Minimal singularities in orbit closures of matrix pencils

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Matrix pencils are a classical and important theme of linear algebra. They have already been studied by Weierstrass and Kronecker who described an algorithm to find the normal form of an arbitrary matrix pencil (see [5]). In modern language, a matrix pencil is just a finite dimensional representation of the double arrow $K = 1 \rightrightarrows 2$, or equivalently a finite dimensional module over the corresponding path algebra $A = kK$, where k is an algebraically closed field of arbitrary characteristic. The structure of the category of such modules is well understood (see [7, 1]) and also some geometric properties of matrix pencils are known like the degenerations of the orbits ([6, 3]). Here we now analyze the singularities of the minimal degenerations using the methods from [4], but now in contrast to the case of Dynkin quivers the singularities heavily depend on the modules and several infinite series of singularities show up.

We idenitfy the occurring types of singularities and show that they are Cohen-Macaulay and regular in codimension one, hence normal.

References

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