THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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Monomial Bases for quantum affine sl_n (25-30)

Co-author: Jie Du Department of Mathematics, Beijing Normal University, Beijing; University of New South Wales) Let $\mathbf{U} = \mathbf{U}_v(\widehat{\mathfrak{sl}}_n)$ be the quantized enveloping algebra of affine $\mathfrak{sl}_n$ with generators $E_i, F_i, K_i^{\pm 1}$ $(i = 1, \cdots n)$. It is well known that $\mathbf{U}$ admits a triangular decomposition

$$\mathbf{U} = \mathbf{U}^{-}\mathbf{U}^{0}\mathbf{U}^{+}$$

where $\mathbf{U}^+$ (resp. $\mathbf{U}^-$, $\mathbf{U}^0$) is the subalgebra generated by the E_i's (resp. F_i's, $K_i^{\pm 1}$'s). By the definition, the monomials $K_{\mathbf{a}} = K_1^{a_1} \cdots K_n^{a_n}$ with $\mathbf{a} = \sum_i a_i i \in \mathbb{Z}I$ form a basis for $\mathbf{U}^0$. It would be interesting to know monomial bases for $\mathbf{U}^+$ (and hence for $\mathbf{U}^-$). More precisely, let Ω be the set of words on the alphabet $I = \{1, 2 \cdots, n\}$. For each word $w = i_1 \cdots i_m \in \Omega$, let

$$E_w = E_{i_1} \cdots E_{i_m}, \quad F_w = F_{i_1} \cdots F_{i_m}.$$

Our main purpose is to find subsets $\Omega' \subset \Omega$ such that the set $\{E_w\}_{w \in \Omega'}$ forms a basis for $\mathbf{U}^+$. In the investigation [2] of realizing the generic composition algebra of a cyclic quiver as the positive part of a quantum affine $\mathfrak{sl}_n$, Ringel constructed certain monomial bases over some so-called condensed words whose definition is rather long and complicated. In this paper, we shall describe monomial bases for a quantum affine $\mathfrak{s}l_n$ in a more general and satisfactory way. Namely, we prove the following monomial basis theorem. **Theorem.** *Let $\mathbf{U} = \mathbf{U}_v(\widehat{\mathfrak{sl}}_n)$. Then there is a partition $\Omega = \cup_{\pi \in \Pi^s} \Omega_\pi$ such that, for any subsets $\Omega^\pm \subset \Omega$ with $|\Omega^\pm \cap \Omega_\pi| = 1$ for every $\pi \in \Pi^s$, the set*

$$\{F_y K_{\mathbf{a}} E_w \mid y \in \Omega^-, w \in \Omega^+, \mathbf{a} \in \mathbb{Z}I\}$$

forms a basis for $\mathbf{U}$. The proof of the theorem is based on the structure of generic composition algebra of a cyclic quiver in [2] and the idea of generic extensions in [1].

References

[1] M. Reineke, *Generic extensions and multiplicative bases of quantum groups at $q = 0$*, Represent. Theory **5** (2001), 147-163.

[2] C. M. Ringel, *The composition algebra of a cyclic quiver*, Proc. London Math. Soc. **66** (1993), 507-537.