July 23, Monday

9:30am–10:30am S.-T. Yau (Harvard University)
Mirror Principle

I will talk on joint work with Bong Lian Kefeng Liu, and others on the mathematical treatment of the mirror principle. We give a rigorous way to treat questions arising in string theory, and at the same time, we give a new approach for certain problems arising in enumerative geometry. We also discuss some open questions.

11:00am–12:00 noon James D. Lewis (University of Alberta)
Regulators of higher Chow cycles on Calabi–Yau varieties

In the first part of this lecture, which is intended to be rather introductory, we introduce Bloch's higher Chow groups $CH^k(X, m)$ and describe them in the cases $m = 0, 1, 2$ in terms of the Zariski cohomology of a certain Gersten-Milnor complex. From this point of view, we give explicit formulas for the various regulators from $CH^k(X, m)$ to Deligne cohomology, in those cases. The second part of this lecture surveys what is currently known about the regulator images for Calabi-Yau varieties of dimension ≤ 3. Finally, in the third part, we introduce a twisted version of the Gersten-Milnor complex, and show that the twisted real regulator from a general (algebraic) $K3$ surface to real Deligne cohomology is surjective. This provides some evidence in support of the Hodge-$\mathcal{D}$- conjecture for general (algebraic) $K3$ surfaces.

2:00pm–3:00pm V. Batyrev (University of Tübingen)
Introduction to toric methods in mirror symmetry

Calabi-Yau complete intersections in toric varieties turn out to be very helpful class for testing mirror symmetry. We introduce and illustrate several ideas used for this purpose.

3:30pm–4:30pm R. Schimmrigk (Georgia Southwestern State University)
Arithmetic aspects of Calabi–Yau varieties and conformal field theory

Geometric string theory has to be formulated in ten dimensions in order to be consistent as a quantum theory. One strategy to make these ten dimensions compatible with the observation of four dimensions at low energies is to compactify the theory on a manifold. Supersymmetry then dictates that this manifold is a Calabi-Yau variety. An a priori completely independent formulation is to construct string theory directly in four dimensions, by constructing the additional degrees of freedom as an 'internal' conformal field theory which does not necessarily have any spacetime interpretation. In a very surprising development Gepner observed in 1987 that the models in a large class of such internal exactly conformal field theories seem to be closely related to Calabi-Yau manifolds. The goal of this talk is to explore how certain aspects of this geometry−conformal field theory relation are illuminated by arithmetic considerations on the Calabi-Yau variety.

5:00pm–6:00pm C. Doran (Columbia University)
Variation of the mirror map and algebro-geometric solutions to Garnier systems

Variation of the string-theoretic mirror map q-series in families motivates an investigation of the Picard-Fuchs differential equation of a pencil under deformation. Such variations define special algebraic solutions to isomonodromic deformation equations. Explicit Painleve VI and Garnier system solutions coming from elliptic pencils are determined. These include the first known algebraic solutions to Garnier systems. An arithmetic question of Mazur is addressed by reinterpreting these results as describing specially parametrized Hurwitz spaces.

July 24, Tuesday

9:30am–10:30am S. Hosono (University of Tokyo)
Introduction to GKZ hypergeometric system

In this talk I will review some important properties of GKZ hypergeometric systems in mirror symmetry of Calabi-Yau manifolds. Let A be an $(n+1)\times(p+1)$ integral matrix, $A=(\bar{\nu}_0,\bar{\nu}_1,\cdots,\bar{\nu}_p)$ with $\bar{\nu}_i=1\times\nu_i\in\mathbf{Z}\times\mathbf{Z}^n$. GKZ hypergeometric system, A-hypergeometric system due to Gel'fand, Kapranov and Zelevinski, is an infinite set of differential equations (with fixed $\beta\in\mathbf{C}^{n+1}$):

$$\Big\{\Big(\frac{\partial}{\partial a}\Big)^{l_+}-\Big(\frac{\partial}{\partial a}\Big)^{l_-}\Big\}\Phi(a)=0\qquad(l\in \mathrm{Ker}A\cap\mathbf{Z}^{p+1})$$
$$\Big\{\sum_i\bar{\nu}_i a_i\frac{\partial}{\partial a_i}-\beta\Big\}\Phi(a)=0$$

for a function $\Phi(a)$ on $a=(a_0,a_1,\cdots,a_p)\in\mathbf{C}^{p+1}$. Period integrals of Calabi-Yau hypersurfaces in toric varieties satisfy GKZ system [2], which is reducible and also resonant[3].

