Preservice teachers' mathematical knowledge and values – threads and connections

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Brief Framework Summary

- Preservice teachers' knowledge of mathematics is generally poor (Ma, 1999; Ball, 1990; Ambrose, 2004 …)
- The relationship of procedural knowledge and conceptual knowledge is important in studying knowledge of mathematics for teaching (Hill and Ball, 2004; Rittle-Johnson and Kroedinger, 2002, Hiebert, 1999, Lloyd, 1998)
- Beliefs and values play a role in teachers' learning and knowing of mathematics (Ambrose, 2004; Stipek et al, 2001; Foss, 2000)

Teacher Beliefs and the Relationship to Practice

- One's conceptualization of the nature of mathematics as well as mathematical knowledge itself relate to beliefs about teaching (Thompson, 1992)
- Influencing teachers' beliefs may be essential to changing teachers' practice (Stipek et al, 2001) *particularly* beliefs about mathematics itself (Raymond, 1997)
- Teachers' self-reported beliefs relate to student achievement (Ross, McDougall, Hogaboam-Grey and LeSage, 2003)
- Early and continued reflection about mathematics beliefs and practices, beginning in teacher preparation, may be the key to …minimizing inconsistency between beliefs and practice. (Raymond, 1997, p.574)

Types of Mathematical Understanding

- Procedural knowledge is a sequence of actions while conceptual knowledge is rich in relationships (Hiebert, 1992, p. 78), for example the relationship between appropriate physical materials and written symbols. A mathematical idea is understood thoroughly if it is linked to existing networks with stronger or more numerous connections (Ibid. p. 67)
- Procedural knowledge refers to computational skills, while conceptual knowledge refers to the underlying mathematical structure (Eisenhart et al, 1993, p. 9)

Conceptions of Mathematics Learning and Knowing

- Instrumental vs. relational (Skemp, 1978)
- Traditional, mixed, or non-traditional (Raymond, 1997)
- Collection of rules, unified body of knowledge, or problem-driven view (Ernest, 1989)
- Content-focused with procedures, content-focused with [conceptual] understanding, learner-focused based on constructivist view (Kuhs and Ball, 1986, unpublished paper quoted from Ernest, 1989 and Thompson, 1992)

‘Math Reform’

- What kinds of conceptions *of* mathematics are assumed?
- Does it include valuing and knowing about *both* connected ideas as well as procedures?
- Or are the conceptions used mainly to facilitate procedural leaning?
- (Where does the Ontario curriculum fit?)

Measuring Procedural and Conceptual Knowledge

- The *type* of mathematical knowledge held by teachers seems to be more important than courses taken or performance on standard tests (Hill and Ball, 2004; Fennema and Franke, 1992; Foss; 2000)
- The lack of measures of teachers' content knowledge may be a difficulty in determining what features of professional development contribute to teacher learning (Hill and Ball, 2004, p. 330)

Challenges and Importance of Deepening Conceptual Knowledge

- …learners who possess well-practiced rules for manipulating symbols are reluctant to connect the rules with other representations that might give them meaning (Hiebert, 1992, p. 78)
- Some aspects of practice are difficult to measure, such as whether teacher-student discourse probes deep conceptual understanding (Ross et al, 2003, p. 345)

Purpose of Study

- To examine preservice teachers beliefs about the nature of mathematics itself and what is important to these teachers in mathematical learning
- To study changes in preservice teachers' procedural and conceptual knowledge of mathematics
- To look for relationships

Variables

- Procedural knowledge – use of methods which generate correct answers
- Conceptual knowledge – connections to other appropriate mathematical ideas, such as to a suitable diagram with explanation which shows why a solution method is reasonable or makes sense mathematically
- Procedural values – beliefs about the importance of knowing and teaching procedural knowledge
- Conceptual values – beliefs about the importance of knowing and teaching conceptual knowledge

Methodology

- Survey conducted with 145 preservice junior intermediate teachers in a B.Ed. Program
- Survey administered before or after third class as well as to be done at the end of the course
- Methodology influenced by the McMaster survey (Lovric and Kajander, in press)

