THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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On a partial order of tilting modules

This is a report on some joint work with Dieter Happel. Let Λ be an artin algebra over a commutative ring and let modΛ be the category of finitely generated left Λ-modules. For a module $M \in \mathrm{mod}\Lambda$ we denote by $\mathrm{pd}_\Lambda M$ the projective dimension of M.

A module $T \in \mathrm{mod}\Lambda$ is called a tilting module if (i) $\mathrm{pd}_\Lambda T < \infty$, if (ii) $\mathrm{Ext}^i_\Lambda(T,T) = 0$ and if (iii) there exists an exact sequence $0 \to {}_\Lambda\Lambda \to T^0 \to \cdots \to T^r \to 0$ with $T^i \in \mathrm{add}T$ for all $0 \le i \le r$. Here add$T$ denotes the subcategory of modΛ whose objects are direct sums of direct summands of T. We say that a tilting module is basic if in a direct sum decomposition of T the indecomposable direct summands of T occur with multiplicity one.

Following Auslander and Reiten we consider for a basic tilting module the right perpendicular category

$$T^\perp = \{X \in \mathrm{mod}\Lambda \mid \mathrm{Ext}^i_\Lambda(T,X) = 0\}.$$

We consider the set $\mathcal{T}_\Lambda$ of all tilting modules over Λ up to isomorphism. Following Riedtmann and Schofield we define a partial order $\le$ on $\mathcal{T}_\Lambda$. For $T, T' \in \mathcal{T}_\Lambda$ we set $T \le T'$ provided $T^\perp \subset T'^\perp$. Moreover Riedtmann and Schofield defined the quiver $\overrightarrow{\mathcal{K}}_\Lambda$ of tilting modules as follows. The vertices are the elements of $\mathcal{T}_\Lambda$. There is an arrow $T' \to T$ if $T' = M \oplus X$, $T = M \oplus Y$ with X, Y indecomposable and there is an exact sequence $0 \to X \to \widetilde{M} \to Y \to 0$ with $\widetilde{M} \in \mathrm{add}T$. We denote by $\mathcal{K}_\Lambda$ the underlying graph of $\overrightarrow{\mathcal{K}}_\Lambda$.

Riedtmann and Schofield raised the question whether $\mathcal{K}_\Lambda$ is the Hasse diagram for $(\mathcal{T}_\Lambda, \le)$. We outline the proof that this is indeed the case and discuss the existence of minimal elements for $(\mathcal{T}_\Lambda, \le)$.