THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

Poster Abstracts

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Sequential Spectral Method for Solving Semi-Linear Hyperbolic Equations

The Sequential Spectral Method is an iterative method to obtain a spectral approximation to the solution of nonlinear partial differential equations. The method is especially suited to solve problems for which the solution behavior is unknown, where there might be multiple solutions and the dependence on parameters of the problem is of interest. We analyze here the convergence properties of the Sequential Spectral Method applied to semi-linear hyperbolic partial differential equations. We prove convergence of the method in two different convergence regimes and illustrate our analysis with numerical examples. This is joint work with Prof. Martin Gander.

SERGEI BADULIN
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Effect of Coherent Wind-Wave Patterns on EM Scattering by Sea Surface

The problem of identifying and quantifying the presence of coherent wind water wave patterns in "real" random wind wave field is a challenging one, so far a very little progress has been made. To address the problem we develop a new approach enabling to access quantitatively the presence of coherent wave patterns by means of remote sensing. The study is based on unique data set of high resolution time series of spatial reconstructions of water wave surface obtained in the large Luminy wind-wave facility for the various regimes of wind wave development identified by Caulliez and Collard (1999). Electromagnetic scattering is calculated within the framework of the Small Perturbation Approximation (SPA, see Voronovich 1994) applied to analysis of time series of spatial reconstructions of wave field. In our context the main advantage of the SPA is that being sufficiently accurate (its accuracy was checked a posteriori), it enables us to quantify explicitly the *effect of interference* of EM scattering. The effect of interference is due to the presence of persistent phase relations of water wave components in the analysed time series of wave profiles (Shrira, Badulin, Voronovich, 2003). Quantitatively, the effect can be characterized by the ratio of the scattering intensity, averaged over 100 consecutive "snapshots" of the surface, to the intensity of scattering by "a surrogate wave field". The "surrogate wave field" is the hypothetical wave field with the same spatial spectrum as the observed field, but with completely random phases, as it is assumed by the so-called *random phase approximation.* This ratio is found to be a good quantitative indicator of the presence of coherent water wave patterns under study in a wide range of "inner parameters" (lengths and directions) of the incident EM waves. The prevalence of coherent wave patterns indicated by the calculations of EM scattering is found to be consistent with the dynamics of water wave patterns observed by other means by Caulliez and Collard (1999). The observed interference of EM scattering and, hence, persistent water wave phase relations are due to the forced or bound harmonics of water waves and can be treated effectively within the weakly nonlinear approach (Badulin et al. 2003). Our calculations show definite presence of series of the phase-linked harmonics and allows us to find their scales. Thus, our results show effectiveness of the developed approach to wave field data processing for distinguishing the coherent fraction in the wind wave field. The research was supported by INTAS grant 01-234 and Russian Foundation for Basic Research 04-05-64784, 02-05-65140. **References** Badulin, S.I., V.I. Shrira and A.G. Voronovich, 2003, EM scattering from sea surface in the presence of wind wave patterns. Part I. Scattering by coherent wave patterns. Intern.Journ.Remote Sensing.

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Caulliez, G. Collard, F., 1999, Three-dimensional evolution of wind waves from gravity capillary to short gravity range, Eur. Journ.Mechanics B/Fluids, 18, N3, 389-402.

Voronovich A.G., 1994, Scattering from Rough Surfaces, Springer-Verlag. This is joint work with G. Caulliez, (Institut de Recherche de Phénomène Hors Equilibre, Marseille, France) N. Kozhelupova (P.P.Shirshov Institute of Oceanology, Moscow, Russia) V. Shrira (Keele University, Keele, UK).

ANASTASSIA BAXEVANI
Lund University

Velocities for Gaussian Moving Surfaces and their Relation to Global Maximum

For a stationary two-dimensional random field evolving in time, we define different notions of velocity and derive their statistical distributions. The results are based on a generalization of Rice's formula. This work is a continuation of the ideas introduced by Longuet-Higgins (1957) and an extension of the concepts presented for one-dimensional waves in Podgórski, Rychlik and Sj (2000). We discuss the importantance of identifying the correct form of the distribution which accounts for the sampling bias. The derived results are applied to Gaussian fields representing irregular sea surfaces and to the envelope field based on this surface. The envelope field is better fitted to study wave group velocities. We further investigate the physical interpretation of some of these velocities. More presicely, we examine how inclusion of the dynamics of the sea surface affects the distribution of global maximum. We illustrate the results for different choices of sea spectra. This is joint work with Igor Rychlik, Krzysztof Podgórski.

