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Delta-critical quasi-hereditary algebras (25-30)

Let k be a field, Q a finite quiver, ρ an admissible set of relations and let $A := kQ/\rho$. Let $S(i)$ be the simple module for the vertex $i \in Q_0$, $P(i)$ the projective cover and $I(i)$ the injective envelop of $S(i)$. Given an enumeration π of the vertices of Q (a bijective map $\pi : Q_0 \rightarrow \{1, 2, \dots, |Q_0|\}$), define $\Delta_\pi(i)$ as the maximal factor module of $P(i)$ and $\nabla_\pi(i)$ as the maximal submodule of $I(i)$, both with composition factors $S(j)$ with $\pi(j) \leq \pi(i)$.

The enumeration π is called quasi-hereditary if every module $P(i)$ has a Δ_π -filtration and $S(i)$ occurs only once as a composition factor of $\Delta_\pi(i)$ and of $\nabla_\pi(i)$ (see [CPS]). If A is tame concealed, then π will be called Δ -critical if it is quasi-hereditary and all modules $\nabla_\pi(i)$ are preinjective.

In this case, the modules $T_\pi(i)$ (the minimal modules with $S(i)$ occurring exactly once as a composition factor of $T_\pi(i)$ and having a Δ_π - and a ∇_π -filtration) are partial tilting modules, $T_\pi := \bigoplus_{i \in Q_0} T_\pi(i)$ is a tilting module, and $B := \text{End}(T_\pi)^{op}$ is another tame concealed algebra. The category $\mathcal{F}(\Delta_\pi)$ of all A -modules with a Δ_π -filtration has a preprojective component of type A and a preinjective component of type B (see [R]).

Happel and Vossieck classified tame concealed algebras as the path algebras of the quivers of frame types \widetilde{A}_n , \widetilde{D}_n , \widetilde{E}_6 , \widetilde{E}_7 and \widetilde{E}_8 (see [HV]). There are only finitely many tame concealed algebras of the types \widetilde{E}_6 , \widetilde{E}_7 and \widetilde{E}_8 , so for these all Δ -critical enumerations can be computed.

References

- [CPS] Cline, E., Parshall, B. and Scott, L.: Finite dimensional algebras and highest weight categories. *Journal Reine und Angewandte Mathematik* 391, 85-99 (1988)
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- [R] Ringel, C.M.: The category of modules with good filtrations over a quasi-hereditary algebra has almost split sequences. *Mathematische Zeitschrift* 208, 209-223 (1991)