AFTALION, AMANDINE
Universite Paris-VI

Properties of Vortices in Rotating Bose Einstein Condensates

We consider a rotating Bose-Einstein condensate in a harmonic trap and investigate the behavior of the wave function which solves the Gross Pitaevskii equation. We give a simplified expression of the Gross-Pitaevskii energy in an asymptotic regime, which only depends on the number and shape of the vortex lines. Following recent experiments, we study in detail the line of a single quantized vortex, which has either a U or S shape.

AHMAD, NAJMA
University of Toronto

Geometry of Shape Recognition via Optimal Transportation

A Monge-Kantorovich optimal transportation problem between measures supported on the boundaries of domains in $\mathbb{R}^2$ is studied with the intent to get an insight into the underlying geometry of a shape recognition problem in computer vision — where one wants to match two simple closed planar curves. The focus is on investigating (i) uniqueness, (ii) smoothness and (iii) geometrical characterization of the solutions. Optimality of these solutions is measured against a cost function defined between the two curves to be compared. Topological constraints allow (iv) a classification of the cost function that strongly dictates the geometry of the optimal solutions.

ALBERTI, GIOVANNI
Universita di Pisa

Microsctructures in a Model of Di-block Copolymers Melt

I will describe some mathematical features of a variational model for the description of micro-phase separation in di-block copolymer melts. In dimension one, minimizers of this energy functional present periodic patterns on a certain microscopic scale. In higher dimension, however, very little is known on the structure of minimizers. In a joint work with R. Choksi and F. Otto we have proved a uniform energy bound (on the right microscopic scale), and shown that the admissible patterns should arise as local minimizer on the entire space of the unscaled functional.

ALI, ISMAIL
Kuwait University

A Partial Differential Equation Related to a Problem of Atmospheric Pollution

We investigate a generalized form of a partial differential equation governing the diffusion of heavy pollutants into the atmosphere. In an earlier treatments of the equation, the vertical componentof turbulent exchange coefficient was assumed to be linear. Our generalization takes into account the nonlinear case of this component. Furthermore, two general identities involving the confluent hypergeometric function of the second kind are derived in the course of solving the given PDE.

BAUMAN, PATRICIA
Purdue University

Variational Methods for Analyzing Phase Transitions in Chiral Liquid Crystals

We introduce the Landau-de Gennes free energy used to model the transition between chiral nematic and smectic A liquid crystal phases. Within this mathematical framework, the physically observed growth behavior of the twist and bend Frank constants in the energy play a major role in bringing about the transition. We rigorously establish a transition regime separating the two phases, using variational techniques to analyze two competing effects: the layer formation of the smectic phase and the twist tendency of the chiral nematic phase. Our discussion will illustrate the analogies as well as the discrepancies in modeling and behavior between smectic A liquid crystals and superconducting materials described by the Ginzburg-Landau theory.

BETHUEL, FABRICE
Universite Paris VI

A Survey on Some New Results for Travelling Waves of the Gross-Pitaevskii Equation

We present some joint work with G. Orlandi and D. Smets as well as some new results by P.Gravejat concerning the existence problem and qualitative propoerties of travelling waves of the GP equation.

BLANK, IVAN
Rutgers University

Eliminating Mixed Asymptotics in Obstacle Type Free Boundary Problems

We show a method to eliminate a type of mixed asymptotics in certain free boundary problems and give two examples of its application. It appears that these problems cannot be handled by the monotonicity formula of Alt, Caffarelli, and Friedman (1984), or by the more recent monotonicity formula of Caffarelli, Jerison, and Kenig (2002).

BRENIER, YANN
CNRS, Laboratoire Dieudonne, Nice France

Asymptotic Analysis of the Born-Infeld Electromagnetism

Born and Infeld introduced in 1934 a non linear version of the Maxwell equations, which is still for use in high energy physics. Remarkably enough, the Born-Infeld system can be enlarged as a 10x10 system of hyperbolic conservation laws, quite similar to the classical MHD equations, with a nearly quadratic conserved energy. This allows us to perform some asymptotic analysis by using a relative entropy method going back to Dafermos.

CHAUDHURI, NIRMALENDU
Max-Planck Institute

Rigidity Estimates for Two Incompatible Wells.