H. BREDMOSE
School of Mathematics, University of Bristol, UK

Extreme Wave Impact Pressures and Compressible flow

When steep waves impact on breakwaters, the water is most often aerated. This is due to entrainment of air from the breaking process and/or entrapment of air pockets at the wall by overturning waves.

The presence of air affects the associated impact pressures. The cushioning of the water by the air can make the pressures smaller (Bullock et al 2001), while the bounce-back effect associated with a compressed air pocket can increase the pressure impulse, i.e. time integrated pressure history at the wall (Wood et al 2000). A full understanding of the impact mechanism of aerated fluid is thus still to be gained. Moreover, proper scaling laws for the aeration effects between laboratory and prototype size are still to be found.

In this poster, experimental and numerical results of the BWIMCOST project (Breaking Wave Impacts on steep fronted COastal STructures) are presented. The project addresses the above problems through collection of field data, laboratory experiments and mathematical/numerical modelling. Laboratory pressures up to 3.5 MPa have been recorded. We model the compressible flow following the lines of Peregrine and Thais (1996), extended to 2D unsteady flow. The fluid is described as a mixture of incompressible water and ideal gas undergoing adiabatic compression. The conservation equations for mass, momentum and gas mass are solved in finite volume form using the software package CLAWPACK (LeVeque 2002). Simulations with idealised initial conditions reveal interesting features of the flow, hereunder oscillating pressure waves in the fluid. Such observations have also been observed experimentally. More detailed comparisons with laboratory data is work in progress.

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JOHN D. CARTER
Seattle University

Short-Wavelength Instabilities of Solitary Wave Solutions to the Two-Dimensional Cubic Nonlinear Schrödinger Equation

The two-dimensional cubic nonlinear Schrödinger equation (NLS) can be used as a model of phenomena in physical systems ranging from waves on deep water to pulses in optical fibers. In this poster, we establish that if the coefficients of the linear dispersion terms have the same sign (elliptic case), then the solitary wave solutions of NLS are only unstable with respect to perturbations that have wavelength longer than a well-defined cutoff. If the coefficients of the linear dispersion terms have opposite signs (hyperbolic case), then there is no such cutoff and as the wavelength decreases, the maximum growth rate of the instability approaches a well-defined limit. This is joint work with Bernard Deconinck (University of Washington).

DMITRY CHALIKOV
ESSIC/University of Maryland

Numerical Simulation of Wind Drag Created by High-Frequency Waves

The main difficulty of experimental and theoretical investigation of small-scale ocean atmosphere interaction is a presence of multi-mode (and occasionally, not single-value) nonstationary interface. It makes many types of measurements in close vicinity of physical surface impossible, and complicates the construction of numerical models. The proposed study is an application of innovative technique for investigation of broad range of processes in vicinity of air-sea interface. The main advantage of this approach is that it is based on primitive Navier-Stokes equations transformed by averaging procedure with a use the closure hypotheses. All previous numerical models considered the air flow above single-mode stationary wave surface. This approach assumed that processes investigated for single mode may be generalized for real multi-mode surface using a principle of linear superposition. This assumption is generally incorrect, because: (1) wave surface due to presence of strong nonlinearity (leading to formation of bound waves, focusing of energy in physical space and wave breaking) phases; (2) dynamic interactions of waves through air (for example, short waves create form drag which affects the overflow of large waves); (3) energy input to steep waves is concentrated in a physical space, and Fourier image of this field is often meaningful. Previous models were based on finite grid low accuracy approximation. For better simulation the wind-wave interaction mechanism the approach should include the essentially new features: (1) problem should be formulated as multi-modes nonstationary problem; (2) waves should be the object of modeling, so, problem should be formulated as two-layer problem with variable interface, and (3) well developed Fourier transform method should be applied. This extended formulation of problem became possible after invention of nonstationary surface-following finite-depth conformal mapping and derivation of principally new form of potential wave equations (Chalikov, Sheinin, 1996). Next step was a generalization of conformal mapping to upper (air domain) and construction of air counterpart of the model based on 2nd order Reynolds equations (Chalikov, 1998). This model has been used for investigation of wind-wave drag created by high frequency waves. It is shown that these waves absorb all incoming momentum flux, so, tangential stress created by molecular viscosity for wind waves in insignificant. These results will be used for formulation the surface boundary conditions for simulating wind-wave interaction for large waves. Perspectives and preliminary results obtained with 3-D coupled model are discussed.