I will try to introduce GKZ system following the 'A- philosophy' due to GKZ ("Discriminants, Resultants and Multidimensional Determinants", Birkhäuser (1994)). This talk is based on the results in [4][5][3].

References:

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[7] Smooth case is studied also in J. Stienstra, *Resonant hypergeometric systems and mirror symmetry* in "Integrable Systems and Algebraic Geometry" (M-H. Saito et al., eds.), World Scientific(1998), pp.412-452.

11:00am–12:00 noon X. de la Ossa (University of Oxford)

Counting $\mathbf{F}_p$-rational points for Calabi–Yau manifolds

This talk is a report on joint work with P. Candelas and F. Rodriguez-Villegas. Two expressions for the number, N, of rational points of a family of Calabi-Yau threefolds will be derived and discussed. The first of these expresses N in terms of the periods and semiperiods of the manifold. The second expresses N in terms of Gauss sums and provides an effective means of computation. Mirror symmetry enters the discussion in virtue of the fact that this second expression makes reference to the divisors of the mirror manifold.

2:00pm–3:00pm P. Cancdelas (University of Oxford)
Observations on the form of the local L-function for the quintic threefold

This is a continuation of the previous talk. Based on a combination of analytical and numerical computations we discuss the form of the local L-functions for a one parameter family of quintic threefolds. Of interest is the fact that the part of the L-function that one would like to ascribe to the divisors of the mirror manifold can also be identified with the contribution of certain auxiliary curves.

3:30pm–4:30pm Y. Goto (Hokkaido University of Education)
$K3$ surfaces over finite fields with symplectic group actions

Calabi-Yau varieties over a field of positive characteristic have various particular properties that do not exist in characteristic 0. In this talk, we look at 2-dimensional Calabi-Yau varieties, i.e. $K3$ surfaces, in positive characteristic with symplectic group actions.

A smooth projective surface X is called $K3$ if it has a trivial canonical sheaf and irregularity 0. A finite group action G on X is called symplectic if every element of G fixes the nowhere vanishing 2-form on X. By now, one knows a large number of results about $K3$ surfaces with symplectic group actions; in particular, using such actions, one can consider mirror symmetry for $K3$ surfaces in weighted projective 3-spaces. Yet, there have not been many results exclusively in positive characteristic.

Here we use symplectic group actions to investigate $K3$ surfaces over finite fields. Among other properties, we discuss (1) their Picard numbers, (2) the height of their formal Brauer groups, (3) their Artin invariant, and (4) the number of their rational points. We also look at special types of $K3$ surfaces to give more explicit computational data.

5:00pm–6:00pm F. Rodriguez-Villegas (University of Texas at Austin)
Arithmetic of some hypergeometric threefolds

In this talk I will discuss work in progress on the arithmetic of several examples of Calabi-Yau manifolds whose periods are hypergeometric.

July 25, Wednesday

9:30am–10:30am S. Müller-Stach (GHS Essen)
Picard–Fuchs equations and algebraic K-theory

We discuss inhomogeneous differential equations of Painleve type which arise from normal functions in

the context of algebraic K-theory resp. motivic cohomology on complex algebraic varieties. These equations are related to Picard- Fuchs equations and some non-trivial examples come up e.g. on certain mirror families of K3 surfaces (joint work with del Angel).

11:00am–12:00 noon I. Dolgachev (University of Michigan)
Calabi–Yau manifolds and Hermitian symmetric bounded domains

It is well-known that the coarse moduli space of K3 surfaces with fixed Picard lattice polarization is isomorphic to an arithmetic quotient of a Hermitian symmetric bounded domain of type IV. In this talk we shall discuss our search for moduli spaces of Calabi-Yau threefolds which are isomorphic to arithmetic quotients of Hermitian symmetric domains. In particular, we shall speculate on the existence of a 27-dimensional moduli space isomorphic to an arithmetic quotient of the exceptional domain of type E_7.