A classical theorem due to Liouville says that if a smooth mapping $u : \Omega \to \mathbb{R}^n$, $\Omega \subset \mathbb{R}^n$, satisfies $\nabla u \in SO(n)$, then it is affine, $u(x) = Rx + c$. There are numerous generalizations of this fundamental result, of which the most general is by Resetnjak. If a sequence $u^{(j)}$ converging weakly in $W^{1,2}(\Omega, \mathbb{R}^n)$ satisfies $\nabla u^{(j)} \to SO(n)$ in measure, then $\nabla u^{(j)}$ converges strongly in $L^2(\Omega)$ to a single matrix on $SO(n)$. This result plays a pivotal rule in solid mechanics. Recently, Friesecke, James and Müller has obtained the precise rate of convergence. They proved that the L^2-distance of ∇u from a single rotation matrix is bounded by a multiple of the L^2-distance from $SO(n)$. This result has played a fundamental rule in studying nonlinear plate/shell theory from the three dimensional elasticity. Here we present extension of this result for to two strongly incompatible wells. To be precise we will prove that the L^2 distance of ∇u from a single matrix in K is bounded by a multiple of L^2 distance from the set $K := SO(n) \cup SO(n)\, H$, $H := \mathrm{diag}(\lambda_1, \cdots \lambda_n)$, $\lambda_i > 0$ with $\displaystyle\sum_{i=1}^{n}(1-\lambda_i)(1-\det H/\lambda_i) \,>\, 0$.

DELELLIS, CAMILLO
Max-Planck-Institute, Leipzig

Nonlinear Versions of the BV Structure Theorem and of Vol'pert Chain

In the last fifteen years some physical models have raised the issue of understanding the singular limit of certain families of smooth functionals which involve first and second derivatives. It turned out that these problems lead naturally to the study of (nonsmooth) divergence free $m : \mathbf{R}^2 \to \mathbf{S}^1$ such that $\operatorname{div} \Phi(m)$ is a Radon measure for any Φ belonging to appropriate classes of vector fields. When m is a function of bounded variation, $\operatorname{div} \Phi(m)$ can be computed by using Vol'pert chain rule. Though general m's are far (in terms of linear function spaces) from having bounded variation, in a joint work with Felix Otto we have shown that the pointwise behavior of m is similar to that of BV functions. Hence it would be natural to expect that $\operatorname{div} \Phi(m)$ can be computed in a similar fashion.

It turns out that very similar questions arise naturally in different areas of PDE's. We will give a brief overview and we will show recent results giving affirmative answers to some of them.

DOLZMANN, GEORG
University of Maryland

Nonconvex Variational Problems and Minimizing Young Measures

Variational integrals modeling solid-to-solid phase transformations often fail to be weakly lower semicontinuous because the energy densities f are not quasiconvex in the sense of Morrey. In this talk we analyse properties of minimizing Young measures generated by minimizing sequences for these variational integrals. We prove that the moments of order $q > p$ exist if the integrand is sufficiently close to the p-Dirichlet energy at infinity. A counterexample related to the one-well problem in two dimensions shows that one cannot expect in general L^∞ estimates, i.e., that the support of the minimizing Young measure is uniformly bounded.

FRIESECKE, GERO
University of Warwick

Variational Methods in Quantum Chemistry

Recent successes of variational methods in quantum chemistry include (i) a new and simpler proof of Zhislin's fundamental structure theorem on the spectrum of many-particle Schr"odinger operators,(ii) a generalization of Zhislin's result to a central approximate method of quantum chemistry (the multiconfiguration self-consistent field method, which may be viewed as a closure assumption on higher oder correlations in terms of lower order correlations), (iii) a rigorous derivation of the celebrated $vanderWaals 1/r^6$ law for long range interatomic forces from the many-electron Schr"odinger equation. The last result is joint work with Phil Gardner (Warwick).

GHOUSSOUB, NASSIF
University of British Columbia and The Pacific Institute for the Mathematical Sciences

A Variational Principle for Dissipative Evolution Equations

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GIORGI, TIZIANA
New Mexico State University

Superconductors Surrounded By Normal Materials

We study questions related to existence in suitable weighted Sobolev spaces, and to properties of minimezers of a generalized Ginzburg-Landau energy functional, which models a bounded superconductor surrounded by a normal material. The model in consideration is of interest as the effects of superconducting electron pairs diffusing into the normal region are here represented.

GRABOVSKY, YURY
Temple University

A Generalized Theorem of Chandler Davis

A polycrystal is a mixture of anisotropic materials (crystals) where each material may participate in a composite in any orientation. The effective conductivity tensor of such a composite depends on the microstructure of the composite. The set of effective properties one can obtain by mixing the same set of materials in different ways is called the G-closure of the original materials. The G-closure set has two important qualities: SO(3) invariance and a certain convexity property. In order to understand the interplay between these two properties we would like to understand SO(3) invariant functions with the convexity property. The first such result is due to Chandler Davis. In our case we examine what happens when the group action in Davis's theorem is non-linear. In the process we uncover a simple abstract mechanism behind the Davis's classical theorem. Our generalization features arbitrary groups, non-linear group actions and infinite dimensional vector spaces. We also gain extra flexibility to prove convexity of some G-invariant convex functions even though the theorem does not hold for all such functions. Even in the case of linear group actions on finite dimensional spaces we achieve a new generalization of Davis's result.

