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Realizability of Modules over Tate Cohomology (50-60)

Given a finite group G and a field k , one can ask when a graded module over the Tate cohomology ring of G can be realized as the Tate cohomology of a kG -module. In this generality, the question seems to be difficult to answer. However, there is a simple homological criterion which tells when a module over the Tate cohomology ring is a direct summand of a realizable module. In my talk I will discuss an obstruction in the third Ext-group of a module which controls the realizability as a direct summand of a realizable module. This obstruction arises from a global class in the third Hochschild cohomology group of the Tate cohomology ring of G . The realizability problem for modules over Tate cohomology generalizes to the question when a graded module over the cohomology ring of a differential graded algebra arises as the cohomology of a dg-module. Our answer will be provided in this general setting. When time allows, I will discuss as well the question when maps between realizable modules are realizable. Moreover, our obstructions will be calculated in some small examples. This is a report on some joint work with Dave Benson and Stefan Schwede. The discussion of realizable maps is based on joint work with Apostolos Beligiannis.