ANDREY BIRYUK
The Fields Institute and McMaster

NLS With Small Viscosity. Lower Bounds for the Space Derivatives of the Solutions

In 1998 S.Kuksin discovered a mechanism that responsible for the growth of the derivatives. The lower bounds is given in terms of (negative) powers of the viscosity. We review this result and show that his method allows us to get a slightly better lower bounds.

JERRY BONA
Illinois

Two and Three Dimensional Water Waves

We will discuss models for surface water waves in both two and three dimensions. Included will be derivation of a class of models and some rigorous estimates of how well they track solutions of the full Euler equations. We will also present the outcome of numerical experiments on some of these models.

NICOLAS BURQ
Paris Sud Orsay

Nonlinear Schrödinger Equations on 3-d Compact Manifolds

We present some new result about existence in energy space of subquintic nonlinear Schrödinger equations on some 3-d manifolds. These results are obtained by combining an $X^{s,b}$ approach and some multilinear spectral projector estimates.

STEPHEN GUSTAFSON
British Columbia

Scattering for the Gross-Pitaevskii Equation

The Gross-Pitaevskii equation, a nonlinear Schröedinger equation with non-zero boundary conditions, models superfluids and Bose-Einstein condensates. Recent mathematical work has focused on the finite-time dynamics of vortex solutions, and existence of vortex-pair traveling waves. However, little seems to be known about the long-time behaviour (eg. scattering theory, and the asymptotic stability of vortices). We address the simplest such problem – scattering around the vacuum state – which is already tricky due to the non-self-adjointness of the linearized operator, and "long-range" nonlinearity. In particular, our present methods are limited to higher dimensions.

SLIM IBRAHIM
McMaster and The Fields Institute

On the Local Solvability for a Quasilnear Cubic Wave Equation

This work is concerned with local solvability of the Cauchy problem for a quasilinear cubic wave equation in dimension $d \geq 3$. Here, we improve the index of regularity of the initial data compared to the one given by classical energy methods.

NIKY KAMRAN
McGill

Long Term Dynamics of Dirac and Scalar Fields in the Kerr Geometry

We consider the Dirac and scalar wave equations in the Kerr geometry of a rotating black hole in equilibrium, with Cauchy data in L^2, bounded near the event horizon. We derive integral spectral representations for the propagators in terms of the radial and angular decompositions arising from Chandrasekhar's separation of variables. These are then used to investigate the long term dynamics of Dirac and scalar waves in Kerr geometry. This is joint work with Felix Finster, Joel Smoller and Shing-Tung Yau.

MARKUS KEEL
Minnesota

Regularity and Scattering for the Critical Defocusing Nonlinear Schröedinger Equation

We describe recent work with Jim Colliander, Gigliola Staffilani, Hideo Takaoka, and Terry Tao on the following nonlinear Schrödinger equation in three space dimensions,

$$\partial_t \phi + i\Delta\phi = |\phi|^4 \phi$$

where we start the evolution from given data $\phi_0(x)$ at time zero. This equation is "critical" in the sense that the natural scaling of the equation leaves the conserved energy norm of solutions unchanged. (So for example, we can't reduce to the case of small energy data simply by scaling.)

Our result is that if the initial data has finite (and possibly large) energy, then there is a unique solution that exists for all time and asymptotically approaches a linear solution. Our work here starts from the results and methods obtained in the context of radially symmetric solutions by J. Bourgain in JAMS 1999, and also the work for radial data by M. Grillakis in CPAM 2000.

DAVID LANNES
Bordeaux

Well-Posedness of the Water-Waves Equations

We prove the well-posedness of the water-waves equations in finite depth. We first reduce the equations to a system of two equations on $\mathbf{R}^d$ involving a Dirichlet-Neumann operator. We give tame estimates on this DN operator and compute an explicit expression of its shape derivative. This allows us to give a simple expression of the linearized equations and to conclude by a Nash-Moser iterative scheme.

FELIPE LINARES
IMPA

On a Degenerate Zakharov System

We will discuss a local well-posedness result regarding the initial value problem associated to a Zakharov system arising in the study of laser-plasma interactions. In contrast with the usual Zakharov system this one has a degenerate dispersion relation.

