THE FIELDS INSTITUTE

FOR RESEARCH IN MATHEMATICAL SCIENCES

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New estimates on the solutions of first order Hamiltonian systems possessing super-quadratic potentials and some applications

We study the following first order Hamiltonian system

$$\mathcal{J}\dot{u} + \nabla H(t, u) = 0, \quad \forall (t, u) \in S_T \times \mathbf{R}^{2N}, \tag{1}$$

where the potential H is super-quadratic increasing at infinite. We prove two newestimates for the C^0 -norm of the solutions (periodic solutions orhomoclinic solutions) of the system (1) for H satisfying: (H4).There is a constant c > 0, such that

$$|\nabla W(t, u)| \le c(\nabla W(t, u), u), \forall |u| \ge 1.$$

or

(H5)
$$\limsup_{|u|\to\infty} \frac{W_t(t,u)}{|u|^{\mu}W(t,u)} = 0, \text{ or } \liminf_{|u|\to\infty} \frac{W_t(t,u)}{|u|^{\mu}W(t,u)} = 0, \text{ uniformly in } t.$$

Then we use these estimates to show the existence of periodic solutions and homoclinic orbits of the system (1) for H symmetric in u-variable and H without any symmetry respectively.