Submission to: the Ontario College of Teachers Review of Teacher Qualifications
From: The Fields Mathematics Education Forum

In this paper we, members of the Fields Mathematics Education Forum, outline our concerns and offer recommendations with respect to four important issues: the need for additional mathematics preparation for elementary teachers that focuses on inquiry learning and the development of conceptual understanding; *the need to develop university mathematics courses that provide extensive opportunities for future secondary mathematics teachers to experience inquiry-based learning; the need for mathematics requirements for special education teachers; and the need for additional opportunities and incentives to encourage practicing teachers to extend their knowledge of mathematics content and mathematics pedagogy*

Our recommendations draw on the results of mathematics education research, and our knowledge of mathematics practices in schools and in the workplace; they represent our sincere desire to improve mathematics education in Ontario. We would welcome the opportunity to discuss them further with the committee.

The Mathematics Education Forum of the Fields Institute for Research in Mathematical Sciences brings together participants from high schools, elementary schools, school boards, faculties of education, mathematics departments in universities and colleges, and the private sector. It holds monthly meetings to discuss issues of mathematics education at all levels.

The Forum members welcome the Ontario College of Teachers' review of teacher qualifications currently underway and would like to be actively involved in the process. We have extensive knowledge of issues related to the teaching and learning of mathematics as well as the preparation and support of teachers. Our members are involved in research on a) preservice and inservice programs, b) attitudes and abilities of elementary and secondary teacher candidates, c) analysis of factors that affect student success in mathematics, d) transitions from elementary to secondary, secondary to university, and secondary to college, and e) the changing role of the mathematics teacher. This submission outlines some of our concerns about the present regulations and offers suggestions for change.

Mathematics education researchers stress that learning mathematics means learning to think mathematically, and that mathematical knowledge is partly a product of the activity, context and culture in which it is developed and used (Saxe, 1995; Schoenfeld, 1994).

The current Ontario curriculum is built on these ideas. It requires that teachers go beyond 'telling', to help students develop understanding.

> This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group. (*Ontario Secondary School Curriculum, Grades 9 and 10*, 1999, p. 4).

> Students in a mathematics class typically demonstrate diversity in the ways they best learn. It is important, therefore, that students have opportunities to learn in a variety of ways – individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. (*Ontario Secondary School Curriculum, Grades 9 and 10*, 1999, p. 6).

In general, changes in an education system, such as implementation of a new curriculum, succeed or fail depending on what teachers choose to do in the classroom (Elmore, Peterson, & McCarthey, 1996; Fullan, 2001). For an innovation to be successful, practicing teachers need to learn new skills—and may need to unlearn beliefs about students or instruction that have dominated their professional careers (Ball, 1988; Darling-Hamilton & McLaughlin, 1996); those entering the profession need renewed programs that will prepare them to meet the demands of a more rigorous curriculum and a diverse student population.

Research shows that the use of an inquiry-oriented approach, which encourages students to explore mathematical ideas, pose questions, and discuss solutions, improves achievement for all learners (Boaler, 2002). The Ontario mathematics curriculum is developed around an inquiry approach; however, teachers who have a weak or procedural understanding of mathematics have difficulty implementing the curriculum effectively. Current Ontario Ministry of Education initiatives in numeracy, and reports such as the Early Math Strategy, Teaching and Learning Mathematics, and Leading Math Success recognize the important role that teachers play in the development of mathematical literacy and are providing opportunities for professional growth. Yet, we know that more needs to be done to sustain the development of sound mathematics teaching and learning. We believe that the current Ontario College of Teachers review offers an unparalleled opportunity to positively impact the mathematics classrooms of the province by improving preservice preparation, strengthening requirements for special education teachers, and providing incentives for practicing teachers to extend their mathematical knowledge.

Preservice – Elementary

Elementary teachers must be able to draw on a deep understanding of fundamental mathematical ideas as well as a knowledge of teaching and learning to perform the diverse tasks required in teaching mathematics (Ball, 2000; Ma, 1999). Research indicates that such tasks

include choosing and managing representations of mathematical ideas, selecting and formulating questions, analyzing and responding to student interpretations, deciding among alternative courses of action, and leading discussions. A sound understanding of mathematics helps teachers to re-cast a mathematics concept in multiple ways and to analyse the thinking of pupils as they grapple with new concepts.

