THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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On Modules with Minimal Annihilators (30)

Let $\mathfrak{g}$ be a semi-simple complex, finite-dimensional Lie algebra with a fixed Cartan subalgebra $\mathfrak{h}$ and $\mathfrak{p}$ be a parabolic subalgebra of $\mathfrak{g}$ containing $\mathfrak{h}$ with Levi factor $\tilde{\mathfrak{a}}$. Fix some simple $\tilde{\mathfrak{a}}$-module V with minimal annihilator. For a Lie algebra L and L- module M let F_L be the category of finite-dimensional L-modules and coker $F_L \otimes M$ be the full subcategory of $\mathfrak{g}$-mod whose objects are cokernels of the $\mathfrak{g}$-module maps $E_1 \otimes M \to E_2 \otimes M$, where $E_1, E_2 \in F_L$. We introduce new abelian structure on the category $\mathrm{coker}(F_{\tilde{\mathfrak{a}}} \otimes V)$ $(\mathrm{coker}(F_{\mathfrak{g}} \otimes V))$ (which we will call "rough"). This abelian structure "deals with" submodules with minimal annihilators. Establishing an equivalence of these categories to certain subcategories of Harish-Chandra bimodules or representation categories of quasi-hereditary or properly stratified finite dimensional associative algebras, we derive the following results:

1. Computation of the number of simple subquotients with minimal annihilator in $E \otimes V$ for $E \in F_{\tilde{\mathfrak{a}}}$. Note that in general $E \otimes V$ is not artinian. Corresponding example for $\mathfrak{g} = \mathfrak{sl}_2 \times \mathfrak{sl}_2$ was constructed by Stafford in 1985.
2. A criterion for parabolically induced $\mathfrak{g}$-module $M_{\mathfrak{p}}(V)$ to be simple.
3. Computation of the number of simple subquotients of type $L_{\mathfrak{p}}(V')$ in $M_{\mathfrak{p}}(V)$, where V' is a simple $\tilde{\mathfrak{a}}$- module with minimal possible annihilator and $L_{\mathfrak{p}}(V')$ is the simple quotient of $M_{\mathfrak{p}}(V')$. This was obtained by reducing the problem to computation of certain simple subquotients in Verma modules.
4. "Rough" classification of categories $\mathrm{coker}(F_{\mathfrak{g}} \otimes M_{\mathfrak{p}}(V))$ in the case when semisimple part of $\tilde{\mathfrak{a}}$ is isomorphic to $\mathfrak{sl}_2$ and V is simple.

The essential part of this work was done in collaboration with V.Mazorchuk.