

THOMAS BRUESTLE  
University Bielefeld

*Tame Tree Algebras (50-60)*

We consider finite-dimensional basic algebras over an algebraically closed field  $k$ . Such an algebra  $A = kQ/I$  is called a **tree algebra** if the quiver  $Q$  is a tree. Tree algebras can also be characterized as those algebras which are simply connected and whose ideal  $I$  admits a set of generators which consists of monomials in the arrows of  $Q$ . Let  $q_A$  denote the Tits form of  $A$ . During the last two decades, in many situations the representation type of certain classes of algebras could be related with properties of the Tits form. In the case of tree algebras, we confirm in the following theorem conjectures by de la Peña and Skowroński: **Theorem.** *Let  $A$  be a tree algebra. Then  $A$  is tame precisely when the Tits form  $q_A$  is weakly non-negative, that is,  $q_A$  has non-negative values when evaluated at non-negative vectors.* This theorem is based on a number of earlier results: It is well-

known that the Tits form of an arbitrary tame algebra is weakly non-negative, so only the converse has to be shown. Using the investigations of the polynomial growth case by Skowroński and others, one can restrict to study those algebras containing a so-called pg-critical algebra. In this case, we classify the sincere tree algebras completely: They are quotients of  $\mathbb{D}$ -algebras. Here, the class of  $\mathbb{D}$ -algebras is defined by local properties, roughly speaking they locally look like a pg-critical algebra. We also determine which quotients of  $\mathbb{D}$ -algebras do actually arise: The extra relations are all passing through some  $\tilde{\mathbb{D}}$ . In order to prove tameness, we finally show that every  $\mathbb{D}$ -algebra degenerates to a gentle algebra, which ensures tameness by Geiss' degeneration theorem.