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ABSTRACTS 1.2

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A generalization of the Auslander formula

A generalization of the Auslander Formula is provided under the Auslander condition. Let Λ be a left and right Noetherian ring and modΛ the category of all finitely generated left Λ-modules. Let the grade of a module $M \in \mathrm{mod}\Lambda$; $\mathrm{grade}_\Lambda M := \inf\{i : \mathrm{Ext}^i_\Lambda(M, \Lambda) \neq 0\}$. (Au) For all integers $i \geq 0$ and all Λ^{op}- submodules $N \subset \mathrm{Ext}^i_\Lambda(M, \Lambda)$, it holds that $\mathrm{grade}_{\Lambda^{\mathrm{op}}} N \geq i$. We assume that all Λ-modules and Λ^{op}-modules satisfy the condition (Au). Let $M \in \mathrm{mod}\Lambda$. Put $M_k := \mathrm{Ext}^k_{\Lambda^{\mathrm{op}}}(\mathrm{Tr}\Omega^{k-1}M, \Lambda)$ for $k \geq 1$ and $M_0 = M$. **Theorem** *Let $g := \mathrm{grade}_\Lambda M < \infty$. Then there exists a filtration of Λ-submodules of $M : M_0 = \cdots = M_g \supset \cdots \supset M_k \supset \cdots$ such that i) in case $k = g$, the following sequence is exact,*

$$0 \to M_{g+1} \to M_g \to \mathrm{Ext}^g_{\Lambda^{\mathrm{op}}}(\mathrm{Ext}^g_\Lambda(M, \Lambda), \Lambda) \to \mathrm{Ext}^{g+2}_{\Lambda^{\mathrm{op}}}(\mathrm{Tr}\Omega^g M, \Lambda) \to 0,$$

ii) if $\mathrm{Ext}^k_{\Lambda^{\mathrm{op}}}(\mathrm{Ext}^k_\Lambda(M, \Lambda), \Lambda) \neq 0$, then $\mathrm{grade}_\Lambda M_k = k$, $M_k \neq M_{k+1}$, M_k/M_{k+1} is pure of grade k, iii) if $\mathrm{Ext}^k_{\Lambda^{\mathrm{op}}}(\mathrm{Ext}^k_\Lambda(M, \Lambda), \Lambda) = 0$, then $M_k = M_{k+1}$. Moreover, if G- dim ${}_\Lambda M = d < \infty$, then iv) $M_{d+1} = 0$ and $M_d = \mathrm{Ext}^d_{\Lambda^{\mathrm{op}}}(\mathrm{Ext}^d_\Lambda(M, \Lambda), \Lambda)$.

References

[1] M. Hoshino and K. Nishida, *A generalization of the Auslander formula*, preprint 2002, http://math.shinshu-u.ac.jp/ kenisida/