THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

CLAUDE BARDOS
Universite Paris VI

Relevance of the Time Dependent Hartree Fock Approximation in the Mean Field Scaling?

YANN BRENIER
Universite de Nice

Derivation of Particle and String Motions from the Born-Infeld Electromagnetic Field Theory

The Born-Infeld system is a nonlinear version of Maxwell's equations. We first show that, by using the energy density and the Poynting vector as additional independent variables, the BI system can be augmented as a 10x10 system of hyperbolic conservation laws. The resulting augmented system has some similarity with MHD equations and enjoy remarkable properties (existence of a convex entropy, galilean invariance, full linear degeneracy). In addition, the propagation speeds and the characteristic fields can be computed in a very easy way, in contrast with the original BI equations. Then, we investigate several limit regimes of the augmented BI equations, by using a relative entropy method going back to Dafermos, and recover, the Maxwell equations for low fields, some pressureless MHD equations (describing string motion) for high fields, and pressureless gas equations for very high fields.

ALAIN J. BRIZARD
Saint Michael's College

Variational Formulations of Exact and Reduced Vlasov-Maxwell Equations

A new Eulerian variational principle is presented for the exact Vlasov-Maxwell equations [1]. This variational principle is based on constrained variations of the Vlasov distribution in an eight-dimensional extended phase space. The standard energy-momentum conservation law is then derived explicitly by the Noether method. This new variational principle can be applied to various reduced Vlasov-Maxwell equations in which fast time scales have been asymptotically eliminated (e.g., low-frequency gyrokinetic Vlasov-Maxwell theory [2]).

[1] A.J. Brizard, Phys. Rev. Lett. 84, 5768-5771 (2000).

[2] A.J. Brizard, Phys. Plasmas 7, 4816-4822 (2000).

STEVE COWLEY
Imperial College, London

Transport in Tangled Astrophysical Magnetic Fields

In many astrophysical plasmas the field is tangled on scales smaller than the mean free path. However we would like to describe the plasma on scales much bigger than the mean free path. I will describe the problem of calculating average transport coefficients in this limit and present some recent results.

PIERRE DEGOND
Universite Paul Sabatier, Toulouse

Quantum Hydrodynamics and Quantum Diffusion Models Derived from the Entropy Principle

This work addresses the question of deriving hydrodynamic and diffusion models from a macroscopic limit of quantum kinetic models. This question is of key importance in a certain number of fields such as plasma or semiconductor mesoscopic modeling. The major difficulty to solve when investigating hydrodynamic limits is that of the closure relation (i.e. finding the equation-of-state of the system). This problem is resolved in the classical framework by assuming that the microscopic state is at local thermodynamical equilibrium. Such a state realizes the minimum of the entropy functional subject to local constraints of mass, momentum and energy. We propose an extension of this method to quantum systems. This leads to hydrodynamic models with non-local closure relations. These models preserve the monotony of the entropy functional. The same approach leads to a proposal for quantum extensions of the classical Boltzmann or BGK collision operators. Finally, it allows the investigation of diffusion limits of quantum systems (which are distringuished from hydrodynamic limits by the nature of the scaling) ands lead to quantum extension of the well-established drift-diffusion and energy-transport models.

LAURENT DESVILLETTES
Ecole Normale Superieure, Cachan

Some Results About the Smoothness of the Solutions of Boltzmann and Landau Equations

We consider kinetic equations with collision kernels of the form

$$\partial_t f + v \cdot \nabla_x f = Q(f,f),$$

where $f \equiv f(t,x,v)$ is the density in the phase space, and Q is the (quadratic) Boltzmann or Landau operator.

We present the available results about the smoothness with respect to both x and v variables. Those results are obtained using

1. Results of smoothness for the spatially homogeneous equations
2. Averaging lemmas (more precisely, their variant without Fourier transform in time)
3. A procedure of convolution in the v variable which enables to transfer the smoothness in the x variable from the averages in v to the whole function

Propagation of singularities and regularity are shown to hold for the Boltzmann equation with an integrable cross section, while smoothing effects are proven in the case of the Landau equation.

We also discuss the applications of the smoothness properties to the study of the large time behavior of the solutions.

PATRICK H. DIAMOND
University of California, San Diego

Granulation Formation and Turbulent Trapping in Wave Kinetics

Plasma physics has contributed two notable problems to nonlinear science, namely the problem of Vlasov turbulence and the nonlinear modulation instability or "collapse" problem, typified by the Langmuir turbulence problem. Here, we combine the ideas and methods of these two problems to describe turbulent trapping in nonlinear modulational dynamics.