Currently many elementary teachers in Ontario do not possess a deep understanding of the fundamental math concepts that they will teach, and this directly affects their students. As the Early Math Strategy document states:

> Teachers are essential in influencing their students' attitudes towards mathematics, since those teachers who understand and enjoy mathematics generally provide positive experiences for their students. Teachers can facilitate attributes such as curiosity, creativity, enjoyment, flexibility, and perseverance. These characteristics are likely to lead to positive attitudes towards mathematics, and contribute to the students' continued enjoyment, confidence, and success in the subject. However, teachers who have developed a distaste for mathematics because it has eluded them or appeared abstract or confusing are likely to convey these negative feelings about mathematics to their students (Clements et al., in press). (*Early Math Strategy: The Report of the Expert Panel on Early Math in Ontario,* 2003, p. 13)

We can point to several possible causes for the problem:

1. Teacher preservice programs for K-8 include minimal mathematics instruction – typically only one course in mathematics pedagogy, which ranges in length from 24 to 39 hours.
2. Many elementary teachers have less than OAC/Grade 12U mathematics. Although they have "met" topics such as ratio they have limited understanding and are often anxious about teaching these topics. Elementary teacher candidates who have taken a minor or a major in mathematics are prepared for continued study of advanced mathematics, but the content and approach of their programs may not have prepared them to "unpack" fundamental ideas for students.

There are several ways in which these problems could be addressed, including: increased mathematics requirements to enter a teacher education program; increased time for mathematics preparation within the elementary preservice program; or additional courses in mathematics and mathematics pedagogy throughout the elementary teacher's career. In this section we will address the first two. The third will be addressed in the section: *Inservice – K-8 and Special Education*.

In our view, increasing the mathematics requirements to enter a teacher education program (or for certification) would necessitate the development of university courses appropriate for future teachers. Grade 11 and 12 courses do not help students unpack fundamental ideas. In fact,

they often feed the fear and dislike of mathematics – an exception may be Mathematics for Data Management which we consider the most useful of the Grade 12 courses for those planning to teach K-8. Most entry level university mathematics courses are service courses – designed as preparation for future work in mathematics, business, engineering, or science. Such courses do not address the elementary teacher candidate's need for deep understanding of key concepts. They also do not use a pedagogy that helps elementary teacher candidates develop an experience of inquiry- oriented mathematics. Ideally, a course designed for mathematics majors/minors interested in elementary education would involve advanced mathematics but would encourage exploration of the connections among topics related to the elementary curriculum (e.g., geometry, number theory, probability). For those taking degrees in areas other than mathematics, courses could be designed to offer students the chance to re-experience mathematics and its rich connections.

Another alternative is to increase the time for mathematics preparation during the elementary preservice program by adding a mathematics content course. Current programs provide one course (approximately 24 – 39 hours) in mathematics pedagogy; however, that course must address much more than pedagogy. In many cases, prospective elementary teachers have a profound mathematics anxiety coupled with a weak understanding of mathematics. The course must help these teacher candidates develop new attitudes and beliefs towards mathematics, new understandings of mathematics, and sound pedagogical approaches to facilitate mathematics learning. It is difficult to accomplish all of this within the allotted time frame.

Over the past decade in Ontario there has been considerable work to address the problem of inadequate teacher preparation in mathematics. Several university mathematics departments and faculties of education have designed specific courses and programs to provide increased opportunities for prospective or current teacher candidates to extend their mathematics content knowledge and to develop inquiry-oriented practices. Brock, Lakehead, Queen's, Trent, and York universities each offer a course to prospective elementary teachers through the mathematics department. The University of Western Ontario and OISE/UT each offer a course through the faculty of education; and last year the University of Ottawa initiated a mathematics "summer camp" for prospective elementary teachers (see Appendix A for further information on many of the University courses and projects). All of these programs provide opportunities for participants to use diverse strategies in reasoning about mathematical situations, and to make connections between the 'big ideas' of mathematics. Research on several of these programs suggests that they are effective in helping participants extend their content knowledge and develop confidence in

mathematical reasoning (see Appendix A for some examples of research). However, not all elementary teacher candidates have access to such additional programs and because there is no requirement for elementary teacher candidates to take them, these courses and programs touch only a small fraction of those who graduate each year.

We therefore recommend that:

1. The mathematics pedagogy course within the present elementary teacher education programs be extended to approximately 48 hours.
2. That all elementary teachers be required to take the equivalent of one course (i.e., 72 hours) in mathematics, in addition to the pedagogy course offered in the preservice program, whose content and approach is relevant to the needs of the elementary educator, and whose enrollment is low enough to permit explorations and discussion. This course could be part of an extended preservice program, or be taken as a pre-requisite for certification as part of an undergraduate degree, or as a separate course. To meet the very different needs of mathematics majors/minors and those with only Grade 11 mathematics a variety of such courses could be developed by mathematics departments and faculties of education.

Preservice – Secondary

Secondary mathematics teachers are required to have at least two full courses (i.e., 12 credits) in university mathematics. We believe that this minimum number of courses is inadequate preparation for the teaching of secondary mathematics.

