THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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Irreducible components of varieties of modules

We prove some basic results about irreducible components of varieties of modules for an arbitrary finitely generated associative algebra. Our work generalizes results of Kac [2] and Schofield [4] on representations of quivers, but our methods are quite different, being based on deformation theory.

Let k be an algebraically closed field, and let A be a finitely generated k-algebra (associative, with 1). By $\operatorname{mod}_A^d(k)$ we denote the variety of d-dimensional A-modules.

Given irreducible components $C_1 \subseteq \operatorname{mod}_A d_1(k)$ and $C_2 \subseteq \operatorname{mod}_A^{d_2}(k)$ let

$$\operatorname{ext}_A^1(C_1, C_2) = \min\{\dim \operatorname{Ext}_A^1(M_1, M_2) \mid (M_1, M_2) \in C_1 \times C_2\}.$$

For irreducible components $C_i \subseteq \operatorname{mod}_A^{d_i}(k)$,$1 \le i \le t$, we consider all modules of dimension $d = d_1 + \cdots + d_t$, which are of the form $M_1 \oplus \cdots \oplus M_t$ with the M_i in C_i, and we denote by $C_1 \oplus \cdots \oplus C_t$ the corresponding irreducible subset of $\operatorname{mod}_A^d(k)$. The following theorem is proved in [1]:

Theorem. *If $C_i \subseteq \operatorname{mod}_A^{d_i}(k)$, $1 \le i \le t$, are irreducible components and $d = d_1 + \cdots + d_t$, then $\overline{C_1 \oplus \cdots \oplus C_t}$ is an irreducible component of $\operatorname{mod}_A^d(k)$ if and only if $\operatorname{ext}_A^1(C_i, C_j) = 0$ for all $i \neq j$.*

This theorem has numerous applications.

The case of Lusztig's nilpotent variety is particularly interesting, since by work of Kashiwara and Saito [3] its irreducible components are in 1-1 correspondence with the elements of the canonical basis of the positive part of the quantized enveloping algebra of a Kac-Moody Lie algebra. Because these varieties arise as module varieties, one can hope to use decomposition properties of modules, and homological algebra techniques, to study the irreducible components.

References

[1] *W. Crawley-Boevey, J. Schröer*, Irreducible components of varieties of modules. J. Reine Angew. Math. (to appear).

[2] *V. G. Kac*, Infinite root systems, representations of graphs and invariant theory II. J. Algebra **78** (1982), 141–162.

[3] *M. Kashiwara, Y. Saito*, Geometric construction of crystal bases. Duke Math. J. **89** (1997), 9–36.

[4] *A. Schofield*, General representations of quivers. Proc. London Math. Soc. **65** (1992), 46–64.