Mathematical Romance and the Professional Development of Elementary Mathematics Teachers[1]

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Abstract: What elementary mathematics teachers need most of all in professional development sessions are: (1) Romantic experiences with mathematics, where they fall in love with mathematical thinking; (2) the incentive to try similar experiences for their students; and (3) opportunities to meet and discuss their experiences with other teachers.

Issues in elementary mathematics education

The following are key issues that need to be addressed in professional development sessions for elementary mathematics teachers (McGowen & Davis 2001a, 2001b; Stipek et al 2001):

- It is not uncommon for elementary teachers to have negative aesthetic associations with mathematics. Many openly and sometimes proudly admit that they do not like mathematics.
- Despite recent curriculum and assessment reform, many elementary teachers conceive mathematics as learning procedures and getting right answers. Curriculum documents are like inkblots where teachers see their personal conceptions of mathematics.
- Many elementary teachers do not have a background in mathematics and do not feel confident about their own mathematical ability.

Teachers need critical experiences

The above issues relate to deep-seated beliefs about mathematics and can only change through critical professional development experiences. Critical experiences may be defined as those experiences of epiphany that cause us to reflect on our knowledge and beliefs and see mathematics and mathematics teaching in new light (Gadanidis, Hoogland & Hill 2002b). When such moments of epiphany occur for teachers, mathematics education artifacts – such as curriculum documents, classroom experiences, ideas from professional development workshops, journal articles, and so forth – they can be thought of as inkblots where the image appears to shift and something new is seen, something that was not apparent before. As one teacher in one of our studies commented "I feel like [this experience] has cleaned my spectacles". Similar findings are reported by McGowen & Davis (2001) where teachers noted that course experiences "opened [my] eyes to a new outlook on mathematics" (444).

I believe the core component of a critical experience is a romantic mathematical experience – an intellectually romantic experience. I also believe that teachers are drawn to such experiences.

In one of our research projects we are looking at pre-service elementary teachers' reactions to interviews with mathematicians where mathematicians express affection towards mathematics (Labelle 2000; Sinclair 2000; Yeats 1999). It is interesting that almost all of the teachers in the study – most of which entered the faculty of education experience with very negative attitudes

towards mathematics – expressed positive aesthetic reactions to the mathematicians' views of mathematics and they shared personal examples of mathematics experiences that they found aesthetically pleasing, as reflected in the two excerpts below.

> My initial response to the question regarding the beauty of mathematics was one of disbelief! I honestly never considered such an adjective as applicable to the subject of math. Yet with a little introspection I can remember …
>
> After reading the interviews with the mathematicians, particularly Nathalie Sinclair's, I felt a creeping desire to tackle math again. The language that she used to describe her love of math was inspirational, as well as, poetic, appealing to my senses, and the possibility of actually loving math again. ... Maybe it is possible, after all, to enjoy math again. We'll see …

Romantic experiences with mathematics – An example

The typical view of numbers and operations with numbers focuses on students solving questions to get answers.

For example, the following is a typical math question:

1. 4 + 6 = ___

But what would happen if we turned this around so that students were given answers and they had to make up the questions? We can reverse the flow of the above question as follows:

2. ___ + ___ = 10

In terms of practicing addition, both the first and second questions provide this.

However, I have found that both teachers and students enjoy solving the second question much more than the first (Gadanidis 2002).

The second question has a number of advantages:

- It has many solutions.
- It allows students to find creative ways of making ten.
- It allows students to engage at a level that is appropriate to their mathematical knowledge and ability.

The second question also leads to more mathematics.