In addition, we have concerns about the procedural nature of secondary teachers' mathematics knowledge. We believe that in order to help students develop conceptual understanding of mathematics, secondary teachers need to move towards inquiry-oriented approaches, and we further believe that the best way for them to learn these approaches is through their own mathematics learning experiences. In Ontario, some mathematics departments offer undergraduate courses that engage students in this type of learning (e.g., see Appendix A – York University); however, the requirements for a mathematics teachable are usually satisfied by taking combinations of calculus, linear algebra, and statistics, in classes that are too large for any meaningful inquiry work, and too packed with material to allow time for reflecting on the 'big ideas' of mathematics.

We therefore suggest:

1. That those intending to teach mathematics at the secondary level be required to take at least one full undergraduate course (or equivalent) in mathematics that provides

opportunities to examine topics in mathematics and/or mathematics education using exploratory approaches to better facilitate the development of conceptual understanding.

2. That the minimum number of courses for a secondary mathematics teachable be raised to the equivalent of 3 full mathematics courses.

Inservice – K-8, and Special Education

In the last decade, a consensus has emerged that professional development is most effective when it is long-term, collaborative, school-based, focused on students' learning, and linked to curricula (Hiebert, Gallimore, & Stigler, 2002). More specifically, in the field of mathematics education, numerous studies point to the need for long term professional development opportunities that focus on improved knowledge of school-related mathematics content and mathematics pedagogy (Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Boaler, 2002; Lampert, 2001).

We have the following concerns regarding the opportunities for mathematics inservice support:

1. Although many elementary teachers lack competence in mathematics there is little incentive for them to upgrade their knowledge of mathematics content and pedagogy;
2. Current prerequisites for the intermediate AQ course prevent some Grade 7 and 8 teachers from deepening their knowledge of mathematics pedagogy and their understanding of fundamental mathematics concepts;
3. A Masters degree/diploma with a focus on mathematics or mathematics education receives little recognition as an additional qualification;
4. There is no requirement for special education teachers to have any background in mathematics.

Although many elementary teachers lack competence in mathematics there is no requirement for them to take an AQ in mathematics; if they do decide to upgrade their knowledge of content and pedagogy there are too few courses available. There is an additional problem for intermediate teachers. There are teachers of grades 7 and 8, who did not study mathematics at the undergraduate level, but who became interested in mathematics through the experience of teaching and who would like to take an intermediate specialist qualification. These teachers are frustrated at the current regulations. At present, a Grade 7 or 8 teacher who has no university background in mathematics cannot take the intermediate AQ course in mathematics. They can take a primary AQ in mathematics, but most examples and topics in the course will not be

applicable their work in the classroom. They can take undergraduate courses in mathematics, but mathematics education research indicates that it is inappropriate for them to do so unless the courses are specific to the needs of the mathematics teacher.

Another critical issue is the need to provide incentives for practicing teachers. Throughout the province many dedicated teachers are working on graduate degrees and diplomas - some in the area of mathematics education (see example, Appendix B). They receive little recognition for this work unless they are willing to leave the classroom and take an administrative position. Incentives for those who take additional qualifications in mathematics would provide experienced teachers, who may have reached "top category" by taking AQ courses in an area other than mathematics, significant opportunities for growth.

Statistics show that failure in mathematics marginalizes many students; in fact, student difficulties in the Grade 9 applied mathematics course have been blamed for an increased drop out rate in secondary schools in the province (King, 2003). Some students who have problems in mathematics are eligible for assistance, yet very few special education teachers have a background in mathematics. As a result, the focus in many resource departments is on helping students with procedural questions instead of helping them carry out "sense-making" activities.

We summarize our recommendations for inservice support of elementary mathematics teachers, and special education teachers as follows:

1. Develop additional AQ courses for elementary teachers, and provide some incentive for teachers to take them.
2. Develop an intermediate level AQ path for Grade 7 and 8 teachers who have no math background.
3. Designate a mathematics qualification (e.g., "elementary math practitioner"), which can be attained through earning a Masters of Education with a focus on mathematics education, a Graduate Diploma in Mathematics Education, or other equivalent course of study.
4. Require special education teachers to have completed the mathematics AQ courses for their division.

Inservice – Secondary

Some of our concerns around the teaching of secondary mathematics have been mentioned in the discussion of preservice preparation, but there are several others specific to the professional development of practicing teachers. Namely:

1. Many secondary mathematics teachers have the minimum number of undergraduate courses required to teach mathematics. These teachers have only one option to extend their mathematics content knowledge – take additional undergraduate mathematics courses.
2. There is lack of recognition for those who take a Masters degree/diploma with a focus on mathematics or mathematics education; thus, there is little incentive for teachers to upgrade their knowledge of mathematics pedagogy.

