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*$\ell^1$ -spreading models in mixed Tsirelson spaces*

Joint work with Wee Kee Tang We consider the problem of determining the existence of higher order  $\ell^1$ -spreading models in mixed Tsirelson spaces. Let  $(\theta_n)$  be a nonincreasing null sequence in  $(0, 1)$  such that  $\theta_{m+n} \geq \theta_m \theta_n$  for all  $m, n$ . Argyros, Deliyanni and Manoussakis proved that if  $\lim \theta_n^{1/n} = 1$ , then every block subspace of  $T[(\theta_n, \mathcal{S}_n)_{n=1}^\infty]$  contains an  $\ell^1$ - $\mathcal{S}_\omega$ -spreading model. We have the following results.

1. The mixed Tsirelson space  $T[(\theta_n, \mathcal{S}_n)_{n=1}^\infty]$  contains an  $\ell^1$ - $\mathcal{S}_\omega$ -spreading model if and only if  $\lim_n \limsup_m \theta_{m+n}/\theta_m > 0$ .
2. Let  $X$  be a block subspace of the mixed Tsirelson space  $T[(\theta_n, \mathcal{S}_n)_{n=1}^\infty]$ . The following are equivalent.
  - (a) The Bourgain  $\ell^1$ -index of  $X$  is  $\omega^{\omega \cdot 2}$ .
  - (b)  $X$  contains an  $\ell^1$ - $\mathcal{S}_\omega$ -spreading model.
  - (c) Every normalized block sequence in  $X$  has a further normalized block  $(x_n)$  that is equivalent to the sequence  $(e_{k_n})$ . Here  $(e_k)$  is the unit vector basis of  $T[(\theta_n, \mathcal{S}_n)_{n=1}^\infty]$  and  $k_n = \max \text{supp } x_n$ .

Moreover, if every block subspace  $X$  has the equivalent properties above, then  $T[(\theta_n, \mathcal{S}_n)_{n=1}^\infty]$  is arbitrarily distortable.

Note that the result of Argyros et al. can be deduced from (2).