THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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New properties and applications of Gelfand- Zetlin modules

For a positive integer k let $\mathfrak{g}_k$ denote the Lie algebra $\mathfrak{gl}(k, \mathbb{C})$. The embeddings $\mathfrak{g}_k \subset \mathfrak{g}_{k+1}$ with respect to the left upper corner give rise to the *Gelfand- Zetlin subalgebra* Γ of $U(\mathfrak{g}_n)$, which is generated by all centers $Z(\mathfrak{g}_k)$, $1 \leq k \leq n$. The algebra Γ is commutative and generalized Γ-weight $U(\mathfrak{g}_n)$-modules with finite-dimensional generalized Γ-weight spaces are called *Gelfand-Zetlin modules*. This name comes from the fact that the action of Γ separates the elements of the Gelfand-Zetlin basis in a finite- dimensional $U(\mathfrak{g}_n)$-module. Gelfand-Zetlin modules were introduced by Drozd, Ovsienko and Futorny approx. 15 years ago together with the precise construction of the so-called generic Gelfand-Zetlin modules, which form the biggest known family of simple $U(\mathfrak{g}_n)$-modules (it depends on $n(n+1)/2$ complex parameters). In this talk we would like to report on various new properties and applications of Gelfand- Zetlin modules obtained recently by several people. This will include the following:

1. An analogue of the Harish-Chandra theorem on the intersection of annihilators of finite-dimensional modules for generic Gelfand-Zetlin modules.
2. Existence of a simple $U(\mathfrak{g}_n)$-module with a given Gelfand-Zetlin character. Some sufficient results for the uniqueness of this module.
3. Description of the category of Gelfand-Zetlin modules and some finiteness results for this category.
4. Application to maximality of Γ.
5. Admissible categories of Gelfand-Zetlin modules and their connection with Enright-complete modules in the category $\mathcal{O}$ and with Harish-Chandra bimodules.
6. Applications of these categories to the study of generalized Verma modules.
7. Application of Gelfand-Zetlin modules to the construction of an embedding of $U(\mathfrak{g})$ in the localized Weyl algebra.
8. Application of Gelfand-Zetlin modules to the study of annihilators of Verma modules(Duflo Theorem).