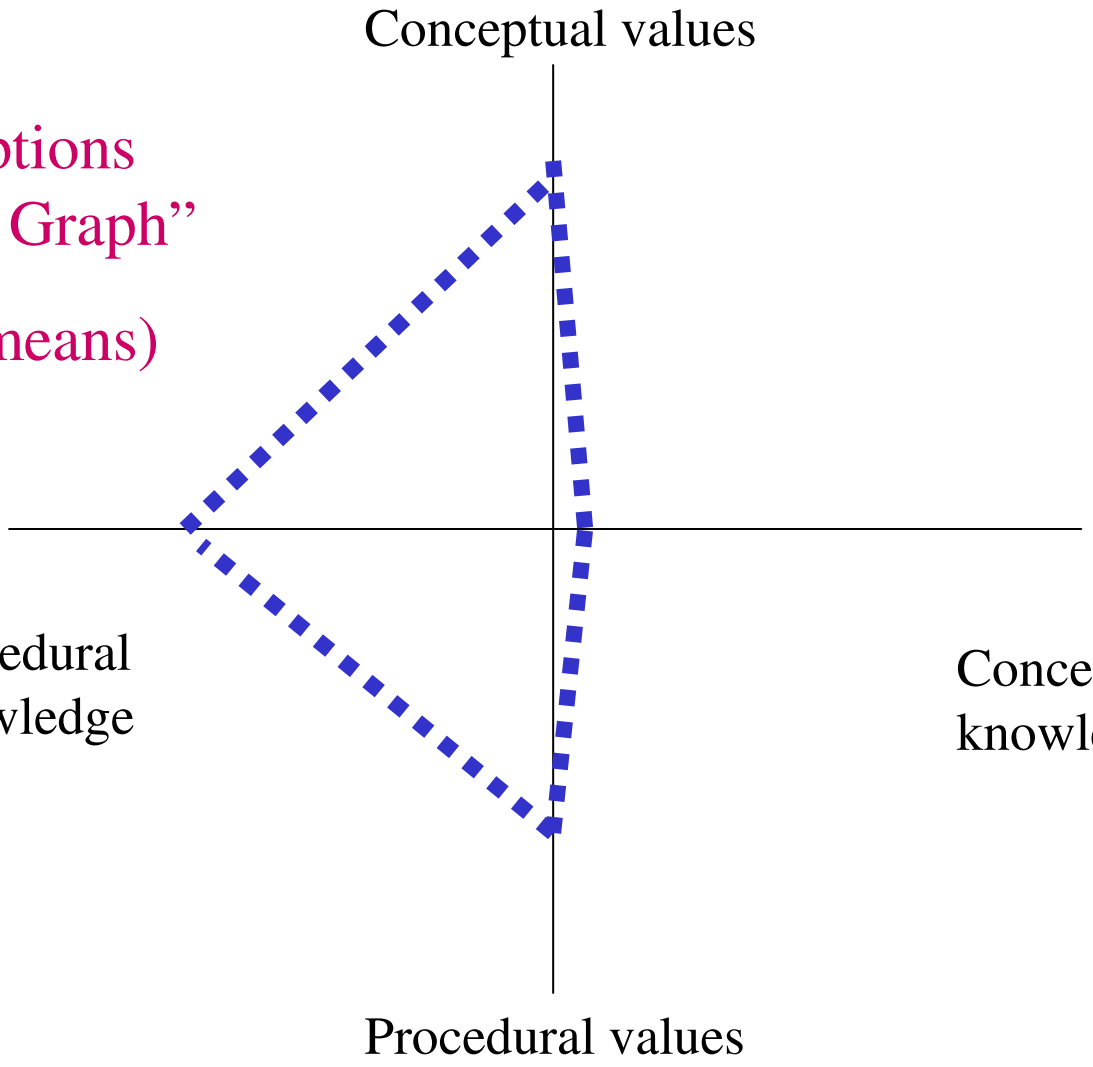
Mean scores from first survey (out of a maximum score of 10)

