Speakers' Abstracts

SERGEI BADULIN
P.P. Shirshov Institute of Oceanology, Moscow

Self-Similar Solutions for the Hasselmann Equation and Experimental Scaling of Wind-Wave Spectra

The kinetic Hasselmann equation for wind waves

$$\partial N_k/\partial t + \vec{C}_g \nabla N = S_{nl} + S_{in} + S_{ds}$$

is a theoretical basis for wave-prediction models. It prescribes changes of wave action spectral density to the effects of resonant nonlinear interactions, the wind generation and the wave dissipation (mostly by white-capping). While the nonlinear term is known from the 'first principles', our knowledge of generation term S_{in} is based on experimental results, which are not accurate enough to make a dispersion in measurements of the wave growth-rate less than the growth-rate itself. Less is known about the dissipation term S_{diss}. This uncertainty hinders the development of well-justified prediction models.

We modified the Resio-Tracy code for numerical solution of the Hasselmann equation and made it approximately ten times faster, more accurate and stable. With this new code we performed a massive duration-limited simulation of the Hasselmann equation using different versions of generation and dissipation terms, also choosing a small white noise as initial conditions. In all cases we observed the same effect: soon after beginning of computation the nonlinear term S_{nl} becomes dominating over S_{in} and S_{diss}, while the spectral evolution tends to a self-similar regime.

The Hasselmann equation in absence of S_{in} and S_{diss} has a rich family of self-similar solutions. At the fronts of these solutions propagating to low frequencies strong nonlinearity is balanced by non-stationarity or non-homogeneity, while the rare faces of the solutions are described by power-like weak-turbulent exact Kolmogorov solutions of the stationary Hasselmann equation $S_{nl} = 0$.

The whole scope of experimental data on wind-driven sea implies the self-similarity of the spectral shape. Also, the self-similarity hypothesis corroborates with the evidence of power-like dependence of main energy and peak frequency on dimensionless fetch. We propose to describe experimental spectra of wind-driven sea by self-similar solution of the 'conservative' Hasselmann equation. The parameters of these solutions can be found from averaged by $\vec{k}$-space balance of wave action. The balance equation includes the generation and dissipation terms in an integral form, which can be parameterized by a detailed comparison with experiment.

So far the shapes of our self-similar spectra display perfect coincidence with the spectra measured in the JONSWAP and other major fetch-limited studies.

MICHAEL L. BANNER
The University of New South Wales,Sydney

On the Onset and Strength of Breaking of Two-dimensional Deep Water Waves

The recent numerical study of Song and Banner (JPO, 2002, henceforth SB02) proposed a generic breaking onset threshold parameter based on the wave energy convergence rate at the maximum of an evolving two-dimensional nonlinear gravity wave group in deep water. In a companion paper, these authors found that this mechanism for breaking onset was not modified significantly by strong wind forcing and shear. SB02 also suggested that the parametric growth rate just prior to breaking onset might control the strength of breaking events. This presentation describes the results of a detailed observational study aimed at validating their proposals.

For the nonlinear wave groups investigated, the observations reproduced the evolutionary trends of the SB02 model predictions in regard to key evolution times and confirmed the breaking threshold level of 0.0015 for their proposed parametric growth rate. In regard to strength of breaking, the proposed dependence of breaking strength on the level of their parametric growth rate just before breaking onset was not found to be valid. However, by using a slight modification of the SB02 parametric growth rate, it appears possible to achieve a consistent predictor for the observed breaking strength.

J. THOMAS BEALE
Duke University

Analytical Issues in Boundary Integral Methods for 3D Water Waves

Boundary integral methods have long been used to simulate fully nonlinear, time-dependent water waves. For surface waves in 3D it appears that choices of discretization can strongly affect the numerical stability as well as accuracy. We will discuss the formulation, accuracy, and stability in the case of a doubly periodic surface with Lagrangian markers. One version of the method, not too different from ones in use, has been shown to converge to the actual solution as long as it is smooth. In the method to be described, the normal velocity at the surface is determined from the potential by solving an integral equation. The singular integrals are computed using a simple quadrature rule with a regularization and then adding a correction. A sign condition for the discrete single layer potential seems to be important in maintaining wavelike behavior. The error analysis leading to convergence depends on preserving a structure in the linearized discrete equations which appears in the original system.