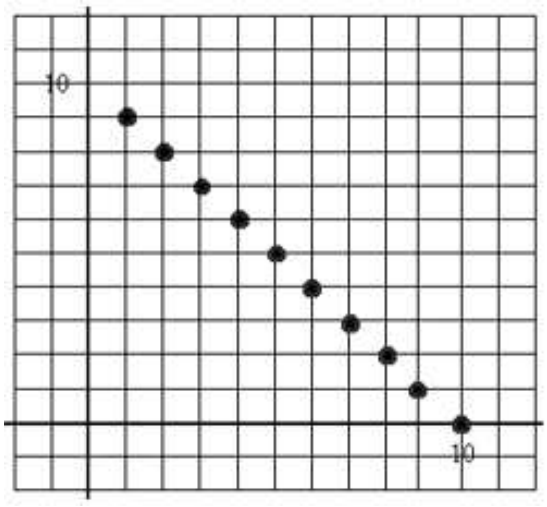
For example, students could explore patterns and in fact use patterning to generate solutions, as shown in the table on the right.

First number	Second number
1	9
2	8
3	7
4	6
5	5
6	4
7	3
8	2
9	1
10	0

We could also consider extending the pattern. What would happen for example if we extended the pattern in the first

column, using the numbers 11, 12, 13 and so forth? What numbers do we need in the second column so that we still get a sum of ten? Young students already know about thermometers and have little difficulty extending the pattern in the second column using negative numbers.

We could also see the pairs of numbers as ordered pairs and graph them on a grid, as shown on the right.



Isn't that interesting! The points form a line.

I wonder if we would also get a similar pattern for a question like ___ + ___ = 5?

How could we control the direction of the line?

It is interesting that teachers who have positive attitudes towards mathematics often have had open-ended mathematics experiences like the one above.

As one teacher commented, "I LOVE math. I always have. I, like many of you, had many problem-solving car trips. I still get excited when I see a license plate that I can make ten with (using any means)." Note the teacher's verbal expression of delight and the use of capital letters to convey her emphasis. If this had been a face-to-face dialogue the teacher might have smiled or raised her voice pleasurably. The example shared by this teacher involves the open-ended mathematical problem of "making ten" using "any means" which contrasts sharply with the more traditional problem of finding the answer, to, say, 5 + 5. The person "making ten" has the opportunity to use her imagination and to find personal, creative ways of looking at mathematically-combined digits on license plates (Gadanidis & Hoogland 2002a, 2002b).

Professional development

In my four years as math coordinator for the Durham District School Board, we structured all our mathematics professional development using a ‘sandwich’ approach:

- Session 1: Mathematics-based experiences for teachers. Ideas to try out in the classroom.
- Between Sessions: Teachers tried ideas from Session 1 in their classroom. They collect samples of student work. They reflect on their teaching.
- Session 2: Sharing of experiences. Analysis/comparison of teaching experiences and samples of student work. More ideas to try out in the classroom (from presenters and from participants).

This structure offered teachers the opportunity and the incentive to try out ideas in their classrooms, to reflect on them and to talk to other teachers about them.

A last word

The focus of mathematics professional development for elementary teachers has to be on mathematics (it often isn’t). In the first few mathematics experiences I provide for teachers in

professional development sessions and in education courses I teach, I ensure that the starting point is mathematics they teach[1]. I also ensure that they experience this mathematics in new ways, where they have opportunities to explore, to wonder, to notice the beauty of mathematics and mathematical thinking.

Notes

[1] This paper is a summary of a presentation made to the Fields Mathematics Education Forum on 21 September 2002.

[2] If you see the example I shared above as having to do with linear functions you may wonder where it would fit in the elementary curriculum. I was recently invited by two grade four teachers to do a lesson on algebra in their classrooms. The topic was finding missing values in simple equations. I started the lesson by giving each student a die that they rolled to find the first missing number in ___ + ___ = 10. Then they calculated the second missing number. They kept going until they exhausted all possibilities. Students recorded results in a table of values, plotted them and were surprised that they lined up. So we tried a few more, with different sums … and the mathematical romance continued. This gave them practice finding missing values, but unlike typical classroom problems, the equations were related. Most teachers and students see algebra as the area of study where you develop skills for finding the 'unknown'. Is this algebra? It's a very small part of what is algebra. Mainly algebra is the study of relationships among quantities that vary. The above activity is a good starting point for teachers and young students to experience the beauty of algebra.

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