As mentioned earlier: For an innovation to be successful, practicing teachers need to learn new skills—and may need to unlearn beliefs about students instruction, or the nature of mathematics that have dominated their professional careers (Ball, 1988; Darling-Hamilton & McLaughlin, 1996). But teachers need encouragement and incentives to begin the journey towards change. Thus, we recommend the following:

1. Initiate a number of AQ courses to focus on inquiry-based approaches and extended understanding of mathematics concepts in the secondary curriculum. Designate a qualification (e.g., specialist in mathematics pedagogy) in connection with taking a number of these AQ courses.
2. Provide additional recognition for secondary mathematics teachers who complete a Masters degree or diploma with a focus on mathematics or mathematics education, e.g., designate them as mentors to guide groups of first year mathematics teachers.

Conclusion

In this paper we have outlined our concerns and offered recommendations with respect to several important issues: the need for additional mathematics preparation for elementary teachers that focuses on inquiry learning and the development of conceptual understanding; the need to develop university mathematics courses that provide extensive opportunities for future secondary mathematics teachers to experience inquiry-based learning; the need for mathematics requirements for special education teachers; and the need for additional incentives to encourage practicing teachers to extend their knowledge of mathematics content and mathematics pedagogy. Many of these recommendations (such as additional mathematics for future elementary teachers) are in line with practices in other jurisdictions. The recommendations are also supported by mathematics education research, and by the current initiatives at Ontario Universities (see Appendix A), which could be expanded with further support.

We would welcome the opportunity to engage in a further conversation about these critical issues and possible solutions.

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Appendix A

Lakehead University
Education Courses and the *Topics in Mathematics* undergraduate course
Ann Kajander, Faculty of Education

Curriculum and instruction courses in our Faculty of Education are 36 hours for the Bachelor of Education program. The Primary/Junior course includes a content exam which must be passed in order to gain credit for the course, although the exam grade is not included in the course grade. Students who are unsuccessful are allowed to take an extra math tutoring program and re-write the content exam. Junior/Intermediate students do not have a content exam, but work on learning mathematics conceptually during parts of their methods course; they also take two tests during the course on mathematics understanding that do form part of their course grade.

Students in the Concurrent program (B.A./B.Ed.) must take a mathematics or science course during their undergraduate program. If they choose the mathematics course, they generally take *Topics in Mathematics* which is designed for PJ and JI students and offered by the Mathematics Department. Historically however, the faculty member teaching this course has been one with an education background, either cross-appointed from Education or hired as a contract lecturer. The connections between the Mathematics Department and the Faculty of Education are seen as very important in the design and teaching of such a course.

In 1997 the *Topics in Mathematics* course was completely redesigned, based on the new Ontario Elementary curriculum content and pedagogy. The faculty member doing this had a formal background in Mathematics Education (myself). The format of the course has stayed largely the same since then. The most important goals of the course are to change the conception of mathematics among these students including their attitude to mathematics, and deepen their conceptual understanding. The mathematics content is exactly the content of the Ontario Elementary Curriculum and its five strands, but based on the kind of deeper and broader understanding referred to in the research as appropriate for teachers. Reform based learning is modeled as much as possible; students are assigned to math groups at the start of the year, and regularly work in these groups with manipulatives during classes. The main emphasis of the course is on concrete modeling and making connections to support deeper understanding, rather than on procedural fluency. For example, a typical test or assignment question might be *Draw a diagram and use it to explain the operation 3 ½ ÷ ¾. Also give an appropriate example of a situation which could give rise to a problem to solve such as this one.*

A relatively large class size makes returning weekly assignments as well as providing manipulatives to everyone more challenging. The use of designated groups with monthly leaders has greatly helped alleviate these problems. These groups are also used for the class hands-on activities.

The deep connection with the mathematics content is felt to be highly important for these students. As well, attitudes to mathematics have been seen to dramatically improve over the duration of this course. Anecdotal evidence does indicate that this course has a significant impact on students' success in their Methods courses; more formal research to establish this point is currently underway.

OISE/UT

Research on a Graduate/Preservice Course Aimed at Building Math Confidence and Reducing Anxiety

Rina Cohen, Faculty of Education

Based on a pilot study with a group of twelve math-anxious elementary teachers who participated in a series of eight Math Empowerment Workshops (Cohen & Green, 2002), a new Graduate/Preservice course was developed by the author at OISE/UT, titled: *Gaining Confidence In Mathematics: A Holistic Approach to Overcoming Mathematics Anxiety.* The one-term course which has been taught four times, provides a reform-based learning environment in which hands-on math explorations, mental math with invented strategies and group problem solving are combined with journal writing, group reflections, relaxation and guided visualization activities. The initial study took place when the course was taught for the first time in summer 2003 to a class of 13 in-service and 5 preservice teachers. Data consisted of participants' journal entries, math work and final reflection papers, results of three questionnaires, and researchers' field notes. In-depth, qualitative data analysis focusing on the experiences of nine of the most anxious class members was conducted. For a discussion of findings see Cohen & Leung (2004).

