THE GENERALIZED BANACH CONTRACTION THEOREM

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Abstract. Assume that (X,d) is a complete metric space and $T : X \to X$ is a self map of X with the following property: There is a positive integer J and an $M \in [0,1)$, so that for any couple $x, y \in X$,

$$\min\{d(T^i(x), T^i(y)) : i = 1, 2, \dots J\} \leq M d(x,y).$$

The assumption of the well known Banach Contraction Theorem, is the case where $J = 1$. In this case it can be proved that T is uniformly continuous and has a fixed point.

In the general case where $J > 1$, T need not be continuous. However, the Generalized Banach Contraction Theorem states that even in this case T also has a fixed point.

We shall present a proof of this theorem, which is mainly of combinatorial nature.

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