We argue that it is convenient to classify approaches to the well-known modulation dynamics problems according to their location in a 2D space parametrized by the eikonal ray Chirikov parameters (which exceeds unity when group-phase resonances overlap) and the ray Kubo number K (which exceeds unity when a trapped ray bounces several times in a single structure). The regime where $s > 1$ and $K \gtrsim 1$ is the regime of *turbulent trapping*. We show that both the 1D Langmuir and 2D drift wave-zonal flow interaction problems in this regime may be mapped to the 1D Vlasov turbulence problem in the regime of turbulent trapping. We then apply the theory of phase space density granulation, developed for turbulent trapping in Vlasov systems, to describe the turbulent trapping of waves in modulation structures. We focus on three specific issues, namely:

i.) the modification of the dynamics of relaxation of the wave population $< N >$ induced by granulations. In particular, the diffusive quasi-linear wave-kinetic equation is replaced by a Lenard-Balescu type equation, with a drag, induced by incoherent fluctuations in N (i.e. granulations).

ii.) the appearance of novel modulational instability mechanisms related to granulations.

iii.) the role and significance of caustics in the theory. These are interesting as they have no counterpart in the Vlasov turbulence problem.

Special attention will be given to turbulent trapping in drift wave-zonal flow turbulence.

YVES ELSKENS
CNRS-universite de Provence, Marseilles

From N Particles Interacting with M Waves, to Vlasov and to Quasilinear Equations–Chaos and Granularity in the Kinetic Limit

The Vlasov equation is the standard description of many-body systems with mean-field coupling [1,4,5]. However, chaotic dynamics imply a departure from the Vlasov behaviour on relatively short time scales, so that the many-body evolution often looks more stochastic than predicted by the Vlasov equation. We consider two examples. First, for a plasma beam with N particles interacting with a wave, finite-N effects (granularity of the empirical, microscopic distribution function) have been shown to drive the system over long times towards a thermal equilibrium, which is not the (metastable) state corresponding to the saturation of the beam-wave instability in the Vlasovian picture. [1,3] Second, quasilinear theory was developed in 1962 to describe the saturation of the weak warm beam-plasma instability, which involves the development of a Langmuir turbulence and the formation of a plateau in the electron velocity distribution function. The original derivations assume that particle orbits are weakly perturbed (quasi linear description), though the plateau formation is the result of a strong chaotic diffusion of the beam particles. Over two decades a controversy has developed about the validity of quasilinear equations in the chaotic saturation regime within the Vlasovian description of the problem, and is not yet settled. It is worth noting that, in contrast to the Vlasov equation, the quasilinear equations imply irreversible behaviour, with an H-theorem. We derive the quasilinear equations in the strongly nonlinear chaotic regime without resorting to the previous description. Instead, the Langmuir wave-beam system is described as a Hamiltonian system with a finite number of degrees of freedom. A new technique enables one to derive the quasilinear evolution in the limit of a continuous wave spectrum (i.e. of infinite Chirikov resonance overlap parameter s). The close to rigorous argument takes advantage of the fact that the motion of any particle and the evolution of any wave depend only weakly on each other. [1,2]

[1] Y. Elskens and D. Escande, Microscopic dynamics of plasmas and chaos (IoP Publishing, Bristol, 2002).

[2] D. Escande and Y. Elskens, Phys. Lett. A 302 (2002) 110-118 ; Phys. Plasmas 10 (2003) 1588-1594 ; Plasma Phys. Contr. Fusion 45 (2003) A115-124.

[3] M-C. Firpo, F. Doveil, Y. Elskens, P. Bertrand, M. Poleni and D. Guyomarc'h, Phys. Rev. E 64 (2001) 016411.

[4] M-C. Firpo and Y. Elskens, J. Stat. Phys. 93 (1998) 193-209.

[5] H. Spohn, Large scale dynamics of interacting particles ((Springer, Berlin, 1991).