Findings indicated that the teachers went through significant positive changes in their math related affect (McLeod; 1992; Ho, 2000) during the course. By reflecting on their negative early math experiences, and on assigned readings on math anxiety such as Ashcraft (2002), teachers gained a better understanding of how their anxiety interfered with their mental functioning during math activities. Such reflection, along with various relaxation and guided visualization activities, helped them learn how to stay more focussed and relaxed while solving problems. The group mental math and problem solving sessions helped unleash the teachers' mathematical creativity as they finally overcame their blocks and started to invent their own strategies. As the course progressed, teachers' sense of self efficacy and confidence grew significantly.

By the end of the course teachers' math anxiety was significantly reduced but not fully overcome yet, as one teacher wrote: "I am still anxious with certain types of problems or math concepts, however, it doesn't prevent me from attempting math questions." Having been exposed to reform-based teaching which emphasized conceptual understanding and mathematical thinking, the teachers' beliefs about and conceptions of math have also started to shift. But probably the deepest change was witnessed in the teachers' self perceptions and self efficacy. As one of them wrote: "In just six weeks, my perception of my math skills has changed from me as a dud to me as a mathematician." While not all of them perceived themselves as mathematicians, they did perceive themselves as capable math learners who don't shy away from math any more.

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Queen's University

Math 010 – Fundamental Concepts in Elementary Mathematics for Teachers

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Prospective elementary school teachers are invited into a genuine mathematical experience - one that appeals even to those who have had a bad experience with mathematics, and one that is accessible even to those most fearful of the subject.

The appeal is created by structuring the course around a mathematics enrichment programme for Grade 7 and 8 students in local schools. This programme is delivered by the students registered in the course. Working in pairs, they visit their assigned school once a week over a ten week period, where they present a one-hour lesson to a group deemed by the school to be ready to benefit from the enrichment.

The enrichment programme alternates between geometry one year and number patterns the next. This allows us to combine grades 7 and 8 in the same class. University class time is used to discuss the mathematics and the pedagogy together. In class we try to emulate the kind of problem-centred environment that is ideal for the elementary school classroom. Knowing that the material must be presented to bright, eager 12 and 13 year olds is the best possible incentive for learning it well, and assimilating it in the form of pedagogical content knowledge (Shulman (1987), Ball (1991)). Other performance measures include weekly assignments and a final exam.

To illustrate course content, the Geometry year centres on the Theorem of Pythagoras, three-dimensional shapes, areas and volumes. Published instructor's notes are used as the text. The mathematics in the course does not make use of any knowledge beyond elementary school, and pushes it in directions that do not allow Math 010 students to rely on formulaic recall of high school mathematics for the solutions, instead encouraging fresh investigation. For the math phobic students in the course, elementary school is typically where attitudes are formed, so mathematics at this level is appropriate for a new experience.

In addition to the enrichment mathematics, more standard elementary mathematics concepts are discussed in the final third of the course. These include integers, fractions, forms of reasoning, and mathematics as a language. The enrolment is capped at 50.

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Philosophy of Mathematics Course for Beginning Teachers

Geoffrey Roulet, Faculty of Education

Researchers (Ball & McDiarmid, 1990; Pajares, 1992) have argued that teachers' beliefs, and in particular their beliefs or conceptions of the nature of a discipline, impact on their choices of instructional approaches. For mathematics, this beliefs-practice link has been described by Ernest (1991) and Thompson (1992) and demonstrated in multiple studies (see Roulet, 1998). Ball and McDiarmid (1990) report that university students studying mathematics, including those planning on teaching, can graduate "without developing a conceptual understanding of the subject matter" (p. 444). This situation appears to hold in Ontario where two-thirds of secondary school mathematics teacher candidates were found to hold very narrow instrumentalist "toolkit" images of mathematics (Roulet, 1995). These in turn, as the beliefs-practices studies show, lead to teacher-centred transmissive modes of instruction; approaches that run counter to those advocated in mathematics education reform literature (NCTM, 2000) and the new Ontario curriculum documents (Ministry of Education, 1997, 1999, 2000, 2003).

Recognizing the situation described above, the Faculty of Education at Queen's University has established an elective Philosophy of Mathematics course, in which teacher candidates can explore the substantive and syntactic structures (Schwab, 1964) of mathematics; knowledge that generally has not been addressed in their undergraduate courses in the discipline. This course is not designed to promote any particular image of mathematics, but encourages beginning teachers to examine their own beliefs concerning the subject and exposes them to a range of possible conceptions of the nature of mathematics: Platonism, Formalism, Instrumentalism, and Constructivism (Ernest, 1989, 1991).