HANS LINDBLAD
UCSD

The Weak Null Condintion and Global Existence for Einstein's Equations

We prove global stability of Minkowski space for the Einstein vacuum equations in harmonic (wave) coordinate gauge for the set of restricted data coinciding with Schwartzschild solution in the neighborhood of space-like infinity. The result contrasts previous beliefs that wave coordinates are "unstable in the large" and provides an alternative approach to the stability problem originally solved (for unrestricted data, in a different gauge and with a precise description of the asymptotic behavior at null infinity) by D. Christodoulou and S. Klainerman. Using the wave coordinate gauge we recast the Einstein equations as a system of quasilinear wave equations and, in absence of the classical null condition, establish a small data global existence result. In our previous work we have introduced the notion of and shown that the Einstein equations satisfy the weak null condition. The result of this paper relies on this observation and combines it with the vector field method based on the symmetries of the standard Minkowski space.

MICHAEL LOSS
Georgia Tech

A Sharp Analog of Young's Inequality on S^N and Related Entropy Inequalities

We prove a sharp analog of Young's inequality on S^N, and deduce from it certain sharp entropy inequalities. The proof consists of constructing a nonlinear heat flow that drives the inequality to its sharp value, while transporting the functions to the optimizers. This strategy also works for the general Young inequality on R^N and yields a fairly simple proof of the fact (due to Brascamp and Lieb) that it suffices to optimize over Gaussian functions in order to find the sharp constant.

NADER MASMOUDI
Courant Institute

Hydrodynamic Limits of the Boltzmann Equations

We will overview different recent results on the subject. Indeed, after the work of C. Bardos, F. Golse and D. Levermore of 1989 a lot of difficulties were left open about a rigourous derivation and we will overview how some of them were solved. Then, we will explain more the case of a bounded domain. In particular, we will consider the Boltzmann equation in a bounded domain with different types of (kinetic) boundary conditions and derive the Stokes-Fourier system with different type of (fluid) boundary conditions when the main free path goes to zero. This is a joint work with Laure Saint-Raymond.

KEN MCLAUGHLIN
North Carolina at Chapel Hill

Asymptotic Analysis of the Integrable Nonlinear Schrödinger Equation

For initial value problems associated to linear partial differential equations, we know very well that the Fourier transform provides an avenue towards a complete description of solutions. Long time behavior, regularization of singularities in initial data, and other singular limits, are reduced to analysis of integrals. For initial value problems associated to pdes that are not linear, much less detail is afforded by Fourier based methods, and frequently we are left with results which, in essence, explain that solutions often behave like solutions to linear pdes. The integrable nonlinear Schroedinger equation (a nonlinear partial differential equation) is an example which may be integrated via scattering and inverse scattering theory. I will describe some asymptotic analyses of this equation which exhibit how solutions differ from solutions to linear equations.

FRANK MERLE
Cergy Pontoise, IAS

Blow-up Behavior for Critical NLS

We describe blow-up behavior and related problems.

PETER MILLER
Michigan

Semiclassical Asymptotics for the Focusing Nonlinear Schroedinger Equation

An approach to the semiclassical limit of the initial-value problem with real, "single-lobe" initial data for the integrable focusing nonlinear Schroedinger equation will be described. Although based on integrability rather than WKB asymptotics, analyticity of the data plays an important role. The technique to be described allows for the identification of phase transition curves (nonlinear caustics) in the (x,t) plane, and for the calculation of strong asymptotics as well as weak limits.

ANDREA R. NAHMOD
Massachusetts, Amherst and IAS, Princeton

The Cauchy Problem for the Hyperbolic-Elliptic Ishimori System

We show an improved local in time existence and uniqueness result for the hyperbolic-elliptic nonlinear system proposed by Ishimori in analogy with the 2d CCIHS chain. The proof uses fairly standard gauge geometric tools and energy estimates in combination with a new method devised by C. Kenig to obtain a priori $L^q L^\infty$ estimates for classical solutions to certain dispersive equations.

VLADISLAV PANFEROV
Victoria

Maxwellian Upper Bounds for the Boltzmann Equation and Applications to Equations of Granular Media

I will present an approach for obtaining pointwise upper bounds for solutions of Boltzmann equations for rarefied gases and granular media. In the elastic collisions case the bounds take the form of Maxwellians, while for the inelastic model in a heat bath we derive a "stretched exponential" "3/2" asymptotics. The method is based on a variant of a maximum principle and using the properties of the Carleman form of the collision terms.This is a joint work with I. Gamba, C. Villani and A. Bobylev.