LASZLO ERDOS
Georgia Institute of Technology

Towards the Quantum Brownian Motion

The mystery of the erratic motion of pollen grains suspended in water, named after its explorer, Robert Brown, was solved by Einstein in 1905. His kinetic theory, based upon lightwater molecules continuously bombarding the heavy pollen, provided not just an explanation of diffusion from the Newtonian mechanics, but also the most direct evidence yet for the existence of atoms and molecules. Since the discovery of quantum mechanics it has been a major challenge to verify the emergence of diffusion from the Schrödinger equation. In this talk I will report on a mathematically rigorous derivation of a diffusion equation as a long time scaling limit of a random Schrödinger equation in a weak, uncorrelated disorder potential.

IRENE M. GAMBA
The University of Texas at Austin

Sharp Estimates to Solutions of Homogeneous Boltzmann Type Problems

We have recently derived sharper form of the Povzner Lemma which essentially provides control of the gain operator for classical bilineal collisional integral forms for hard spheres with rather general differential cross sections. This estimate provides the tool, combined with recent maximum principle for this Boltzmann type of equations and analysis of the Carleman representation of the gain operator, to obtain pointwise bounds to stationary smooth solutions of these type of Boltzmann equations by integrable functions with exponential decay. The decay exponent depends only on the balance between the forced term and the loss operator corresponding to the problem under consideration.

Examples and applications range from elastic Boltzmann for hard spheres to inelastic Boltzmann with diffusive forcing, selfsimilar inelastic Boltzmann and inelastic shear flow. The corresponding stationary states for inelastic interactions are rigorously shown to be overpopulated with respect to classical Maxwellians.

I will present the new sharp Povzner inequality and the linking to the pointwise bound for tail decay.

These techniques may aid a better understanding to the regularity properties of the solutions to the underlying Boltzmann eqaution as well as the possible corrections to asymptotics for classical hydrodynamic limits.

ROBERT GLASSEY
Indiana University

The Vlasov-Maxwell System

The Vlasov-Maxwell equations are the equations of motion for a collisionless plasma: a high temperature, low density ionized gas in which electromagnetic forces dominate collisional effects. In this lecture we will probe the major open question: does the initial-value problem have a smooth global solution for smooth data of unrestricted size in 3 space dimensions? That is, are there shocks in collisionless plasmas? Partial answers and known results will be surveyed, including weak solutions, solutions with small data, the resolution of the large-data problem in two space dimensions, classical vs. relativistic formulations and the use of symmetry.

ISAAC GOLDHIRSCH
Tel-Aviv University

Inelastic Kinetic Theory: The Granular Gas

The talk will consist of two parts. The first part will comprise a description of the main features of granular gases and their theoretical description. This part will include a brief description of clustering, collapse, the Maxwell demon effect, floating clusters, the question of the lack of scale separation in granular gases, the Brazil nut effect, granular hydrodynamics and (time allowing) granular boundary conditions. Should the audience be familiar with the first part, emphasis will be put on the second part, which consists of recent results, mostly on the theory of granular gases with weak frictional restitution. The latter case is an example of a system whose analysis requires the generalization of the Champman-Enskog method. Time allowing, the question of moderately dense granular gases will be briefly described. A list of some open question will conclude the talk.

FRANCOIS GOLSE
Universite Paris VII

The Boltzmann-Grad limit for the periodic Lorenz gas

The Lorentz gas is a gas of point particles that move freely in some environement of spherical obstacles, with elastic collisions at the surface of the obstacles. In 1905, H.A. Lorentz considered this simple dynamical sytem as a model for the motion of electrons in metals,and proposed to use the methods of kinetic theory to describe it in some appropriate scaling limit. While this has been rigorously justified by Gallavotti (1972) in the case of a random distribution of obstacles, the case of a periodic distribution of obstacles remains to be understood. This talk will review the latter case.

ALEX GOTTLIEB
University of Vienna

Quantum Effects in Mean Field Electron Dynamics

Vlasov's equation for plasmas can be derived from a model of quantum N-electron dynamics: one considers "jellium" electrons with a truncated Coulomb interaction and derives the classical Vlasov equation in the thermodynamic limit [1]. Recent work indicates that the time-dependent Hartree equation (a weakly nonlinear Schroedinger equation) provides a kind of semiclassical correction to Vlasov's equation in the same thermodynamic limit [2]. Thus, essentially classical behavior arises in the thermodynamic limit. However, if there is some confinement of electrons at the nanoscale, as there is in semiconductor quantum wells, then quantum behavior must survive the limit N tends to infinity. In practice, time-dependent Schroedinger-Poisson equations are used to simulate such systems. Although there is still no fully satisfactory derivation of such approximations from an N-electron model, the Schroedinger-Poisson equation can be derived rigorously if a mean field scaling is assumed [3]. I will review the results of [1] and [2], and speculate upon the application of [3] to semiconductor quantum wells.