Philosophical issues are often viewed as being remote from school classroom life, but employing K-12 teaching activities that illustrate and challenge various subject images helps bridge the theory-practice gap. For instance the Platonist position is introduced through a technology supported exploration of daylight and trigonometric functions. Class members gather daily sunrise and sunset data from the Web, transfer this to graphing calculators, and construct function models with curves that approximate the plotted hours of daylight data. As a viable high school activity this exercise captures beginning teachers' interest, and serves to raise the question of where does mathematics begin: in the rising and setting of the sun, the observation of yearly patterns, the collection of data, the plotting of a graph, or the construction of a function model. Debate on this question helps class members reflect on and question their personal positions.

Undergraduate mathematics courses include few experiences that suggest that "mathematics is primarily a creative activity, and this calls for imagination, geometric intuition, experimentation, judicious guessing, trial and error, the use of analogies of the vaguest sort, blundering and fumbling" (Kline, 1970, p. 271). To counter this, a major component of the course involves pairs of teacher candidates in extended explorations of open-ended mathematics problems. Participants are asked to record and reflect on their mathematical and emotional journeys – stories that provide opportunities to explore the genesis of mathematical ideas.

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Trent University
Mathematics 280 – Mathematics for Teacher Education
Cathy Bruce, School of Education and Professional Learning

In the School of Education and Professional Learning (Trent University's consecutive Bachelor of Education program), a 36 hour methods course is mandatory for all Primary/Junior teacher candidates. This course in teaching Primary/Junior Mathematics for prospective teachers focuses on developing a community of mathematics learners. Current theories related to teaching and learning mathematics are examined within the context of rich mathematical investigations. A variety of manipulatives and technologies are explored as tools for learning and assessment.

In addition, at Trent, a specific mathematics course has been designed by the department of mathematics in consultation with experts in the field of education to address the mathematics education needs of prospective elementary teachers. Mathematics for Teacher Education (Mathematics 280) is an undergraduate course in mathematics and mathematical thinking. This dynamic, introductory course is designed to increase participant confidence with concepts and procedures normally associated with elementary grade mathematics instruction. It includes an examination of number systems and counting, graphs and networks, probability and statistics, measurement and growth, symmetry, and computer based mathematics. The course is appropriate for those undergraduates who intend to apply to Bachelor of Education programs but have not taken university math courses, and it is particularly suitable for math anxious students.

The demand for Mathematics 280 is very high since many course participants describe it as essential to their preparation for the Bachelor of Education program. Unfortunately we are unable to admit all applicants. We now require that students compose a letter explaining why they want to take the course.

Current research at Trent (conducted by the course coordinator and primary instructor) is examining those features of the methods course and the overall program which contribute to increasing teacher efficacy in mathematics. Students who have previously taken Mathematics 280 or a similar course at another university have consistently demonstrated greater confidence as preservice teachers entering the mathematics methods course.

University of Ottawa
Summer Math Program for Prospective Elementary Teachers

Barbara Graves & Chris Suurtamm, Faculty of Education

Facilitating effective mathematics inquiry poses different challenges for preservice and new teachers at both the elementary and secondary levels. For elementary teachers, the research suggests that their knowledge of mathematics is limited and more procedural than conceptual making it difficult for them to facilitate open-ended problem solving situations and ask probing questions that deepen student thinking (Ball, 1988, 1990; Ball, Lubienski & Mewborn, 2001). Their weak understanding of mathematics also prevents them from recognizing and furthering the important concepts that are inherent in mathematical activity (Ball, 1999; Kahan, Cooper, & Bethea, 2003). For secondary teachers some of the same concerns exist about the procedural nature of their mathematics knowledge, but the greater challenge is to help them shift their teaching practices from traditional delivery models to more inquiry-oriented approaches (Stigler & Hiebert, 1999)

A one-week, intensive *Summer Mathematics Program* initiated in August 2004 is designed as a learning environment for mathematical inquiry for prospective elementary teachers with the goal of deepening their mathematical understanding and developing a more open-ended perspective of mathematics. Each day of the program (9:00 a.m.-3:30 p.m.) consists of in-class problem solving experiences centred on foundational domains of elementary mathematics in a supportive and collaborative learning environment. The facilitators/instructors for the summer program are recently graduated secondary mathematics teachers for whom the summer program serves as an initial teaching setting that is closely aligned with the theories of inquiry-based mathematics learning.