2:00pm–3:00pm S. Tankeev (Vladimir State University)
On the Brauer group of a Calabi–Yau variety

Let $X \to Spec(A)$ be an arithmetic model of a Calabi-Yau variety V of dimension > 1 over a number field k. Assume that V has a rational k-point and the Neron- Severi group of V coincides with the geometric Neron-Severi group. If a prime number l does not divide the order of the torsion subgroup of the Neron-Severi group of V, then the l- primary components of $Br'(V)/Br(k)$ and $Br'(X)$ are finite.

3:30pm–4:30pm M. Saito (Kobe University)
Relative Lefschetz action and MPS state counting

This is a joint work with S. Hosono and A. Takahashi. We propose a mathematical definition of a new "numerical invariants" of Calabi–You 3-folds from stable sheaves of dimension one, which is motivated by the Gopakumar-Vafa conjecture in M- theory. Moreover, in the process of the definition, we show that for any projective morphism $f : X \longrightarrow Y$ of normal projective varieties, there exists a natural $sl_2 \times sl_2$ action on the intersection cohomology group $IH(X, \mathbf{Q})$ which fits into the perverse Leray spectral sequence.

5:00pm–6:00pm P. Berglund (University of Southern California)
On hybrid phases in the extended Kähler moduli space of Calabi-Yau threefolds

We discuss aspects of certain types of non-geometrical phases, known as hybrids, in the extended Kähler moduli space of string compactifications on Calabi-Yau three-folds. These are vacua to which one cannot give a purely geometric interpretation. Instead, the structure is that of a fibration in which the base is a complex projective space and the fiber is given by an abstract conformal field theory. In particular, we compute the metric on the moduli space and show that the hybrid phase is at an infinite distance from the smooth manifold.

July 26, Thursday

9:30am–10:30am S. Hosono (University of Tokyo)

Monodromy transforms of hypergeometric series in (local) mirror symmetry

The most general form of the mirror symmetry of Calabi-Yau manifolds is now understood as an equivalence of the derived category of coherent sheaves on X and that of Fukaya's Lagrangian submanifolds on the mirror $X^\vee$ (the homological mirror symmetry conjecture due to Kontsevich). From this viewpoint, some monodromy transformations on hypergeometric series for the period integrals are identified with Fourier-Mukai transformations on the coherent sheaves. In this talk I will present a conjectural formula which connects the period integrals with the Chern classes of the corresponding coherent sheaves (D-brane charges in physics). I will also review a recent result by Horja and interpret his result in local mirror symmetry.

References:

[1] S. Hosono, *Local mirror symmetry and type IIA monodromy of Calabi-Yau manifolds*, Adv. Theor. Math. Phys. 4(2000) 335-376.
[2] R.P. Horja, *Hypergeometric functions and mirror symmetry in toric varieties*, math.AG/9912109.

11:00am–12:00 noon V. Batyrev (University of Tübingen)
Local mirror symmetry and McKay correspondence

Crepant resolutions of Gorenstein singularities can be considered as examples of noncompact Calabi-Yau varieties. One expects that some properties of (local) mirror symmetry may be observed also in this case. The geometry of crepant resolutions of Gorenstein quotient singularities $\mathbf{C}^n/G$ is related to representations of G via so called McKay correspondence. We suggest an application of the McKay correspondence to mirror symmetry using K-theory.

12:10pm–1:10pm A. Todorov (University of California Santa Cruz)
Moduli of Calabi–Yau manifolds

In this talk we will prove some theorems about the global structure of the Teichmuller space of Calabi–Yau manifolds. Namely we will prove that the Teichmuller space of polarized Calabi–Yau manifolds exists. Moreover there exists a subgroup of the mapping group such that if we take the quotient of the Teichmuller space by this group, we will get an open and everywhere dense subset in the domain of holomorphy (which is a complex vector space of dimension equal to $H^{(n-1,1)}$ where n is the dimension of the Calabi–Yau manifold) and the complement of the image is a countable union of codimension one divisors.

July 27, Friday

9:30am–10:30am B. Lian (Brandeis University)
Monodromy of Calabi–Yau varieties

We discuss a certain degeneration of Calabi-Yau manifolds into a configuration whose monodromy takes on a special form. The monodromy operator is maximally unipotent, and has been studied extensively in classical Hodge theory, as well as in mirror symmetry more recently. We give a simple computational criterion for the occurance of this form of monodromy.