[1] G. Sewell, Quantum statistical derivation of the hydrodynamics of a plasma model. J. Math. Phys., 26 (9): 2324-2334 (1985)

[2] A. Elgart, L. Erdos, B. Schlein, H.T. Yau. Nonlinear Hartree equation as the mean field limit of weakly coupled fermions. Preprint: http://lanl.arxiv.org/abs/math-ph/0311043

[3] C. Bardos, L. Erdos, F. Golse, N.J. Mauser and H.T. Yau. Derivation of the Schroedinger-Poisson equation from the quantum N-particle Coulomb problem. C. R. Acad. Sci. Ser. I Math, 334: 515–520 (2002)

YAN GUO
Brown University

A Nonlinear Energy Method in Boltzmann Theory

A nonlinear energy method has been developed recently to construct global in time smooth solutions near Maxwellians for the Boltzmann equation. As the key feature of such a method, the linearized collision operator L can be shown to be positive definite for solutions to the Boltzmann equation close to Maxwellians, even in the presence of spatial dependence. The main idea and applications of such a method will be discussed.

GREG HAMMETT
Princeton Plasma Physics Lab

Non-local Fluid Closure Approximations to Model Long Mean-Free-Path Dynamics

I will review work in plasma physics over the last decade on extensions of fluid equations beyond the collisional Chapman-Enskog regime. These extensions have employed various closure approximatons that model effects such as collisonless phase-mixing and Landau-damping. Two approaches to deriving these models will be illustrated, by a fitting procedure or by a renormalization-like procedure (by Parker and Carati). These models can be shown to give n-pole Pade approximations to the exact linear kinetic response. We will describe some of the limitations of these fluid models, and compare with fully kinetic computational approaches, which are rapidly advancing.

REINHARD ILLNER
University of Victoria

Three Exotic Applications of Kinetic Equations

Analytical and computational challenges arise in a remarkable variety of contexts, and some of the favourite hard analytical questions from classical kinetic theory arise over and over again. In this talk I will present three examples:

1. Coal dust in an incinerator, modelled as an inelastic rarefied particle system in a diffusive background. The steady boundary value problem associated with the corresponding Boltzmann-Fokker-Planck equation is very challenging from an analytical point of view; a potential solution is likely to require combinations of variational techniques for the Fokker-Planck case, pioneered by Baouendi and Grisvard, and structural information about the inelastic Boltzmann collision term as investigated by Gamba, Panferov and Villani.

2. Fokker-Planck equations modeling the evolution of the average orientation of fiber pieces moving in Stokes flow. This is a problem relevant in the industrial problem of plastic moulding, where the inclusion of glass or steel "fibers" in liquid plastic is used to change the elastic properties of the finished product. The equations driving the orientation of the individual fiber are a system of ODEs known as Jeffery's equation (derived by Jeffery in 1922), and we are going to discuss the Fokker-Planck equation driving the distribution of many such orientations on the surface of the unit sphere.

3. Coupled systems of Fokker-Planck equations as models of multilane traffic flow. It will be shown how reasonable assumptions on lane change probabilities, reaction times and braking behaviour give rise to bifurcated fundamental diagrams, traffic synchronization, and stop-and-go waves.

DAVE LEVERMORE
University of Maryland

Gas Dynamics Beyond Navier-Stokes

Compressible fluid dynamical systems are traditionally derived from a kinetic theory by either a Hilbert or Chapman-Enskog expansion in small Knudsen number. These derivations fail to produce formally well-posed systems beyond the compressible Navier-Stokes system, which arises as a first order correction to the Euler system. Here we offer an alternative derivation that produces a family of compressible fluid dynamical systems. The first two systems are again the Euler and Navier-Stokes systems, but one can go further. Every system in the family dissipates entropy and is formally well-posed over domains without boundary. The validity of these systems formally extends into transition regimes. These systems extend the compressible Navier-Stokes system and also extend a class of fluid dynamical systems developed by Maxwell, Kogan, Sone, and others that are not derivable from the Navier-Stokes system.