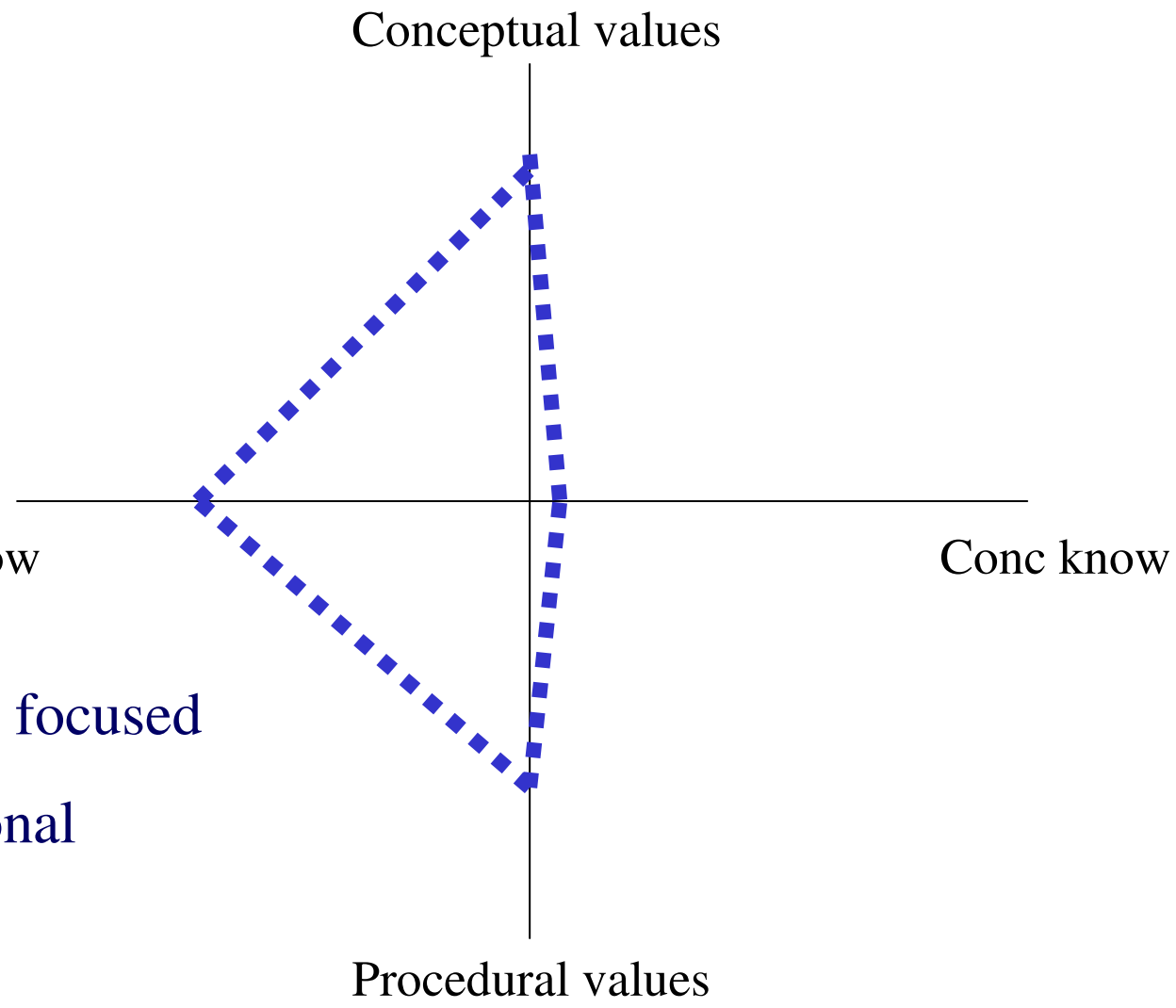
- Procedural knowledge 5.9
  [revising a flaw on the survey would make this 1 – 2 points higher]
- Conceptual knowledge 1.1
- Procedural values 6.3
- Conceptual values 7.3

- Survey done third week of classes. Students mentioned that their conceptual values would not have been so high had the survey been done on the first day of class

"Conceptions of Math Graph"