GUILLEMETTE CAULLIEZ
Institut de Recherche sur les Phenomenes Hors Equilibre

Modulation of Small-Scale Wave Roughness by Long Waves

Modulation of small-scale wind waves by long waves, for example, by swell or dominant waves, is a phenomenon which is central for the radar image formation at the sea surface. Recent theoretical models suggest that the modulation is dependent not only on direct wave straining by long-wave orbital motions, but also on variation of the wind-exerted stress along long waves. Owing to the complexity of long wave-short wave-air flow interaction processes at the origin of these phenomena, well-controlled observations made in a wide range of wind and wave parameters are needed to validate the models. An experiment was then designed in the large IRPHE-Luminy wind-wave tank aiming at simulating two-scale wave fields composed of mechanical long waves and narrow-band wind-generated short waves. Measurements of water surface heights and mean air flow parameters were made simultaneously by high-resolution capacitance wave gauges and hot X-wire probes. To estimate the modulation transfer function, the long wave signal and the short wave energy component modulated by the long wave are extracted from the wave signal by means of a continuous wavelet decomposition. The potentiality of this method to analyze two-scale wave fields as well as more complex wind wave fields is examined in detail and compared with the classical methods based on the phase average or the windowed Fourier transform. The phase and the amplitude of the modulation transfer function are thus determined for the different wind speed, wave steepness and frequency range conditions, and the results obtained are analyzed within the framework of the recent coupled air-water flow theories.

DMITRY CHALIKOV
ESSIC/University of Maryland

Numerical Investigations of Nonlinear Properties of 1-D Wave Field

By a nonstationary conformal mapping, the principal equations are rewritten in a surface-following coordinate system and reduced to two simple evolutionary equations for the elevation and the velocity potential of the surface; Fourier expansion is used to approximate these equations. High accuracy of the method was confirmed by validation of the nonstationary model against known solutions and by comparison between the results obtained with different resolution in the horizontal. The method developed is applied to simulation of evolution of wave fields with different initial conditions. Numerical experiments with initially monochromatic waves with different steepness show that the model is able to simulate breaking conditions when the surface becomes a multi-valued function of the horizontal coordinate; an estimate of the critical initial wave height that divides between non-breaking and eventually breaking waves is obtained. Simulations of nonlinear evolution of a wave field represented initially by two modes with close wave numbers (amplitude modulation) and a wave field with a phase modulation both result in appearance of large and very steep waves, which also break if the initial amplitudes are sufficiently large. Statistical properties of multimode linear and nonlinear wave fields have been compared.

JAMES H. DUNCAN
University of Maryland

An Experimental Study of the Effects of Surfactants on Spilling Breakers

The dynamics of spilling breakers in the presence of surfactants were studied experimentally. The breakers were produced from Froude-scaled mechanically generated dispersively focused wave packets with average frequencies of 1.15, 1.26 and 1.42 Hz. Separate experiments were performed with the same wave maker motions in clean water and in water with various bulk concentrations of the soluble surfactants Sodium Dodecyl Sulfate, Triton X-100 and Hemicyanine. For each surfactant condition, the surface pressure isotherms, equilibrium surface elasticity and surface viscosity were measured in situ in order to characterize the dynamic properties of the free surface. The histories of the crest profiles during breaking were measured with a laser-induced fluorescence technique and a high-speed digital camera. In clean water, all waves examined herein break without overturning of the free surface. This breaking process begins with the formation of a bulge on the forward face of the wave crest and with capillary waves forming upstream of the leading edge of the bulge (called the toe). After a short time, the flow separates under the toe and a turbulent flow is developed while the toe moves rapidly down the wave face. During the toe motion, a train of ripples appears between the toe and the crest and this train of ripples is swept downstream. For most surfactant conditions, the "clean-water" breaking process is modified quantitatively but not qualitatively. In these cases, the bulge shape changes and its size generally decreases with increasing surfactant concentration. The capillary waves found upstream of the toe in the "clean" water case are dramatically reduced at even the lowest concentrations of surfactants used herein. The transition to turbulent flow is still initiated when the toe begins to move down the forward face of the wave. The pattern of ripples generated between the toe and the crest of the wave during this phase of the breaking process varies with the concentration of surfactant. Various geometrical parameters of the wave crest profiles are analyzed by scaling them with the measured dynamic properties of the surfactants. Under a more restricted set of surfactant conditions, the transition to turbulent flow is initiated by the formation of a small plunging jet which sometimes issues from a place below the point of maximum surface elevation. The surfactant conditions under which these "micro jets" appear are examined.