GIOVANNI MANFREDI
Universit de Nancy, France

How to Model Quantum Plasmas

Traditional plasma physics has mainly focused on regimes characterized by high temperatures and low densities, for which quantum-mechanical effects have virtually no impact. However, recent technological achievements (particularly on miniaturized semiconductor devices and nanoscale objects) have made it possible to envisage practical applications of plasma physics where the quantum nature of the charged particles plays a crucial role. In this paper, I shall review different approaches to the problem of modeling quantum effects in electrostatic collisionless plasmas. The full kinetic model is in principle given by the Wigner equation, which is the quantum analog of the classical Vlasov equation. The Wigner formalism is particularly attractive, as it recasts quantum mechanics in the familiar classical phase space. However, this is at the cost of having to deal with distribution functions that can take negative values. Equivalently, the Wigner model can be expressed in terms of N one-particle Schrdinger equations, coupled by Poissons equation: this is the Hartree formalism, which is also the quantum analog of the multi-stream approach, well-known in plasma physics since the works of John Dawson in the sixties. In order to reduce the complexity of the above approaches, it is possible to develop a quantum fluid model by taking velocity-space moments, as is usually done for classical plasmas. Interestingly, this quantum fluid model can be recast as a single nonlinear Schrdinger equation. Finally, certain regimes (particularly at large excitation energies) can be described by semiclassical kinetic models (Vlasov-Poisson), provided that the initial ground-state equilibrium is treated quantum-mechanically using a Fermi-Dirac distribution. Practical applications of the above results to the physics of metal clusters, nanoparticles and thin metal films will be illustrated.

NORBERT J. MAUSER
WPI Wien

Incompressible Euler and E-MHD as Scaling Limits of the Vlasov-Maxwell System

In this talk we sketch the basic ideas and methods of a series of papers of Yann Brenier, Norbert Mauser, Marjolaine Puel and Laure Saint-Raymond. We consider asymptotic limits of the (relativistic) Vlasov-Maxwell system with different scalings concerning the magnetic field. The combined "quasi-neutral" and non-relativistic limits thus lead to two different limit systems. In the case when we keep the magnetic field in the non-relativistic limit, we obtain the so-called electron magnetohydrodynamics equations. Otherwise we obtain the incompressible Euler equations with no more magnetic field left. A key tool of the proofs is the method of modulated energy.

FABRICE MOTTEZ
CETP, Universite de Versailles-St Quentin en Yvelines

Implicit Particle in Cell Plasma Simulation Codes

The space plasmas of the solar wind, of the planetary magnetospheres and other hot and tenuous environnments are governed by two sets of equations : the Vlasov equation that govern the evolution of the particle distribution function (PDF), and the Maxwell equations that govern the electromagnetic field. Particle in cell (PIC) simulation codes describe the particle distribution function, through a set of macro particles whose trajectories evolve according to the Lorentz force. The charge and current densities are interpollated on a rectangular grid. The Maxwell equations are solved on this grid. The electric and the magnetic field are then interpollated on the macroparticle positions to evaluate the Lorentz force. When simulating an electromagnetic plasma with an explicit PIC code, several time scales must be taken into account. They depend on the propagation of light waves, on the electrostatic electron plasma oscilliations, on the electron gyromotion into the magnetic field... This is not necessarily relevant if we are interested in low frequency phenomenons such as ion-cyclotron or Alfvn waves. The aim of an implicit code is to solve the equations of evolution of a dynamical system without keeping the high frequency fluctuations. For instance, time decentered schemes are made implicit by a time-discretisation method, in which the intermediate time level is slightly decentered. In the direct implicit scheme, a few variables of the system (at time step N) are replaced by time averaged implicit variables. These implicit variables depend both on the state of the system in the past (time N-1) and in the future (time N+1). A rough classification of time scales in a plasma can help us to understand which physical parameters must be made implicit, according to the problem to be treated. I will present how implicit variables are chosen, how to deal with them, what are the consequences concerning the other equations, a few pitfalls, and a few ways of avoiding problems. A few examples of applications of implicit PIC codes will illustrate this talk.