GUSTAFSON, STEPHEN
University of British Columbia

On the Dynamics of Vortices and Solitary Waves

We present results describing the dynamics of stable, localized structures in solutions of nonlinear evolutions PDEs. The main examples are superconducting vortices (Ginzburg-Landau equations) and solitary waves (nonlinear Schroedinger equations).

JIANG, HUIQIANG
New York University

Remarks on Singular Elliptic Equations

We consider nonnegative solutions of a singular elliptic equation, which arises in thin film rupture and minimal surface theory. We get a general estimate of the size of singular (zero) set.

KANG, XIAOSONG
University of Toronto

Localization Properties for a Porous Medium Equation with Source Term

We establish the strict localization of a porous medium equation with source, i.e., if the initial data is compactly supported, the unbounded solution will be of uniformly compact support. Our argument works for arbitrary spatial dimension, hence the result extends the well-known one dimensional case.

KINDERLEHRER, DAVID
Carnegie Mellon University

The Mesoscale View of Grain Growth

Most technologically useful materials are polycrystalline, composed of many small crystallites called grains separated by interfaces called grain boundaries. These grain boundaries play a role in many material properties, for example conductivity and fracture toughness, and across many scales. Preparing arrangements or distributions of boundaries suitable for a given purpose is a central problem in materials. It is, indeed, the central problem of microstructure and has an extensive history dating from prehistory. Grain growth is one of the primary microstructural mechanisms. We may ask many questions, for example, to what extent is grain growth like or unlike the growth of soap bubbles. We discuss some of the scientific challenges we encounter in the investigation of these issues. In recent years we have been able to begin simulations at mesoscale which are both accurate and statistically significant, that is, they are very large scale. What is the 'answer' of such a simulation? This is a very pregnant question. We present various results and surprises, but primarily we expose the rich trove of problems this study is unveiling. This is joint work with Florin Manolache, Jeehyun Lee, Irene Livshits, Gregory Rohrer, Anthony Rollett, and Shlomo Ta'asan. [1] Partially supported by the NSF under the MRSEC program.

LIN, CHI-KUN
National Cheng Kung University

Homogenization of the Dirac System

The homogenization of Dirac system is studied. It generates memory effects. The memory (or nonlocal) kernel is described by the Fredholm integral equation. When the coefficient is independent of space, the nonlocal kernel can be characterized explicitly in terms of Young's measure. The homogenized equation can be reformulated in the kinetic form by introducing the kinetic variable.

MENON, GOVIND
University of Wisconsin

Dynamic Scaling in Smoluchowski's Coagulation Equation

Smoluchowki's coagulation equations describe a wide variety of mass agggregation processes in physical chemistry and physics (polymerization, colloidal separation, aerosol physics, gravitational clustering...). They also arise in population genetics and combinatorics. I will describe simple proofs of optimal results on dynamic scaling in these equations. These involve one-parameter families of self-similar solutions with fat tails, and the characterization of their domains of attraction. This is work with Bob Pego (Maryland).

OTTO, FELIX
University Kassel

Multiscale Analysis in Micromagnetism

Domains and walls in ferromagnets are a paradigm for pattern formation in materials science. Domains are subregions of the sample Ω in which the magnetization m is nearly constant; the transition layers separating domains are called walls. We will focus on the technologically important ferromagnetic films. Mathematically speaking, the micromagnetic model is a non–convex, non–local variational problem for the magnetization m. It is characterized by several length scales: On one end, there are the scales given by the sample geometry (film thickness and film diameter) and on the other end, there are the scales which depend only on the material. This set–up drives the pattern formation on intermediate scales. In this lecture, we shall try to explain specific experimental observations on walls and domains in ferromagnetic films starting from the micromagnetic model. First, we shall try to understand domain formation neglecting wall energy. Then, we'll take wall energy into account and will discover that there are different modes of walls. Finally, we'll have to take wall interaction into account. We will use a mixture of heuristic and rigorous arguments and shall present some numerical simulations.

PAKZAD, MOHAMMAD REZA
Max Planck Institute

Sobolev Isometric Immersions

We study isometric immersions with Sobolev type regularity from a domain of $\mathbb{R}^2$ into $\mathbb{R}^3$.