A.I. DYACHENKO
Landau Institute for Theoretical Physics, Moscow

Direct Numerical Simulation of $2+1$ *Dimensional Surface Waves on Deep Water*

Euler's equation describing the potential flow of an ideal incompressible fluid with a free surface can be formulated in a Hamiltonian form. Expansion of the Hamiltonian in powers of nonlinearity up to the terms of fourth order leads to equations that can be solved numerically by the spectral code. We performed the simulation of these equations on the grids of 256×256 and 512×512 harmonics. To stabilize the algorithm and avoid the accumulation of energy in high wave numbers, we introduced an artificial high-frequency dumping to this equation. The cases of gravity and capillary waves we studied separately. In both cases we studied the development of instability for stationary waves as well as the formation of weak-turbulent Kolmogorov spectra.

The stationary capillary waves undergo the first order decay instability. This instability leads to appearing of secondary waves concentrated in K-plane near the curve describing by the resonant condition

$$\omega(\vec{k}_0) = \omega(\vec{k}) + \omega(\vec{k}_0 - \vec{k})$$

Here $\omega(k) = |\sigma_k|^{3/2}$ and $\vec{k}_0$ is the initial wave vector. The secondary waves, in their turn, undergo the decay instability. The process repeats itself several times and leads to the formation of totally chaotic turbulent state.

Stationary gravity waves are unstable with respect to secondary-order (modulational) instability that leads to the formation of waves on the Phillips curve

$$2\omega(\vec{k}_0) = \omega(\vec{k}) + \omega(2\vec{k}_0 - \vec{k})$$

We have observed this process in our experiments.

As the theory of weak turbulence predicts, we observed the formation of weak-turbulent Kolmogorov spectra for both capillary and gravity waves.

MARK GROVES
Loughborough University

Three-Dimensional Solitary Gravity-Capillary Water Waves

The existence of solitary-wave solutions to the three-dimensional water-wave problem with strong surface-tension effects is predicted by the KP-I model equation (Kadomtsev and Petviashvili). The term *solitary wave* describes any solution which has a pulse-like profile in its direction of propagation, and the KP-I equation admits three types of solitary wave. A *line solitary wave* is spatially homogeneous in the direction transverse to its direction of propagation, while a *periodically modulated solitary wave* is periodic in the transverse direction. A *fully localised solitary wave* on the other hand decays to zero in all spatial directions.

In this talk I outline mathematical results which confirm the existence of all three types of solitary wave for the full gravity-capillary water-wave problem in its usual formulation as a free-boundary problem for the Euler equations. The line solitary wave is found by establishing the existence of a two-dimensional invariant manifold containing a homoclinic orbit (Kirchgässner). The periodically modulated solitary waves are created when the the line solitary wave undergoes a *dimension-breaking bifurcation* in which it spontaneously loses its spatial homogeneity in the transverse direction; an infinite-dimensional version of the Lyapunov centre theorem is the main ingredient in the existence theorem (Groves, Haragus and Sun). The fully localised solitary wave is obtained by a variational argument based upon the mountain-pass lemma and the concentration-compactness theorem (Groves and Sun).