MAURIZIO OTTAVIANI
CEA, DSM / DRFC, Cadarache, France

Development of Gyrokinetic Codes using Semi-Lagrangian Methods

This talk will report on the development and the exploitation of gyrokinetic codes carried out at DRFC-Cadarache in collaboration with various laboratories. These codes solve a low frequency limit of the Vlasov equation for magnetized plasmas, obtained by time averaging over the ion cyclotron period. This approximation replaces the motion in the six-dimensional phase space of charged particles in an electromagnetic medium, by the motion in a lower dimensional space parameterized by one (or more) conserved quantities. The equation is then coupled to the Maxwell equations for the evolving fields. Further approximations that depend on the physical problem may also be employed. The main effort is currently on the development of a 4D+1P (four dimensions + one parameter, the magnetic moment) electrostatic code (i.e., the magnetic field is given and static) in cylindrical geometry. Accurate integration and excellent energy conservation was achieved by employing a semi-Lagrangian integration scheme, which exploits the constancy of the distribution function on the characteristics of the kinetic equation. First results on the formation of convective cells driven by the ion temperature gradient will be presented. A second class of codes, where the distribution function depends on two-dimensional phase space variables and one or two parameters, were also developed. Results from these codes, showing, in particular, the competition between the formation of convective cells and that of sheared zonal flows (flows perpendicular to the mean gradients), and their effect on convective heat transfer in steady state temperature gradient driven turbulence, will be reported. A third code that aims at simulating magnetic reconnection is also being implemented. This code treats kinetically the electron dynamics in a cold ion fluid. The goal is to understand the evolution of certain internal disruptions, abrupt changes of the magnetic configuration in the interior of the magnetic confinement devices, and to characterize the phenomenon of particle acceleration during these events.

THIERRY PASSOT
CNRS, Observatoire de la Côte d'Azur

Fluid Description for Dispersive MHD Waves in a Collisionless Plasma

The validity of the mono or multi-fluid descriptions and the determination of closure approximations in a magnetized collisionless plasma are important issues for the simulation of realistic three-dimensional problems for which the integration of the Vlasov-Maxwell system is still impossible.

We present a mono-fluid model with Landau damping, specifically adapted for the description of MHD waves in a collisionless plasma permeated by a strong ambient magnetic field where the distribution functions are close to bi-Maxwellians. This model that can be viewed as an extension of the Landau-fluid model of Snyder, Hammett and Dorland (1997), is able to describe oblique or kinetic Alfvén waves for which the Hall effect as well as finite Larmor radius corrections are relevant. It includes dynamical equations for the gyrotropic components of the pressure and heat flux tensors. The latter equations that contain non-local operators associated with wave-particle resonances are built in a way that reproduces the weakly nonlinear dynamics of Alfvén and magnetosonic waves with a wavelength long compared to the ion Larmor radius, whatever the propagation direction.

This model will be particularly useful to study the formation of coherent structures such as magnetic holes or shocklets and also to perform three-dimensional numerical simulations of dispersive MHD turbulence taking into account realistic dissipation and heating mechanisms. Further applications include the interpretation of spectra and structures observed in the solar wind and the terrestrial magnetosheath.

References

[1] T. Passot and P.L. Sulem, Phys. Plasmas, **10**, 3887; **10**, 3906 (2003).

[2] T. Passot and P.L. Sulem, "A fluid description for Landau damping of dispersive MHD waves", Nonlinear Proc. Geophys., in press.

[3] T. Passot and P.L. Sulem, "A Landau fluid model for dispersive magnetohydrodynamics", submitted to Phys. Plasmas.

[4] P.B. Snyder, G.W. Hammett, and W. Dorland, Phys. Plasmas **4**, 3974 (1997).

FRANCESCO PEGORARO
University of Pisa and INFM, Italy

Magnetic Field Line Reconnection in Dissipationless Regimes and Mixing of the Lagrangian Invariants in Strongly Magnetized, Two-dimensional, Plasma Configurations

Magnetic topology plays an important role in the global dynamics of high temperature plasmas. Within the ideal MHD plasma description, two plasma elements that are initially connected by a magnetic field line remain connected at any subsequent time. This condition introduces a topological linking between plasma elements that is preserved during the ideal plasma evolution. Magnetic linking constraints the plasma dynamics by making configurations with lower magnetic energy, but different topological linking, inaccessible. Magnetic field line reconnection partially removes these constrains by allowing the field lines to decouple locally from the plasma motion and to reknit in a different net of connections. In collisionless magnetic field line reconnection the decoupling between the magnetic field and the plasma motion occurs because of the current limitation due to the finite electron inertia (in the fluid limit) or to thermal effects (in the kinetic plasma description). However, in the absence of dissipation, the plasma response both in the fluid and in the kinetic electron treatment admits generalized linking conditions that in a two-dimensional configuration are preserved during the process of magnetic reconnection in the form of Lagrangian invariants. Here we compare the analytical and numerical results obtained recently [1,2] in the study of the nonlinear development of magnetic reconnection in the fluid and in the drift-kinetic limits of the electron response and establish a clear link between these two regimes by showing that the (two) fluid Lagrangian invariants and the (infinite number of) drift-kinetic Lagrangian invariants evolve in time in an analogous fashion: in both cases the growth and saturation of the magnetic island is accompanied by their spatial mixing in the reconnection plane.

