THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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*Hereditary noetherian categories with a commutative function field (50-60)*

Let $k$ be a field, not necessarily algebraically closed. The talk deals with the classification of (connected) hereditary noetherian categories $\mathcal{H}$ having finite dimensional morphism and extension spaces (over $k$) and satisfying Serre duality in the form

$$D\mathrm{Ext}^1(X,Y) = \mathrm{Hom}(Y, \tau X)$$

for some automorphism $\tau$ of $\mathcal{H}$, where $D$ refers to the formation of the $k$-dual. We call these categories *hereditary noetherian* for short. A typical example is the category of coherent sheaves over a smooth projective curve $C$ defined over $k$; moreover the process of inserting (positive integral) weights (=parabolic structure) in a finite number of points of $C$, yields a hereditary noetherian category as well, to be interpreted as the category of coherent sheaves on a weighted smooth projective curve. Let $\mathcal{H}$ be hereditary noetherian, and let $\mathcal{H}_0$ denote the Serre subcategory of objects of finite length. Then the quotient category (sense of Serre-Grothendieck) is equivalent to the category of finite dimensional vector spaces over a skew field $F$, which is either a finite extension of $k$ or a finite extension of a function field $K$ in one variable over $k$. If $k$ is algebraically closed $F$ is known to be commutative. Hence the following assertions extend recent related results of I. Reiten and M. van den Bergh (J. Amer. Math. Soc. **15**, 2002).**Theorem.** *Let $\mathcal{H}$ be hereditary noetherian, and assume that $F$ is a commutative function field in one variable over $k$. Then $\mathcal{H}$ arises from the category of coherent sheaves on a smooth projective curve $C$ by insertion of weights.* **Corollary 1.** *The Grothendieck group $\mathrm{K}_0(\mathcal{H})$ of $\mathcal{H}$ has the form $\mathrm{K}_0(C) \oplus \mathbb{Z}^n$. In particular, for $k$ algebraically closed, $\mathrm{K}_0(\mathcal{H})$ is finitely generated if and only if $C$ has genus zero.***Corollary 2.** *$\mathcal{H}$ has a tilting object if and only if $C$ has genus zero.*