R.T. GUZA
Scripps Institution of Oceanography

Field Observations of Shear Waves

Obliquely incident waves breaking on a beach drive an alongshore mean current that can exceed 1m/sec. This current, localized to the surf zone and highly sheared in the cross-shore direction, is unstable. The growing instabilities, known as shear waves, are vorticity waves with periods of a few minutes and alongshore wavelengths of a few 100m. Shear waves observed at five cross-shore locations within a few 100m of the shoreline of a sandy beach are compared with numerical solutions of the nonlinear shallow water equations. As in previous model-data comparisons within the surfzone where V is strong and shear waves are energetic (Özkan-Haller and Kirby, JGR 1999), modeled and observed shear wave alongshore phase speeds are similar, and observed and simulated shear wave root-mean-square velocity fluctuations are both 10-20% of V. Farther offshore, where V and shear wave energy levels are reduced, the observed alongshore phase speeds usually decrease to roughly V. A cross-shore decrease in the numerically simulated phase speed is associated with the alongshore advection, by the relatively weak and unsheared V, of features shed from the strong, sheared flow further onshore. Although finite amplitude effects limit shear wave growth, aspects of the fully nonlinear numerical solutions (and the observations) resemble the linearly unstable modes.

JOSEPH HAMMACK
Pennsylvania State University

Solitary-Wave Collisions

Experimental and theoretical results are presented for both head-on and following collisions of two solitary waves. The experimental data provide high-resolution measurements of the water surface at fixed times thereby enabling direct comparisons with predictions by a variety of mathematical models. These models include the Korteweg-de Vries equation, higher-order coupled KdV equations, Eulers equations, and linear superposition of KdV solutions.

TETSU HARA
University of Rhode Island

Observation of Nonlinear Steep Waves

Nonlinearity and directionality of evolving open-ocean wave fields are investigated using wave height records from a triangular wave gauge array. In addition to the frequency wave elevation spectrum, the frequency wave slope spectrum, the bicoherence of the wave elevation time series, and the peak wavenumber estimates at a given frequency are examined. The results suggest that as the wave field matures, the contribution to the frequency spectrum from the directional higher bound harmonics generated by short-crested steep dominant waves becomes increasingly important relative to the contribution from free waves. Next, a wavelet analysis methodology is used to estimate the statistics of steep waves. The method is applied to open ocean spatial wave height data. Results show that high wave slope crests appear over a wide range of wavenumbers, with a large amount being much shorter than the dominant wave. At low wave slope thresholds, all wave fields have roughly the same amount of wave crests regardless of wind forcing. The steep wave statistic decays exponentially with the square of the wave slope threshold, with a decay rate that is larger for the low wind cases than the high wind cases. Comparison of the steep wave statistic with past measurements of the breaking wave statistic suggests a breaking wave slope threshold of about 0.12. Although the steep wave statistic roughly increases with the cube of the wind speed as the breaking wave statistic does, other factors besides the wind speed also affect the level of the steep wave statistic. Comparison of the steep wave statistic to the saturation spectrum reveals a reasonable correlation at high wave slope thresholds. The crest directionality statistic shows that most of the steep wave crests are normal to the direction of the mean wind. This is inconsistent with the Fourier wavenumber spectrum that shows a broad bimodal directional spreading at high wavenumbers. The crest length statistics demonstrate that the wave field is dominated by short-crested waves with small crest length/wavelength ratios.