The accompanying longitudinal, qualitative study that we are undertaking asks whether learning and teaching mathematics in communities of inquiry is enough to transform beginning teachers' knowledge of mathematics to enable them to create learning environments which support mathematical inquiry. The 3-year longitudinal research design addresses the challenge of teacher change in mathematics education and allows us to examine the experiences of both the prospective elementary teachers and the secondary math teachers not only during the summer math program but also into their first years of teaching. Through questionnaires and focus group interviews we are also collecting data on how beginning teachers of mathematics deepen their own understanding of mathematics in the context of teaching. Our specific objective is to determine what experiences help beginning teachers develop a deeper understanding of mathematics and mathematics teaching and learning so that they can facilitate effective mathematics inquiry in their classrooms.

Results of our research are being presented at the International Commission on Mathematics Instruction (ICMI-Study 15), the theme of which is "The Professional Education and Development of Teachers of Mathematics", co-chaired by Deborah Ball and Ruhama Even in Brazil in May 2005.

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University of Western Ontario

Math for Preservice Teachers at UWO

George Gadanidis & Immaculate Namukasa, Faculty of Education

It's not surprising to meet elementary teachers who wear their view that "I'm not a math person" as a badge of honour. At a recent orientation assembly, we asked our in-coming group of 440 elementary preservice teachers how they felt about mathematics. When asked to raise their hands if they loved mathematics, 15-20 hands went up. When asked to raise their hands if they hated mathematics, a sea of hands filled the auditorium. As one preservice teacher commented, "Math is like an iguana. As long as it blends into its environment I don't mind it. But once I have to hold it I'm not so fond of it."

To address this issue, in 2004, we added 9 more hours of contact time with elementary preservice teachers, in the form of 9 large group auditorium sessions, where they worked on small group problem activities (Gadanidis & Kajander 2002; Gadanidis 2005). We would prefer to have small group sessions, but timetabling and funding constraints make this not possible.

Doing mathematics became the starting point for the course. However, we conceptualized the purpose of teachers doing mathematics not in terms of gaining content knowledge but as an experiential therapy (Gadanidis & Namukasa forthcoming). Mathematics experiences were designed (1) to be interesting and challenging enough to capture candidates' interest and imagination, (2) to offer the potential for mathematical insight and surprise (Gadanidis 2004), and (2) to form the basis of critical experiences that help teacher candidates see mathematics and mathematics teaching in new light (Gadanidis, Hoogland & Hill 2002).

We selected 9 interesting problems to form the basis of each of the 9 mathematics sessions and purchased enough materials to allow 440 preservice teachers to work in groups of four. In the last 5 minutes of each session, preservice teachers completed and handed in a sheet outlining what they had learned and what they had felt during the session. These reflections were summarized and then shared and discussed at the beginning of the next session. A website was created to provide extensions and interactive explorations of problems. A new assessment component was added, where in the last workshop of the course, preservice teachers would have thirty minutes to write an 'essay' on one of the problems dealt with in the mathematics sessions.

As the course progressed, preservice teachers started expressing an excitement for teaching mathematics. "I have learned that teachers have the ability to make math engaging. All the math essay activities have changed my view of math education – it can be FUN!"

We have also instituted a similar course for inservice teachers (post-graduate certificate) and offered this for local math teachers in January-February 2005. We are currently conducting research on the 2004-5 Mathematics Course experiences.

Gadanidis, G. (2004). The pleasure of attention and insight. *Mathematics Teaching 186*(1), 10-13.

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York University
Undergraduate Courses for Prospective Teachers

Walter Whiteley, Department of Mathematics and Statistics
Margaret Sinclair, Faculty of Education

At York, we have developed courses and programs to address the mathematics education needs of prospective elementary and secondary teachers. For the math anxious student who is thinking about applying for the education program we offer *Thinking Mathematically I and II.* For mathematics majors and minors we offer several undergraduate courses whose content and approach are appropriate for the development of the deep understanding required to teach mathematics, including *Introduction to Geometries,* and *Topics in Mathematical Education.*

Thinking Mathematically I and II (MATH 2590, 2591). These courses are intended primarily, but not exclusively, for education students in the primary and junior divisions. They are designed to create a positive attitude towards mathematics through an examination of topics relevant to the study of mathematics at the elementary school level.

The main objectives of these two half courses include providing opportunities for students who feel that their background in mathematics is incomplete, or whose past experiences have caused them to avoid mathematics, to achieve success in thinking mathematically and to reflect on the learning and practice of mathematics. Thus, in all work, an exploratory approach is used. Students work in small groups on selected problems and projects, using a mix of hands-on materials, appropriate technology such as Geometer's Sketchpad, and pencil and paper. Throughout, the focus is on developing students' communication skills in written, oral, and visual form both within their groups, and with the larger class.

