Banach spaces with small operator algebras.

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This is joint work with J. Lopez-Abad† and S. Todorcevic‡.

We present a reflexive Banach space X_{ω_1} with a transfinite basis. $(e_\alpha)_{\alpha<\omega_1}$ with the following properties, among others:

1. There is no unconditional basic sequence in X_{ω_1}.
2. Each block sequence $(x_n)_{n<\omega}$ relative to the basis generates an Hereditarily Indecomposable (HI) space.
3. Each operator $T \in \mathcal{L}(X_{\omega_1})$ is of the form $T = T_f + S$, where S is a strictly singular operator and where T_f is defined by $T_f(e_\alpha) = f(\alpha)e_\alpha$ for certain continuous function $f : \omega_1 \to \mathbb{R}$.
4. X_{ω_1} is not isomorphic to any proper subspace.
5. X_{ω_1} is not isomorphic to any non trivial quotient.
6. The only projections are of the form $P_{I_1} + \cdots + P_{I_n} + S$, where I_i are intervals of ordinals, P_{I_i} are the natural projections associated to I_i, and S is strictly singular.
7. The basis $(e_\alpha)_{\alpha<\omega_1}$ is nearly spreading. In particular, $(e_n)_{n<\omega}$ is a nearly spreading basis[1] for the reflexive HI space $\overline{\langle e_n\rangle_{n<\omega}}$, which makes the space considerably different from the HI space by Gowers-Maurey.

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[1]There is some $K > 0$ such that for any finite set $\{k_1 < \cdots < k_n\}$ of integers and any integer N there is some $\{l_1 < \cdots < l_n\}$ with $N \leq l_1$ such that the operator T defined by $T(e_{k_i}) = e_{l_i}$ is a K-isomorphism.