BRIAN K. HAUS
University of Miami

The Modulation of C-Band Radar Reflectivity by Long Waves in a Wind-Wave Tank

Laboratory measurements to quantify the effect of long waves on the modulation of short waves and the effect that this modulation has on radar reflectivity have been conducted at the Air-Sea Interaction Saltwater Tank (ASIST) facility of the University of Miami. The critical parameters for the modulation of radar reflectivity were directly observed using non-intrusive techniques. The local water surface elevation and the slope vector were sampled using laser techniques at a point. The two-dimensional distribution of surface slopes were also imaged at a rate of 120 Hz using an optical imaging slope gauge. Experiments were run with various wind velocities modulated by mechanical long-waves. Co-incident with these measurements, the radar backscatter was sampled over the same region of the surface using a dual-polarized C-band Doppler radar. When there were no long waves, the short wave structure as observed by the 2-D slope gauge was well developed with peaks in surface slopes corresponding to the wavelength of the peak wind waves and having a clear group structure. This organization was in sharp contrast to the variability of the slopes observed when a 1 Hz mechanical long-wave was generated in the same direction as the wind waves. The slopes linked to the wind wave were suppressed, while there was modulation at the 1 Hz long-wave frequency. Significant modulation of the C-band HH polarized radar return was evident at the long wave frequency.

PAUL HWANG
Naval Research Laboratory

Spatial Measurements of Ocean Surface Waves

Spatial measurements of ocean surface waves provide direct wavenumber resolution of the wave spectrum. This is advantageous in avoiding the problems associated with doppler frequency shift in the case of small scale surface roughness studies in relation to ocean remote sensing. If the spatial measurements cover the 2D (x, y) plane, the directionality of the wave field can be derived straightforwardly. The processing of 2D images, however, is still limited by the 180-degree ambiguity due to the axi-symmetric property of Fourier analysis, and may hamper our ability to distinguish between left-right and front-back propagation of the wave components. The ambiguity can be avoided by 3D (x, y, t) measurements, which produce the complete 3D $(k_x,\ k_y,\ \omega)$ spectrum. Examples of 1D, 2D and 3D spatial measurements of surface waves will be presented. The resolved wavelength ranges from 4 millimeters (laser scanning slope sensor) to hundreds of meters (airborne scanning lidar system). Some interesting results derived from spatial measurements include the wavenumber properties of short waves that are important to quantification of the ocean surface roughness, delicate difference in the interpretation of dispersion relation derived from spatial and temporal measurements, bimodal directional distribution of surface waves in equilibrium range that illustrates the role of nonlinear wave-wave interaction in the evolution of surface wave dynamics, and bimodal directional distribution of young waves that poses questions on our present interpretation of the wind wave generation mechanisms.

PETER JANSSEN
ECMWF

Nonlinear Four-Wave Interactions and Freak Waves

In this presentation I will show that there is a close connection between four-wave interactions and the occasional occurrence of extreme sea states, usually referred to as Freak Waves. Four-wave interactions tend to gives rise to focussing of wave energy with the consequence that deviations from the Normal probability distribution of the surface elevation result. These deviations increase with increasing Benjamin-Feir Index. There is a good agreement between prediction of theory and laboratory observations.

When I initially started thinking about a possible connection between four-wave interactions and freak waves, I thought that it would be of vital importance to validate the theoretical results against ensemble averages of results from Monte Carlo simulations with the deterministic equations. Because I realized large ensembles were required I decided to do one dimensional simulations only.

The problem with one-dimensional simulation is, however, the following. While the ensemble averaged results clearly showed an evolution in time of the wave spectrum, the well-known Hasselmann equation for the rate of change of the wave spectrum owing to resonant four-wave interaction gives in one dimension no rate of change of the spectrum.

An extension of the classical picture was thus required. Two possible directions came to my mind. The first one is that inhomogeneous effects (Alber, 1978) should be included, while the second one is that possibly non-resonant interactions should be taken into account. Using the results from the Monte Carlo simulations the size of the inhomogeneity of the ensemble of waves can be estimated and it turns out that inhomogeneities can be safely neglected. On the other hand, extending the Hasselmann approach to include non-resonant interactions gives for large times a favourable agreement with results from the Monte Carlo Simulations. Therefore, in one-dimension non-resonant interactions play an important role and they give rise to an irreversible change of the wave spectrum.

HENRIK KALISCH
Lund University

Uniqueness Results for Periodic Traveling Waves

Consideration is given to periodic traveling waves on the surface of a perfect fluid without the assumption that the flow is irrotational. If the variation in the direction transverse to the propagation of the traveling wave is negligible, the flow can be described by the two-dimensional Euler equations with appropriate boundary conditions. In this situation, two uniqueness results are presented.

