THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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Hall algebras and their relations to quantized enveloping algebras
(five lectures)

Lecture I. In the first lecture, we introduce the definition of the Ringel-Hall algebras, the Green comultiplication and their Drinfeld double forms, and present the theorems of Ringel-Green and Sevenhant-Van den Bergh, namely, the double composition algebras are canonically isomorphic to the quantized enveloping algebras of the corresponding Kac-Moody algebras and the double Ringel-Hall algebras themselves are isomorphic to the quantized enveloping algebras of the corresponding generalized Kac-Moody algebras (in the sense of Borcherds).

Lecture II. In the second lecture, we mainly concern with the tame-affine example: Kronecker quiver and $\tilde{sl}(2)$. We first introduce the module category of Kronecker quiver and its derived category. Then we will show that Ringel-Hall algebra approach over the derived category will give the explicit description of the basis and the structure constants of $\tilde{sl}(2)$ (Frenkel-Malkin-Vybornov and Peng-Xiao). More interesting, the approach will give us a global realization of $U_q(\tilde{sl}(2))$ over the derived category such that Drinfeld new realization are natural relations in Ringel-Hall algebras (Karpranov and P.Zhang).

Lecture III. In the third lecture, we present a new proof of a weak form of the Kac theorem for species over a finite field by using the Ringel- Hall algebra approach. It is quite different from earlier techniques. Then we show that an interesting relation between the Kac conjecture on representations of quivers and the structure of the Ringel-Hall algebras holds for arbitrary species. This relation was first observed by Sevenhant-Van den Bergh in a special case.

Lecture IV. In the forth lecture, we apply the Bernstein-Gelfand-Ponomarev reflection functors to the Drinfeld doubles of Ringel-Hall algebras to re-discover the Lusztig's symmetries and the braid group relations, not only on the double composition algebras but also on the whole double Ringel- Hall algebras and their integrable modules.

Lecture V. In the fifth talk, we will concentrate on Lusztig's geometric approach to construct generic subalgebras of Hall algebras. Lusztig constructed the composition algebra as the graded Grothendieck group of certain categories of semi-simple complexes of sheaves on certain class of the nilpotent varieties. These categories are generated by the simple perverse sheaves appearing in certain intersection complexes over a the generalizations of Springer resolutions. We will construct the parabolic version of the Springer resolution corresponding to a class of absolutely indecomposable representations of the quiver and construct an algebra structure on the graded Grothendieck group and prove that they are generic subalgebras of Hall algebras. In this way, we can construct the canonical bases of certain classes of generic subalgebras for tame quivers.