[1] E. Cafaro, et al., Phys. Rev. Lett, 80, 4430 (1998); D. Grasso, et al., Phys. Rev. Lett., 86, 5051 (2001); D. Del Sarto, et al., Phys. Rev. Lett., 91, 235001 (2003).

[2] T. Liseikina, et al., submitted to Phys. Plasmas, (2003).

BENOIT PERTHAME
Ecole Normale Supérieure, DMA

Kinetic Model for Chemotaxis

Several classical transport-diffusion systems arise as simple models in chemotaxis (motion of bacterias interacting through a chemical signal) and angiogenesis (development of capillary blood vessels from an exhogeneous chemoattractive signal by solid tumors). The name of Keller-Segel is usally attached to these systems which are coupled through a nonlinear transport term depending on the gradient of the chemoattractant and that can exhibit various qualitative behavior (collapse, rings dynamics).

After a presentation of such models, we will focuss on a microscopic picture based on a kinetic modelling of the interaction and introduced by Alt, Dunbar, Ohtmer and studied by Hadeler, Stevens...etc. It has the form of a nonlinear scattering equation. We will show that, in opposition to the parabolic model, the kinetic system admits global solutions that converge in finite time to the Keller-Segel model, as a scaling parameter vanishes.

The kinetic model has the advantage to present a unified picture for various macroscopic models of cell movement and sustend biological assumptions on the individual interactions.

MARJOLAINE PUEL
Commissariat a l'Energie Atomique

Quasi Neutral Limits of the Relativistic Vlasov-Maxwell System

This work comes from two papers, the first one writen with Y. Brenier and N. Mauser and the second one writen with Laure Saint-Raymond. We deal with the proof of the convergence of the relativistic Vlasov-Maxwell system towards the electron Magneto Hydrodynamic by using some modulated energy method.

HONG QIN
Plasma Physics Laboratory, Princeton University

General Gyrokinetic Theory

A fully nonlinear electromagnetic gyrokinetic theory has been developed under the most general conditions. There is no separation of field quantities into perturbed and equilibrium parts, and the electrical field can contain large-amplitude large-scale component as well as small-amplitude small-scale component. In addition, It is valid for arbitrary frequency and general geometry.

GERHARD REIN
University of Bayreuth

The Vlasov Equation as a Matter Model in General Relativity

One of the main issues in General Relativity are the global properties and possible singularities of spacetimes containing matter. We present results where possible singularities in the solutions of the coupled Einstein-Vlasov system are not due to a break-down of the matter model, but are genuine spacetime singularities. Hence the Vlasov equation provides a particularly suitable matter model in this relativistic context.

FRAYDOUN REZAKHANLOU
University of California, Berkeley

A Stochastic Model for Coagulation-Fragmentation and Smoluchowski Equation

We consider a natural model for the coagulation-fragmentation process. In this model we have N Brownian particles that are travelling in the d–dimensional Euclidean space with a diffusion coefficient that is a function of the size of the particle. We regard the location of each particle as the center of a cluster and when two particles are sufficiently close, they coagulate to a larger cluster. Also clusters randomly fragment to smaller clusters. In a joint work with Alan Hammond, We show that if the range of the interaction is scaled like $N^{\frac{1}{2-d}}$, and as the number of particles N goes to infinity, the microscopic particle densities converge to solutions of the Smoluchoski's equation.

ANDREI SMOLYAKOV
University of Saskatchewan

Collisionless Damping in Plasmas and Neutral Gases

Damping of sound in highly rarefied neutral gases exhibits many features similar to the collisionless damping of the electrostatic longitudinal oscillations in plasmas (Landau damping). We discuss various continuum (fluid -like) models that can be used to describe both types of damping.