GUIDO SCHNEIDER
Karlsruhe

Long Time Existence and Blow Up of Modulated Waves in Case of Nontrivial Resonances

The Nonlinear Schrödinger equation can be derived as an amplitude equation describing slow modulations in time and space of an underlying spatially and temporarily oscillating wave packet. It is the purpose of this paper to prove estimates, between the formal approximation, obtained via the Nonlinear Schrödinger equation, and true solutions of the original system in case of non-trivial quadratic resonances. It turns out that the approximation property holds if the approximation is stable in the system for the three wave interaction associated to the resonance. It does not hold if instability occurs for the approximation in this system. Although we restrict ourselves to a nonlinear wave equation as original system we believe that this is a general result. A consequence of the result is the long time existence of modulated waves in case of stable resonances and blow up of modulated waves in case of an unstable resonance.

TAI-PENG TSAI
British Columbia

Asymptotic Stability of Small NLS Solitons in Energy Space

We study a class of nonlinear Schroedinger equations admitting small solitary wave solutions. If the linearized equation has a single bound state, we prove that all solutions small in the energy space H^1 split, asymptotically, into a fixed nonlinear ground state and a solution of the free Schrödinger equation. If the linearized operator has a second "well-placed" bound state, we prove the same thing for data which is an H^1-small perturbation of the nonlinear ground state. In particular, we do not make the usual "localization" assumption on the initial data.

LUIS VEGA
Pais Vasco

On a Pseudo Differential Calculus Related to Non-Elliptic Operators

We shall present a pseudodifferential calculus modelled on second order but not necessarily elliptic operators. This calculus is a generalization of the one developed by W. Craig, T. Kappeler and W. Strauss in the elliptic setting. It arises naturally when trying to compute an integrating factor to avoid first order perturbations. This reduction turns out to be fundamental to solve the initial value problem for quasilinear non-elliptic Schrödinger equations.

STEPHANOS VENAKIDES
Duke

Steepest Descent and the G-Function Mechanism in Rigorous Semiclassical NLS Asymptotics

The inverse scattering approach linearizes the NLS initial value problem by associating it with a linear 2 by 2 first order ODE eigenvalue problem, the Zakharov Shabhat (ZS) equation. The scattering information of ZS constitutes the input to a Riemann-Hilbert problem (RHP) that is to nonlinear integrable systems what the Fourier integral represetation is to the solution of a linear PDE. Solving the RHP provides the solution to NLS together with the dependence of the "eigenfunctions" of ZS in the spectral parameter for any point in space-time. The time evolution of the input is as simple as the evolution of the Fourier transform in a linear PDE. We will outline the asymptotic methods that lead to the following results:

1) Proof of existence and basic properties of the first breaking curve in space-time above which a phase teransition occurs and show that for pure radiation no further breaks occur. 2) Post-break structure of the solution. 3) Rigorous error estimate. 4) Rigorous asymptotics for the large time behavior of the system in the pure radiation case.

DOUG WRIGHT
McMaster and The Fields Institute

Higher Order Corrections to the KdV Approximation for Water Waves

In order to investigate corrections to the common KdV approximation to long waves, we derive modulation equations for the evolution of long wavelength initial data for the water wave and Boussinesq equations. The equations governing the corrections to the KdV approximation are identical for both systems and are explicitly solvable. We prove estimates showing that they do indeed give a significantly better approximation than the KdV equation alone. We also present the results of numerical experiments which show that the error estimates we derive for the correction to the Boussinesq equation are essentially optimal.

JARED WUNSCH
Northwestern

Morawetz and Strichartz Estimates for the Schrödinger Equation on Nontrapping Three-Manifolds

ZHENGFANG ZHOU
Michigan State

Study Nonlinear Wave Equations from the Einstein Universe

We will discuss the global existence and finite time blowup of solutions to the nonlinear wave equations on Minkowski space. The idea is to conformally embed the Minkowski space-time into the Einstein Universe with the image in a compact set, which will eliminate some complicated decay and energy estimates since these propoerties can be seen easily from the covariance of the linear wave equation. This method was used successfully for existence theory. We will show how this method can also be modified to prove the blowup easily. Some Strihartz type estimates in the Einstein Universe will also be established and used to prove some nontrivial global existence.