have time for and access to professional development opportunities.
- Change is encouraged by rewarding innovation.
- Access to business and education partners/experts is facilitated.
- Teachers are provided time to be innovative and creative.

### Summary

This White Paper presents a vision for mathematics education in Ontario, resulting from a sharing of ideas from industry, business and education sectors in working group discussions at the Fields-Nortel Workshop on Mathematics Education for the 21<sup>st</sup> Century. The core of this vision is a profile of the mathematics learner as a creative, thinking, learning citizen. Based on this profile, the White Paper presents recommendations to the Ontario Ministry of Education and Training that integrate six themes: curriculum, implementation plan, integration of technology, professional development, assessment and resources.

The goal of this White Paper is to help bring about the changes that will enable Ontario secondary school mathematics students develop into successful graduates equipped to meet the challenges of life and work in the twenty-first century. Through the achievement of this process, mathematics education in Ontario can be the best in the world.

## Appendix

#### Workshop Format and Program

The Fields-Nortel Workshop on Mathematics Education for the 21<sup>st</sup> Century, held July 15-19, 1997, provided a forum for discussion of the global and technological challenges facing secondary mathematics education in the twenty-first century. The Workshop brought together a group of highly committed mathematics educators and mathematicians with industry representatives, from Ontario and abroad, to explore these themes. The thirty-six participants were drawn from universities, colleges and schools, industry and government, and included members of eight national and provincial professional mathematics organizations.

Approximately equal time was allocated to informative and provacative presentations on the one hand, and to discussion groups on the other. The number of presentations was limited to allow time to explore issues in more depth. The first two days (July 15-16) were devoted to educational technology issues, the current state of the art and projections of the future impact of these technologies. The emphasis was not on learning how to use these technologies; however, a hands-on lab was provided one evening for those desiring this experience. The next two days (July 17-19) explored the educational implications of changes taking place in the global economy and the needs of Canadian business, industry and society for mathematically-skilled employees. Experiences and success in mathematics education in other provinces and countries were considered. The morning of July 19 was devoted to the first draft of this White Paper.

### Speakers and Titles (in order of presentation)

Tony Marsh, CEO, Canadian Microelectronics Corp. Mathematics and Science Literacy for the World of Work

Vince Delisi, The Country Day School, King, ON *The Technological Challenge* 

Robert Corless, University of Western Ontario How Technology Affects the Curriculum in Mathematics Education

Sarah Inkpen, Seneca College Virtual Reality and Mathematics Education for the 21<sup>st</sup> Century

Barbara Morrison, Math. Supervisor, Calgary SSB, and ITP-Nelson Computer Guided Learning for Grade Nine Mathematics Margaret Sinclair, Francis Libermann C.H.S. Integrating Computers in the Mathematics Classroom

Ethel Thayer, Consultant, Canada's Schoolnet Learning Without Boundaries

Linda Hardman, V.P. Educational Technology, Prentice Hall Secondary Mathlab Toolkit: Using Integrated Math Utilities in a Student-Centered Classroom

Earl Woodruff, OISE/UT Knowledge as Object: Technological Supports for a Discursive Approach to Mathematics Education

Richard Stallwood, Norlington School, London, England Curriculum Reform in England

Barbara Morrison, Math. Supervisor Calgary SSB, and ITP-Nelson *The Alberta Mathematics Curriculum* 

Ron Lancaster, St. Mildred's Lightbourne School, Oakville *The NCTM Standards* 

Bill Higginson. MSTE, Queen's University *Tomorrow's Classroom* 

Marcia Cunningham and John Gregory, Cunningham Gregory and Company *Calculator Connections* 

Robert Long, Nortel, Education Interaction School to Work Challenges, A Nortel Perspective

Mary Lou Kestell, Mathematics Coordinator, Wentworth County Board of Education Report on the Findings of the Ontario Ministry of Education and Training's Expert Panel on Mathematics Education

Ron S. Dembo, President and CEO, Algorithmics, Inc. *Mathematics and Financial Risk Management* 

#### List of Participants and Affiliations

Karen Allan, Education Officer, Ontario Ministry of Education and Training Lynda Colgan, Curriculum Coordinator of Mathematics, Scarborough Board of Education Robert Corless, Department of Applied Mathematics, University of Western Ontario Marcia Cunningham, Cunningham, Gregory and Company Shirley Dalrymple, Head of Mathematics, Dr. Denison Secondary School Don Dawson, Director, The Fields Institute for Research in Mathematical Sciences Stan Devitt, Waterloo Maple, Inc. Karen Eck, Facilitator Harry Giles, Principal, The Giles School Paula Goepfert, Director of Publishing, Addison-Wesley Publishers John Gregory, Cunningham, Gregory and Company Gila Hanna, Ontario Institute for Studies in Education/University of Toronto Linda Hardman, Vice-President, Educational Technology, Prentice-Hall Publishers William Higginson, Faculty of Education, Queen's University Jacqueline Hill, Head of Mathematics, Sinclair Secondary School Sarah Inkpen, Professor, Seneca College Paul Jay, Vice-President, Northern Telecom Ltd-Nortel Mary Lou Kestell, Curriculum Coordinator, Wentworth County Board of Education Robert Kirk, Prentice Hall Publishers Ron Lancaster, St. Mildred's Lightbourne School, Oakville William Langford, Deputy Director, The Fields Institute for Research in Mathematical Sciences and Department of Mathematics, University of Guelph Robert Long, Manager, Education Interaction Northern Telecom Ltd.-Nortel Dragana Martinovic, Humber College Douglas McDougall, Ontario Institute for Studies in Education/University of Toronto Barbara Morrison, ITP-Nelson Eithne Murray, Waterloo Maple Inc. Janice Nixon, McGraw-Hill Ryerson Richard Parker, IOTECH Maurice Poirer, National Coordinator, YES Camps Pat Rogers, Faculty of Education, York University Caroline Rosenbloom, Program Leader, North York Board of Education Geoffrey Roulet, Faculty of Education, Queen's University Ron Scoins, Associate Dean of Mathematics, University of Waterloo Margaret Sinclair, Head of Mathematics, Francis Liberman Central High School Richard Stallwood, Head of Mathematics, Norlington School (England) Michael Tabor, Futurist, e-Commerce Inc.

Ethel Thayer, Education Consultant, Industry Canada's SchoolNet Margaret Warren, Program Coordinator, Peel Board of Education Earl Woodruff, Ontario Institute for Studies in Education/University of Toronto Dave Zimmer, ITP-Nelson

#### Program Committee

Judy Crompton, President, Ontario Association for Mathematics Education Gary Flewelling, Queen's MSTE Group, Consultant William Langford, Deputy Director, The Fields Institute for Research in Mathematical Sciences and Department of Mathematics, University of Guelph Robert Long, Manager, Education Interaction Northern Telecom Ltd.-Nortel Douglas McDougall, Ontario Institute for Studies in Education/University of Toronto Ron Scoins, Associate Dean of Mathematics, University of Waterloo Mike Wierzba, President, Ontario Mathematics Coordinators Association

### Co-organizers of the Workshop and Editors of the White Paper

William Langford Deputy Director, The Fields Institute for Research in Mathematical Sciences and Department of Mathematics, University of Guelph Robert Long Manager, Education Interaction Northern Telecom Ltd.-Nortel Douglas McDougall Assistant Professor, Ontario Institute for Studies in Education/University of Toronto

#### Workshop Sponsors

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