THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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Generalized Serre-Relations (50-60)
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To any unit from q we associate a Lie algebra $G(q)$, defined by generators and relations in the following way. For $q : \mathbb{Z}^n \to \mathbb{Z}$, we define $C_{ij} = q(c_i + c_j) - q(c_i) - q(c_j)$, where $c_1, \ldots, c_n$ are the canonical base vectors in $\mathbb{Z}^n$. Then $G(q)$ is generated by $3n$ elements e_i, e_{-i}, h_i $(i = 1, \ldots, n)$ and the following relations:
(R1) $[h_i, h_j] = 0$ for all i, j,
(R2) $[e_{\varepsilon i}, e_{-\varepsilon i}] = \varepsilon h_i$ for $\varepsilon = \pm 1$ and all i,
(R3) $[h_i, e_{\varepsilon j}] = -\varepsilon C_{ij} e_{\varepsilon j}$ for $\varepsilon = \pm 1$ and all i, j,
(R∞) $[e_{\varepsilon_1 i_1}, e_{\varepsilon_2 i_2}, \ldots, e_{\varepsilon_t i_t}] = 0$ whenever $q(\sum_{\ell=1}^{t} \varepsilon_\ell c_{i_\ell}) > 1$.
We prove that $G(q)$ and $G(q')$ are isomorphic if q and q' are equivalent connected non-negative unit forms, further (without assuming connectedness or non-negativity) $G(q)$ and $G(q')$ are isomorphic if q' is obtained from q by a sequence of deflations, inflations and sign- inversions.
If q is positive definite, $G(q)$ is a finite dimensional semisimple (simply laced) Lie algebra and finitely many relations of type (R∞) are sufficient. If q is connected and non-negative of corank one, $G(q)$ is isomorphic to a (simply laced) affine Kac-Moody algebra and if q is connected and non-negative of corank two, $G(q)$ is isomorphic to the Lie algebra associated to a (simply laced) elliptic root system, as investigated by Saito and Yoshii.