THE FIELDS INSTITUTE
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ABSTRACTS 1.2

BIRGE HUISGEN-ZIMMERMANN
University of California at Santa Barbara

Top-stable degenerations by way of Grassmannians (50-60)

Given a finite dimensional algebra A over an algebraically closed field, we introduce and explore Grassmannians parametrizing the isomorphism types of representations with fixed tops and determine the fibre structure of the corresponding representation map in case the tops are squarefree (here a remarkably transparent pattern emerges), as well as the fibre dimensions. These fibres – they are orbits under the operation of the automorphism group Aut of a projective cover of the top – are intimately related to the GL-orbits and their closures in the classical module varieties, but more accessible in the Grassmannian setting, due to the presence of a big unipotent radical in the acting group Aut. In particular, the points of the Grassmannian orbit closures (accessible via an Aut-stable affine cover of the Grassmannian, which can be readily obtained from quiver and relations of A) correspond to degenerations in the classical sense. This leads to an overview of the "top-stable" degenerations of representations with squarefree tops, complete with hierarchy of immediate successors in the degeneration order. One can roughly summarize the picture by saying that the Grassmannian comes "as close as possible" to a geometric quotient of the pertinent subvariety of the classical module variety modulo its GL-action. One reason for this assertion lies in the fact that any given Aut-orbit in the Grassmannian of local modules with fixed top is closed if and only if it is reduced to a point. Another reason is the fact that a geometric quotient of the classical affine module variety modulo its GL-action exists if and only if the Grassmannian has a geometric quotient modulo its Aut-action, a situation which can be characterized. In case of existence, the two quotients coincide.