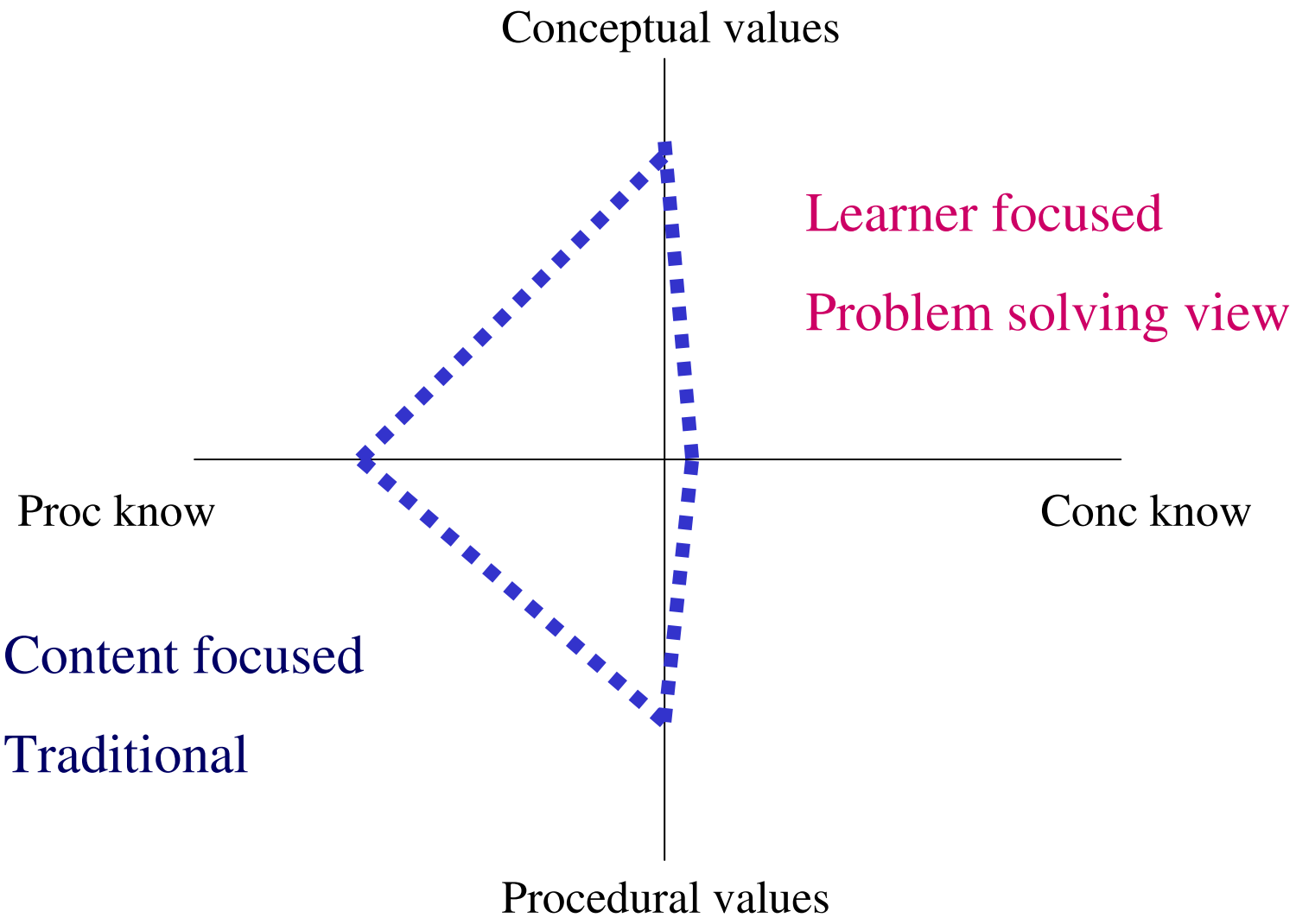
(initial means)