CHRISTOPHE FOCHESATO
Ecole Normale Supérieure de Cachan

Applications of a Numerical Wave Tank Accelerated by a Fast Multipole Algorithm

The numerical wave tank is based on the resolution of fully nonlinear potential flow equations with a free surface. Time updating is realized through a second-order explicit Taylor expansions with adaptive time steps and at each time step, two boundary integral equations must be solved, both for the velocity potential and its time derivative. A boundary element discretization is introduced with specific sliding quadrilateral elements, providing local inter-element continuity of the first and second derivatives. Information on the model can be found in Grilli, Guyenne and Dias article (2001). The discretized equations lead to algebraic linear systems which are full and not symetrical. In order to reduce their computational resolution and to avoid the global matrix assembly, the fast multipole algorithm has been incorporated and adapted in combination with the iterative generalized minimal residual algorithm. This fast algorithm lies on expansions of Green's functions in separate variables and a hierarchical subdivision of space. The limiting $O(N^2)$ complexity of the boundary element method is then replaced by an almost linear complexity. The reduced computational time and memory storage requirements allow us to get new insights on our current applications. First, we study the effect of bottom geometry on the overturning of an incident solitary wave. The presence of a transversal modulation focuses the energy so that a plunging jet is observed. By varying the topography, we can study the difference in the kinematics of these plunging waves. The second application deals with the study of spatial focusing for the occurence of a freak wave. In this case, the waves are generated by a snake wavemaker which focuses the energy at one point of the tank. The parameters are varied in order to learn about the conditions which lead to a high, eventually overturning wave. This is joint work with Frédéric Dias (ENS Cachan).

DAVID R. FUHRMAN
Technical University of Denmark

Numerical Modeling of Water Waves with a Fully Nonlinear and Highly Dispersive Boussinesq Model

This poster will concentrate on applications of a numerical (finite difference) model solving a recently derived high-order Boussinesq formulation in two horizontal dimensions. The method is capable of treating fully nonlinear waves to dimensionless depths of $kh \approx 25$, effectively removing the shallow water limitations conventionally associated with this type of approach. Results will be shown for the modeling of three-dimensional hexagonal and rectangular surface patterns, arising from the nonlinear interaction of wave fronts at oblique incident angles in shallow and deep water, respectively. Results involving nonlinear wave-structure interactions will also be shown. These will include linear and nonlinear diffraction cases, as well as highly nonlinear wave run-up on a vertical bottom-mounted plate. This is joint work with Per A. Madsen, Harry B. Bingham.

DAVID GEORGE
University of Washington

A Numerical Method for the Shallow Water Equations with Topography and Evolving Dry Regions

In addition to the typical difficulties encountered in numerically solving the shallow water equations (shocks, nonuniqueness etc.), the presence of bottom topography and moving dry regions introduces further complications. Many applications involve these features, and require a robust method that can handle them, as well accurately capture moving dry regions. A novel approximate Riemann solver that has been used to develop a robust high-resolution Godunov-type finite volume method for this problem will be shown.

SAMIR HAMDI
University of Toronto

Exact Solutions and Invariants of Motion for General Types of Nonlinear Dispersive Wave Equations

In this work, we derive exact solitary wave solutions for general types of Boussines-like equations and Korteweg-de Vries-like equations with dispersive terms of fifth order and nonlinear terms of any order. We also derive analytical expressions of several conservations laws and invariants of motion for solitary wave solutions. These invariants of motion can be used as verification tools for the conservation properties of numerical models. The new solutions are illustrated with several examples and animations that elucidate salient features of the propagation of solitary waves and undular bores. This is joint work with W.H. Enright, W.E Schiesser, J. J. Gottlieb.