ERIC SONNENDRUCKER
Universite Louis Pasteur

Adaptive Semi-Lagrangian Numerical Methods for the Vlasov Equation

Vlasov solvers, solving directly the Vlasov equation on a grid of phase-space, have been successful in the past years in solving problems as well in plasma physics as in beam physics that were inaccessible to Particle-In-Cell (PIC) methods because of their inherent large numerical noise. However, in simulations where the particle distribution function varies a lot during time, Vlasov simulations on a uniform grid of phase space become very inefficient because large computational resources are waisted on regions where nothing is happening at a given time. For this reason we are investigating methods using a phase-space grid which evolves in time according to the evolution of the distribution function. Two different paths have been taken. The first consists in having a uniform grid at each time step, but which is moving from one time step to the next according to the global evolution of the particles. The second approach, which eventually could be coupled to the first, consists in using an adaptive local mesh refinement scheme. The adaptive method is overlayed to a classical semi-Lagrangian method which is based on the conservation

of the distribution function along particle trajectories. The phase-space grid is updated using a multiresolution technique. In this presentation we shall describe the moving grid and adaptive mesh refinement methods and evaluate their benefits on some applications in beam physics and plasma physics.

GIGLIOLA STAFFILANI
MIT

Global Well-Posedness and Scattering in the Energy Space for Critical Nonlinear Schrödinger Equation in 3D

In this talk I will present the main steps of the proof of global well-posedness, scattering and global L^{10} spacetime bounds for energy class solutions to the quintic defocusing Schrödinger equation in 3D. This proof was recently obtained in collaboration with J. Colliander, M. Keel, H. takaoka and T. Tao and improves upon the results of Bourgain and Grillakis, which handled the radial case. The method is similar in spirit to the induction-on-energy stategy of Bourgain, but we perform the induction analysis in both frequency space and physical space simultaneousely, and replace the Morawetz inequality by an interaction variant. The principal advantage of the interaction Morawetz estimate is that it is not localized to the spacial origin and so is better able to handle non-radial solutions. In particular, this interaction estimate together with an almost-conservation argument controlling the movement of the $L2$ mass in frequncy space, rules out the possibility of energy concentration.

WALTER STRAUSS
Brown University

Some Global Solutions of the Hydrodynamic Model of Semiconductors

The model under consideration is an Euler-Poisson system. It looks like the compressible Euler equations for the electrons together with a coupled electric field and a relaxation term. Both contact and insulating boundary conditions are considered. The unknowns are (ρ, u, ϕ) where ρ is the density, u is the velocity and ϕ is the electric potential. The global solutions are near a steady state (ρ_0, u_o, ϕ_0). I will discuss two results that are joint work with Yan Guo. It is important that we permit $\rho_0(x)$, $\phi_0(x)$ and the doping profile $D(x)$ to have large variation. Our first result is in a bounded 3D domain with insulating boundary conditions. So far as we know, this is the first class of exact global solutions in a multi-dimensional domain. Our second result is in a bounded 1D interval with contact boundary conditions.

R.D. SYDORA
University of Alberta

Kinetic Theory and Simulation of Nonlinear Magnetic Structures

Small-scale processes can determine many aspects of the global behavior of complex plasma physical systems such as magnetically-confined toroidal plasmas, near-Earth space plasmas, and solar plasmas. Examples of this include reconnection of magnetic fields lines in narrow layers near rational magnetic surfaces, which lead to global resdistributions of plasma density, current and temperature, and anomalous transport from microscopic magnetic islands and vortices.

In this work the kinetic nonlinear dynamics of small-scale magnetic perturbations in a magnetized high temperature plasma is considered, including the effects of diamagnetism, ion gyroradius, finite electron mass and magnetic shear. A particle-in-cell simulation model, based on the electromagnetic gyrokinetic Vlasov-Poisson-Ampere system, is utilized to investigate the spontaneous generation of inertial-scale magnetic islands in various regimes of the plasma beta. We focus on two aspects of the physical processes related to the formation of undamped stationary propagating magnetic modes and the cross-scale coupling effects of shorter wavelength gradient-driven fluctuations on the longer wavelength magnetic fluctuations.

EITAN TADMOR
Center for Scientific Computation and Mathematical Modeling, University of Maryland, College Park

Kinetic Formulations and Regularizing Effects in First and Second Order Equations

We discuss the relation between kinetic formulation, the notion of nonlinearity and the regularizing effects of first- and second-order equations. In particular, we highlight the regularizing effect of nonlinear entropy solution operators of quasilinear conservation laws and compare it to recent 2D results based on compensated compactness arguments. We also discuss new regularity estimates for second-order equations with possibly non-isotropic degenerate elliptic part.