11:00am–12:00 noon J. Stienstra (University of Utrecht)

Ordinary Calabi-Yau Threefolds

A Calabi-Yau Threefold X over a perfect field k of characteristic $p > 0$ is, by definition, ordinary if and only if $H^j(X, B\Omega^i) = 0$ for all i, j; here $B\Omega^i = d\Omega^{i-1}$ is the sheaf of exact i- forms.

This talk is about the structure of the cohomology of the universal formal deformation of an ordinary Calabi-Yau Threefold. Among the topics to be discussed are: (1) the canonical coordinates on the formal moduli space, (2) the prepotential of the Yukawa coupling and its behavior under the canonical lifting of Frobenius, (3) the remarkable analogy with the structure of complex Calabi-Yau Threefolds near the point of maximal unipotent local monodromy.

We also present a formula for the zeta function of X if k is a finite field. This formula involves, besides the well-known expression for the so-called unit root, the derivatives of the canonical coordinates.

2:00pm–3:00pm N. Yui (Queen's University)
The modularity of Calabi–Yau varieties

We consider Calabi–Yau varieties defined over number fields, and investigate their arithmetic properties, in particular, their modularity. For this, we first define zeta-functions and L-series of Calabi–Yau varieties. The Langlands Program predicts that there ought to be some "modular" forms (and modular groups) such that the L- series of Calabi–Yau varieties should coincide with the L–series of modular forms. However, there are only few examples of Calabi–Yau varieties for which the modularity has been established. In this lecture, I will look at some examples of Calabi–Yau varieties over number fields in support of the modularity conjecture. We try to understand such results in connection with Beilinson–Bloch conjecture for L-series and algebraic cycles, and also with mirror symmetry.

3:30pm–4:30pm H. Verrill (University of Hannover)
Intermediate Jacobians of certain rigid Calabi–Yau threefolds

A rigid Calabi-Yau threefold X is a Calabi-Yau threefold with two-dimensional middle cohomology, $H^3(X)$. When X is defined over $\mathbf{Q}$, we have on the one hand the $H^3(X, \mathbf{C})$, and on the other hand the l-adic cohomology $H^3(X, \mathbf{Q}_l)$. We have the intermediate Jacobian,

$$H^0(X, \Omega^3)/H_3(X, \mathbf{Z}),$$

and from the l-adic cohomology one has (or expects to have) a weight 4 modular form, from the L-series of the Galois action on $H^3(X, \mathbf{Q}_l)$. One expects these to be closely related, and I will show in some examples how one can very explicitly construct 3-cycles in X corresponding to modular symbols. I will restrict the discussion to the fibre product of the universal family of elliptic curves over $X_0(6)$, which Chad Schoen shows is a rigid Calabi-Yau threefold (after appropriate desingularisation). I relate the monodromy, L-series, and intermediate Jacobian.

5:00pm–6:00pm T. Ito (University of Tokyo)
Birational smooth minimal models have equal Hodge numbers in all dimensions

In this talk, we prove that birational smooth minimal models over the field of complex numbers have equal Hodge numbers in all dimensions. In particular, birational Calabi-Yau manifolds have equal Hodge numbers. Our method is a refinement of the work of Batyrev on Betti numbers using the Weil conjecture. Our key ingredient is the use of subtle arithmetic results such as class field theory and p-adic Hodge theory as well as the Weil conjecture. We note that similar results can be obtained by the theory of motivic integration developed by Kontsevich and Denef- Loeser.

July 28, Saturday

9:30am–10:30am Pedro Luis del Angel (CIMAT, Mexico)
On indecomposable higher cycles in a product of hyperelliptic curves

We construct examples of indecomposable cycles on a product of hyperelliptic curves. We have extended a construction due to Gordon and Lewis, for the case of a product of elliptic curves.

