Stratified algebras arising in Lie theory

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The famous category $\mathcal{O}$ of Bernstein, Gelfand and Gelfand, associated with a semi-simple finite-dimensional complex Lie algebra with a fixed triangular decomposition, was one of the principal motivations and basic examples for the notions of highest weight category and quasi-hereditary algebras, which was introduced in 1987-1988 by Cline, Parshall and Scott (resp. by Scott).

However, the category $\mathcal{O}$ does not exhaust all natural examples of categories canonically associated with Lie algebras. In early 80's Bernstein and Gelfand introduces certain categories of the so-called Harish-Chandra bimodules over the universal enveloping algebras, whose blocks also correspond to finite-dimensional associative algebras. In most cases (regular right action of the center) these blocks are equivalent to blocks of $\mathcal{O}$ and hence are highest weight categories. But if the right action of the center is singular, this is no longer true in general.

The appropriate class of finite-dimensional algebras, sufficient to describe blocks of Harish-Chandra bimodules and many other parabolic generalizations of the category $\mathcal{O}$, appears to be the class of properly stratified algebras, as defined by Dlab in 2000, which can be considered as a subclass of the class of stratified algebras, introduced and studied by Cline, Parshall and Scott in 1996. This connection became clear after an abstract description of Enright's completion functors on the category $\mathcal{O}$ was found. Although properly stratified algebras are sufficient to describe Harish-Chandra bimodules, there are other parabolic generalizations of $\mathcal{O}$, whose blocks are not properly stratified but only admit a stratification in the sense of Cline, Parshall and Scott.

The aim of this talk is to discuss all categories and techniques mentioned above. For all these categories one has an analogue of the BGG-reciprocity. The corresponding finite-dimensional algebras have a kind of "parabolic" decomposition, generalizing the notion of the triangular decomposition, introduced by König. One can study tilting modules and Ringel duals etc.

The information about the parabolic analogues of the category $\mathcal{O}$ is important for example for the study of generalized Verma modules (parabolic analogs of Verma modules).

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