THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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A DOUBLE-TREE REPRESENTATION FOR BOREL SETS

It is a classical result by Kuratowski that for any Borel $\Pi^0_{1+\xi+1}$ set $A \subset \omega^\omega$ there exists a continuous bijection f from ω^ω onto ω^ω with inverse mapping of class ξ+1natorial version of this result: More precisely to any Borel $\Pi^0_{1+\xi+1}$ set $A \subset \omega^\omega$ we associate a decreasing family $(R_\eta)_{\eta \leq \xi}$ of tree relations with domain $\omega^{<\omega}$ (so $R_\eta \subset \omega^{<\omega} \times \omega^{<\omega}$) with R_0 the standard extension relation on $\omega^{<\omega}$, each $R_{\eta+1}$ is "large" in R_η, and moreover the last relation R_ξ is generated by two tree relations R^+ (associated to A) and R^- (associated to A^c). The notion of largeness involved here is purely algebraic; however it insures that the canonical mapping which to any infinite branch α relatively to R_ξ, assigns the unique infinite branch β relatively to R_0 containing α satisfies Kuratowski's Theorem, that is $\alpha \mapsto \beta$ is a continuous bijection and its inverse mapping $\beta \mapsto \alpha$ is of class $\xi + 1$.

This representation is used in a crucial way to prove new results on Borel selectors.