11:00am –12:00 noon A. Collino (University of Torino)
Indecomposable higher Chow cycles on hypersurfaces and on Jacobians

For a smooth complex projective variety X, the subgroup of $CH^k(X,1)$ of decomposable cycles, $CH^k_{dec}(X,1)$, is defined to be the image of $CH^{k-1}(X)\otimes \mathbf{C}^*$. We let

$$CH^k_{ind}(X,1) := CH^k(X,1)/CH^k_{dec}(X,1)$$

be the quotient group of indecomposable cycles. Our aim is to explain two parallel results. One result is Voisin's construction of non-trivial elements in $CH^n_{ind}(X,1)_{\mathbf{Q}}$, where X is a general hypersurface of degree $2n$ and of dimension n in $\mathbf{P}^{n+1}$ (Voisin [V]). The other is our result for Jacobians $J(C)$, see [CF]: we have constructed some natural indecomposable elements of $CH^g(J(C),1)$ with trivial regulator, and in particular, we have proved that $CH^g_{ind}(J(C),1)_{\mathbf{Q}}$ is uncountable for C a generic curve or a generic hyperelliptic curve of genus $g\geq 3$.

We recall next the content of [CF]. In [Ca], a natural element K was constructed in $CH^g(J(C),1)$, C hyperelliptic, and using the regulator map K was shown to give a non-torsion element of $CH^g_{ind}(J(C),1)$ when C is generic hyperelliptic. The difference of K and its translates by elements t of $J(C)$ i.e. $K-K_t$, and more generally the Pontryagin product of K with zero cycles of degree zero, give elements of $CH^g(J(C),1)$ which lie in the kernel of the regulator map. We show that such cycles give uncountably many elements of $CH^g_{ind}(J(C),1)_{\mathbf{Q}}$ if $g\geq 3$. The main technical tool used in the proof is a Hodge theoretic criterion due to J. Lewis which has been used earlier by Gordon and Lewis to construct indecomposable cycles with similar properties in products of generic elliptic curves. For a survey of these and earlier results an excellent source is [GL]. To construct elements of $CH^g(J(C),1)$ for arbitrary curves, we consider divisors

$$D=\sum_{i=1}^n a_i - \sum_{i=1}^n b_i$$

with $2[D]=0$ in $J(C)$ and $n>0$. Associated to such a divisor, we construct a natural subspace of $CH^{n+1}(C^{n+1},1)_{\mathbf{Q}}$. We then complete our results by showing that $CH^g_{ind}(J(C),1)_{\mathbf{Q}}$ is uncountable for a generic curve of genus $g\geq 3$. A result of Nori implies that up to torsion the regulator image of $CH^g(J(C),1)$ is the same as that of $CH^g_{dec}(J(C),1)$ for a generic curve of genus $g=3$, and it seems likely that this is true for higher g. This makes it difficult to use Lewis' criterion to prove indecomposability; we have used a specialisation argument.

References

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2:00pm–3:00pm R. Schimmrigk (Georgia Southwestern State University)
Black hole attractor varieties and complex multiplication

Black holes in compactifications of string theory on Calabi-Yau varieties a priori might be expected to have moduli dependent features. For example the entropy of the black hole might be expected to depend on the complex structure of the Calabi–Yau manifold. This would be inconsistent with known properties of black holes. Supersymmetric black holes evade this inconsistency by having moduli fields that flow to fixed points in the moduli space that depend only on the charges of the black hole. Moore observed in the case of compactifications with elliptic curve factors that these fixed point complex structures are arithmetic, corresponding to curves with complex multiplication. The main goal of this talk is to explore the possibility of generalizing such a characterization to Calabi-Yau varieties with finite fundamental groups.

3:30pm–4:30pm M. Saito (Kobe University)
Deformations of Okamoto–Painlevé pairs and Painlevé equations

This is a joint work with T. Takebe and H. Terajima. We introduce the notion of generalized rational Okamoto–Painlevé pair (S, Y) by generalizing the notion of the spaces of initial conditions of Painlevé equations. After classifying those pairs, we will establish an algebro-geometric approach to derive the Painlevé differential equations from the deformation of Okamoto– Painlevé pairs by using the local cohomology groups. Moreover the reason why the Painlevé equations can be written in Hamiltonian systems is clarified by means of the holomorphic symplectic structure on $S - Y$. Hamiltonian structures for Okamoto– Painlevé pairs of type $\tilde{E}_7 (= P_{II})$ and $\tilde{D}_8 (= P_{III}^{\tilde{D}_8})$ are calculated explicitly as examples of our theory.

5:00pm–6:00pm TBA

July 29, Sunday

9:30am–10:30am
Problem Session

11:00am–12:00 noon
Discussion on the Fields Proposal