Introduction to Geometries (MATH 3050). This year long course explores Euclidean and spherical geometry, and touches on other geometries such as hyperbolic, and affine. It exemplifies the use of the inquiry model through explorations of 'big ideas' such as symmetry as the core of geometric reasoning and the use of hands on group work, dynamic geometry software, writing, and projects. There is a strong emphasis on communication - oral, written and visual – through group work and project presentations. There is also an emphasis on working with multiple representations, reflecting on mathematical experiences, and exploring original 'What if' questions. Students say it is 'unlike any other undergraduate mathematics course'. With a core prerequisite of two semesters of linear algebra, the course is taken by a range of undergraduate students, from applied mathematics and pure mathematics students interested in research, to future teachers who may not be mathematics majors.

Appendix B

York University
Graduate Courses for Teachers

Walter Whiteley, Department of Mathematics and Statistics
Margaret Sinclair, Faculty of Education

York University offers several unique programs for practicing teachers. The MA in Mathematics for Teachers is a long standing, course based, part time Masters program designed for mathematics teachers who want to expand their mathematical experience. The core courses are Fundamentals of Mathematics (focusing primarily on combinatorics, and number theory), and History of Mathematics. Other courses explore a wide range of mathematical topics such as algebra, analysis, geometry, statistics, computational mathematics, and problem solving. The courses make substantial use of appropriate technologies, as they are used in the practice of modern mathematics and in the current mathematics classrooms. The courses also have an emphasis on communication, hands on explorations, and reflection on the big ideas of mathematics. Most of the students in the program are high school mathematics teachers; however, there are also some college mathematics teachers, and a few middle school teachers. Some teachers have full honours degrees while others have mathematics as a teachable, and are working to become honours specialists. Recently, we have expanded the program to include several courses in mathematics education, offered by the Graduate Program in Education. This change allows teachers to explore a range of mathematics topics and to deepen their knowledge of issues in mathematics education.

The growing collaborations between the Graduate Program in Mathematics and Statistics and the Graduate Program in Education at York are also captured in our new Graduate Diplomas in Mathematics Education. The Degree Concurrent Diploma is available with both the MA in Mathematics for Teachers (as preparation for possible Ph.D. work in mathematics education), and the M.Ed and Ph.D. programs in Education. It provides opportunities for candidates to critically examine issues in mathematics education and to complete a survey paper / major research paper on a topic in that area —an important step in helping teachers connect theory with practice. The Direct Entry Diploma will offer the necessary background for practicing teachers who are not interested in pursuing a Masters degree to prepare for mathematics leadership roles in their boards. The first students will be admitted to these diploma programs in the summer of 2005.

(cover letter)

April 6, 2005

Brian McGowan
Deputy Registrar
Ontario College of Teachers
121 Bloor Street East, 6th Floor
Toronto, Ontario M4W 3M5

Dear Mr. McGowan

Please find enclosed a submission of the Fields Institute Mathematics Education Forum, in connection with the Ontario College of Teachers review of teacher qualifications. I and Prof. George Gadanidis (who chairs the forum) had written a letter of intent to you on February 1, indicating that such a submission was in preparation. I enclose a copy of that letter, for your reference.

The forum's submission reflects general discussions held over the past several years. The submission itself was prepared by a subcommittee chaired by Prof. Margaret Sinclair (York University), and was reviewed and discussed at a recent meeting of the forum. It represents our comprehensive recommendations for enhancing the training of mathematics teachers in Ontario, many of which we feel are relevant to the OCT's current review. Failure in mathematics marginalizes many students, and for this reason we feel that the training of mathematics teachers presents particular challenges for Ontario.

I hope you will find this submission a useful input to your review. I and the other subcommittee members would be pleased to provide any additional information you require, or to meet to discuss these issues further.

Sincerely,
Thomas Salisbury
Deputy Director
Fields Institute

On behalf of the Fields Institute Mathematics Education Forum,
Teacher qualification subcommittee:

Cathy Bruce (School of Education & Professional Learning, Trent University)
Rina Cohen (OISE/University of Toronto)
Shirley Dalrymple (York Region District School Board),
George Gadanidis (Faculty of Education, University of Western Ontario)
Leo Jonker (Department of Mathematics and Statistics, Queen's University)
Ann Kajander (Faculty of Education, Lakehead University)
Dragana Martinovic (Sheridan College)
Eric Muller (Department of Mathematics, Brock University)
Pat Rogers (Faculty of Education, University of Windsor)
Geoff Roulet (Faculty of Education, Queen's University)
Thomas Salisbury (Fields Institute)
Riaz Saloojee (Seneca College)
Margaret Sinclair (Faculty of Education, York University)
Chris Suurtamm (Faculty of Education, University of Ottawa)
Peter Taylor (Department of Mathematics and Statistics, Queen's University)
Walter Whiteley (Department of Mathematics and Statistics, York University)

cc. George Gadanidis, Chair, Mathematics Education Forum
Margaret Sinclair, Chair, Teacher qualification subcommittee

encl:
Submission
Letter of intent