DIANE HENDERSON
Pennsylvania State University

Deep-water waves with persistent, two-dimensional surface patterns

Experiments are conducted to generate wavefields in deep water with two–dimensional surface patterns for which two parameters are systematically varied: (i) the aspect ratio of the cells comprising the surface patterns and (ii) a measure of nonlinearity of the input wavefield. The goal of these experiments is to determine whether these patterns persist, what their main features are, whether standard models of waves describe these features, and whether there are parameter regimes in which the patterns are stable. We find that in some parameter regimes, surface patterns in deep water do persist with little change of form during the time of the experiment. In other parameter regimes, particularly for large–amplitude experiments, the patterns evolve more significantly. We characterize the patterns and their evolutions with a list of observed features. To describe the patterns and these features, we consider two models: (a) the standard (2+1) nonlinear Schroedinger equation and (b) coupled nonlinear Schroedinger equations for two interacting wavetrains. Exact solutions of these models provide qualitative explanations for some features. Qualitative stability considerations of the exact solutions provide alternative explanations as well as explanations for the remaining features. This is joint work with Joe Hammack, Harvey Segur.

TOBIAS KUKULKA
University of Rhode Island

Growth Rates of Wind-Driven Gravity-Capillary Waves Estimated from the Air-Sea Momentum Flux Budget

Accurate knowledge of the surface wave growth rate by wind plays a critical role in determining the momentum and energy fluxes between the atmosphere and the ocean. Previously, wave growth rates have been determined from laboratory measurements by focusing on the initial stage of the wave growth when non-linear effects are negligible. In this study, wave growth rates of gravity-capillary waves are estimated based on the momentum flux budget from surface wave spectra and laboratory measurements of total and viscous stress at a wind-driven air-water interface. At the interface the total momentum flux partitions into viscous and wave-induced components. The wave-induced stress is computed via a parameterized wave growth rate, which is proportional to the turbulent stress divided by the wave phase speed squared. The constant of proportionality, c_β, is determined under two different assumptions. The first assumption (sheltering assumption) is that the wave growth rate depends on the local turbulent stress, which is a reduced turbulent stress due to the presence of longer waves. In the second assumption (non-sheltering assumption) the turbulent stress is simply equated with the total wind stress. Both assumptions yield simple closed-form expressions for the stress-partitioning ratio. With the sheltering assumption, c_β agrees with previous theoretical estimates and is close to the lower bound of previous empirical parameterizations. Without sheltering, c_β is significantly lower than previous estimates. Therefore, our results indicate that the growth rate of surface waves is determined by the local turbulent stress rather than the total wind stress. This is joint work with Tetsu Hara.

EMILIAN PARAU
University of East Anglia, UK

Free and Forced Three-Dimensional Capillary Gravity Waves

Steady three-dimensional water waves due to a pressure distribution with compact support moving at a constant velocity at the surface of a fluid of infinite depth are considered. The fluid is assumed to be inviscid, incompressible and the flow is supposed to be irrotational. On the free surface the full nonlinear boundary conditions are applied: the kinematic and the dynamic conditions. Both gravity and surface tension are included in the dynamic condition. The three dimensional problem is formulated as a nonlinear integro-differential equation by using the three dimensional free space Green function. When the velocity of the pressure is close to the minimum of the phase velocity, the pressure is removed and free solitary capillary gravity water waves are computed. These waves have damped oscillations in the direction of propagation, as in the two dimensional case (Vanden-Broeck and Dias, JFM, 1992, Dias and Iooss, Physica D, 1993), and also decay in the transverse direction. This is joint work with Jean-Marc Vanden-Broeck(UEA) Mark Cooker(UEA).

YARON TOLEDO
Technion

A Highly Accurate Boussinesq Type Difference Equation for Water Waves

A new finite-difference numerical method is developed specifically for solving high order Boussinesq equations. The method enables the solving of the water waves flow with much higher accuracy compared with standard finite-difference methods for the same computer resources. It is first developed for the linear water waves' case and afterwards for the nonlinear problem. It is presented for a horizontal bottom, but can be used for varying bottom as well. The method can be developed for other equations as long as they use Pade approximation, for example the parabolic equation for acoustic waves' problems. Finally, the results of the new method are compared with the accurate solution for nonlinear progressive waves over horizontal bottom that is found using the stream function theory. The agreement is found to be excellent.