SANDIER, ETIENNE
Universite Paris 12 Val de Marne

Asymptotics of the Time Dependant Ginzburg-Landau Equations

In a joint work with Sylvia Serfaty we extend previous results on the asymptotics of parabolic Ginzburg-Landau equations for large kappa to the case of an applied magnetic field of the order of log(kappa). This involves a new product estimate useful in both static and time dependent situations.

SERFATY, SYLVIA
Courant Institute

Asymptotics of the Time Dependant Ginzburg-Landau Equations

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SHAFRIR, ITAI
Technion - Israel Institute of Technology

The Logarithmic HLS Inequality for Systems on Compact Manifolds

Let $\mathcal{M}$ be a compact m-dimensional Riemannian manifold. Given a $n \times n$ symmetric matrix $A = (a_{i,j})$ with $a_{i,j} \geq 0$, $\forall i, j$, we give optimal conditions on the vector $\mathbf{M} = (M_1, \ldots, M_n) \in \mathbb{R}^n_+$ which ensure boundedness from below of the functional

$$\Psi(\boldsymbol{\rho}) = \sum_{i=1}^{n} \int_{\mathcal{M}} \rho_i \ln \rho_i + \sum_{i,j=1}^{n} a_{i,j} \int_{\mathcal{M}} \int_{\mathcal{M}} \rho_i(x) \ln d(x,y) \rho_j(y)\, dx\, dy$$

over

$$\mathbf{\Gamma_M} = \big\{(\rho_1, \ldots, \rho_n) \in (\mathcal{L} \ln \mathcal{L}(\mathcal{M}, \mathbb{R}_+))^n,\ \int_{\mathcal{M}} \rho_i = M_i,\ \forall i\big\}.$$

This result generalizes the logarithmic Hardy-Littlewood-Sobolev inequality of Beckner to the systems case. In some cases we also address the question of existence of minimizers. This is a joint work with Gershon Wolansky.

SLASTIKOV, VALERIY
Courant Institute

Geometrically Constrained Magnetic Walls

We show that in a 3-D dumbell shaped domain with a small neck there exists a non-constant local minimizer of the functional closely related to a micromagnetic problem. We also provide a description of the behavior of such minimizers in the vicinity of the neck.

SMETS, DIDIER
Universite de Paris 6

Mean Curvature Flows and the Parabolic Ginzburg-Landau Equation

We will discuss some issues concerning the evolution of the limiting defect measures associated to the parabolic Ginzburg-Landau equation in the whole space.

SPIRN, DANIEL
Brown University

Dynamics and Instability of Elliptical Vortex Patches

We describe the dynamics of elliptical vortex patch by formulating a nonlinear equation for the boundary of a perturbed patch. In the regime for which the linearized equation of motion is unstable, the nonlinear dynamics of a large class of initial perturbations are determined by the fastest growing mode for the corresponding linearized equation. In particular, we show that elliptical patches are unstable in the full nonlinear sense.

STERNBERG, PETER
Indiana University

Stable Vortex Solutions to the Ginzburg-Landau Energy

We establish the existence of locally minimizing vortex solutions to the reduced and full Ginzburg-Landau energy in three dimensional simply-connected domains with or without the presence of an applied magnetic field. The approach is based upon the theory of weak Jacobians and applies to nonconvex sample geometries for which there exists a configuration of locally shortest line segments with endpoints on the boundary. This is joint work with Robert Jerrard, Alberto Montero and William Ziemer.

TARANTELLO, GABRIELLA
Roma-Tor Vergata Universita

Liouville-Type Equations in Gauge Field Theory

We shall discuss the role of Liouville-type equations (and systems) arising in the study of vortices in various gauge firld theories (e.g. Chern Simons theory, Electroweak theory etc). For this class of equations, we present concentration-compactness principles and mass "quantization" properties for the concentration phenomenon that yield to useful existence results, but also to some interesting open problems.

TING, FRIDOLIN
University of Toronto

Stability of Pinned Vortices of the Ginzburg Landau Equations with External Potential

We study the stability of vortex solutions to the Ginzburg-Landau equations with external potential in two space dimensions. For smooth and sufficiently small external potentials, there exists a perturbed vortex solution centered near the critical point of the potential. We show that these perturbed vortex solutions (pinned vortices) are orbitally stable.

WATSON, STEPHEN
Northwestern University

Driven Anisotropic Willmore Flows and Facetted Crystal Growth.

The (driven) anisotropic Willmore flow arises as a model for facetted crystal growth. The flow is a gradient descent with respect to an anisotropic surface energy coupled to a bulk field. We show how the coupling of a bulk and surface energy results in the failure of the Wulff construction for the equilibrium facet angles of the crystal. In addition, we identify the "sharp corner" limit that arises as the hill (valley) corner width tends to zero.