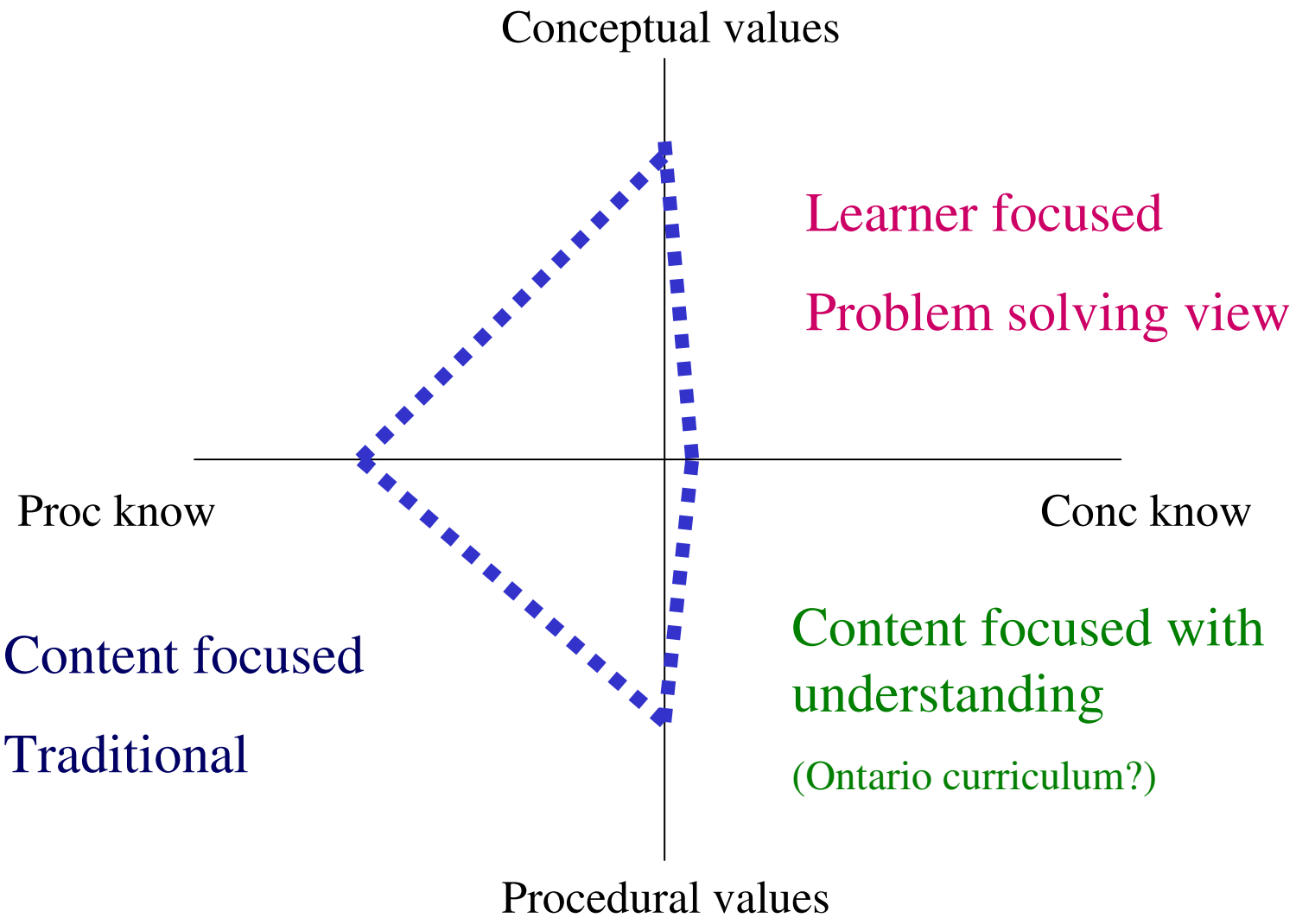


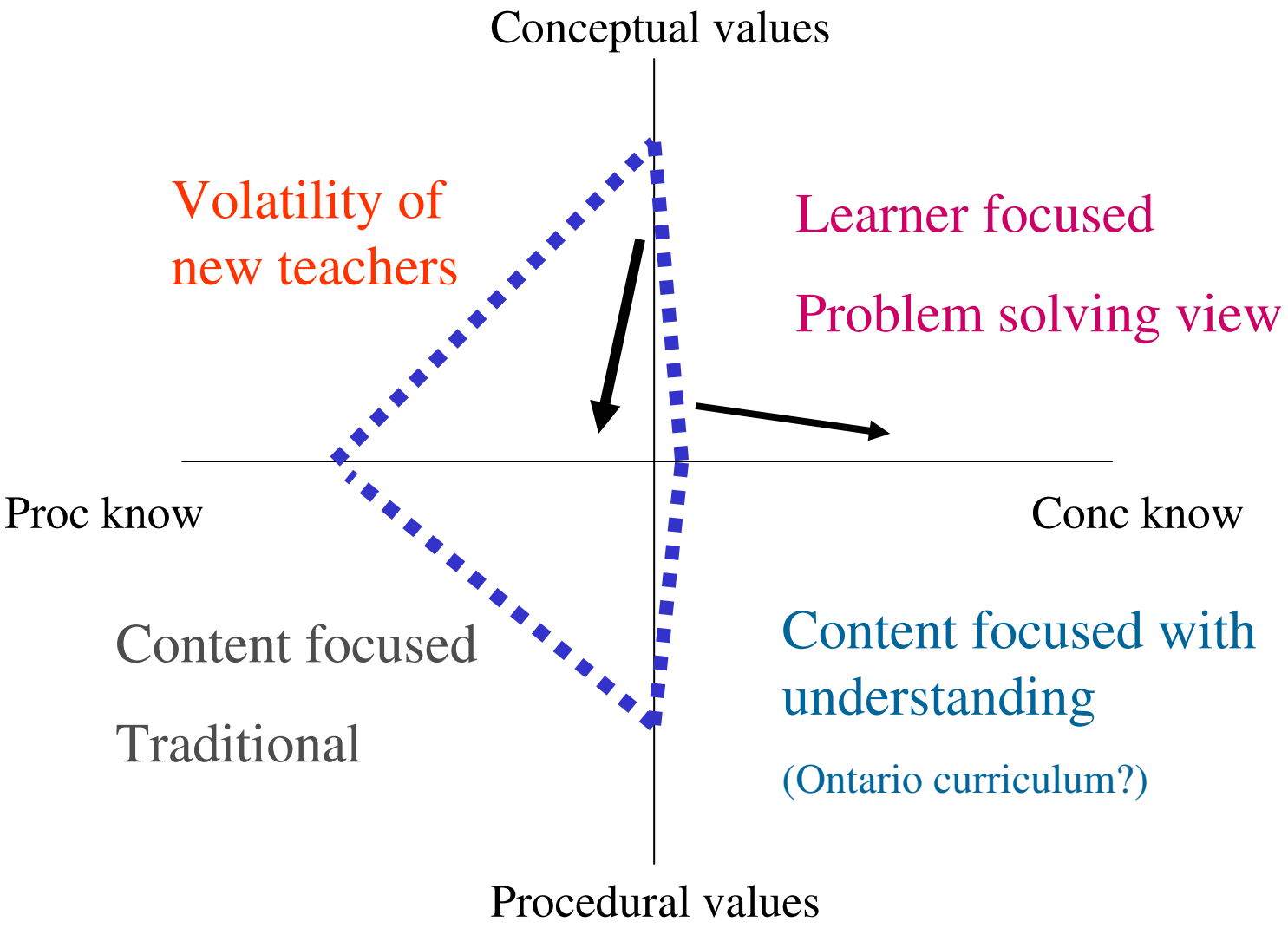


Content focused

Traditional


Conceptual values
Learner focused
Problem solving view
Proc know
Conc know
Content focused
Traditional
Procedural values


Conceptual values
Learner focused
Problem solving view
Proc know
Conc know
Content focused
Traditional
Content focused with understanding
(Ontario curriculum?)
Procedural values


Conceptual values
Volatility of
new teachers
Learner focused
Problem solving view
Proc know
Conc know
Content focused
Traditional
Content focused with
understanding
(Ontario curriculum?)
Procedural values

Final Remarks

- Preservice programs that are short on time need to focus on areas where teachers are weakest
- The current study provides further evidence that the greatest weaknesses are in conceptual understanding
- Beliefs about conceptual learning appear to improve, but their resiliency is unclear
- Preliminary evidence (eg. test results of 80% on items similar to survey) indicate teachers *can* make substantial improvements in conceptual understanding if the learning is focused on needs

In the Future ...

- Further study will be needed to see what happens to the teachers mainly in the "volatile" area:

  - can they continue to improve their conceptual knowledge?

  - can they keep their valuing of conceptual learning high even after experiences in the 'real' classroom?

  - or are these the teachers most likely to revert to valuing traditional conceptions on which to base their mathematics teaching practice?

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