First, it is shown that in water of finite depth, the surface profile of a periodic traveling wave uniquely determines the corresponding flow in the body of the fluid. This holds for rotational flow as long as the vorticity function satisfies appropriate conditions which are also shown to be sharp.

Second, attention is given to the case when the pressure is constant along streamlines. In this case, it is shown that the flow must be given by a special explicit solution which was found by Gerstner.

MICHAEL S. LONGUET-HIGGINS
Institute for Nonlinear Science, University of California, San Diego

Mass Transport by Shoaling Water Waves Over a Rough Sea Bed

In very gradually shoaling coastal water the energy of incident waves appears to be absorbed not by breaking at the upper surface, but predominantly by turbulent dissipation near the rippled sea bed. The question has been asked whether there can be then any wave set-up, that is any increase in the mean water level, at, or close to, the shoreline.

The present investigation is in two parts. In the first, the bottom-boundary-layer is represented by a liquid of higher density and viscosity than the water above, and it is verified that the mean forward motion in the boundary-layer (the "bottom-wind") carries the viscous fluid up a plane beach as far as the breaker-line. There it is blocked and dissipated by the undertow and turbulence in the surf zone. The lower the amplitude of the incoming waves, the higher up the beach can the fluid in the boundary-layer penetrate.

The second part of the paper is theoretical. The usual equations of wave energy and momentum in water of slowly varying depth are generalised so as to include the presence of a dissipative boundary layer at the bottom. It is then shown that the resulting equation for the mean surface slope can be integrated exactly, to give the mean surface depression (the "set-down") in terms of the local wave amplitude and water depth, outside the surf zone. In the special case of a uniform beach slope s, a closed expression is obtained for the wave amplitude in terms of the local depth. It is shown that the waves can theoretically penetrate to the shoreline without breaking, for a sufficiently small value of s. In a practical example, corresponding to swell on the Atlantic coast of North America, it is

found that s must be of order 10^{-3}. The corresponding wave set-up is always negative and tends to zero at the beach.

PER A. MADSEN
Technical University of Denmark

Recent Progress in Modelling of Non-linear Water Waves

A new method valid for highly dispersive and highly nonlinear water waves interacting with a rapidly varying bathymetry has recently been developed. It is based on truncated series solutions to the Laplace equation combined with the exact boundary conditions at the free surface and at the sea bottom. We consider the method to belong to the Boussinesq-type family of methods although it is formulated as six coupled equations involving up to fifth-derivative operators. As a result, linear and nonlinear wave characteristics are very accurate up to wave numbers as high as kh=30, while the vertical variation of the velocity field is applicable for kh up to 12. This presentation will concentrate on the following achievements: a) Velocity profiles in highly nonlinear shallow water and deep water waves; b) Investigation of crescent wave patterns; c) Investigation of class III Bragg scattering. First, velocity profiles in highly nonlinear waves are computed in deep water as well as in shallow water. The results are verified against streamfunction theory and against Tanakas solitary wave solution. With the established range of applicability, the method has an obvious potential for computing the velocity field in highly nonlinear irregular wave fields, which makes it an attractive tool for design purposes. Next, a numerical study of crescent (or horseshoe) water wave patterns is presented. These patterns arise from the instability of steep deep-water waves to three-dimensional disturbances. We focus on the development of the most unstable stationary L2 pattern, discuss the growth rate and the physical processes. Next, we investigate the development of oscillating crescent patterns which no longer propagate in quasi-steady form, but emerge and disappear repeatedly. Quantitative estimates for the oscillation period are given based on a stability analysis following McLean (1982). Finally, we study the interaction of nonlinear waves with an undular sea bottom. As a result we observe class III Bragg scattering, which involves three surface wave numbers and one bottom wave number. Reflection occurs as a sub-harmonic resonance, while transmission occurs as a super-harmonic resonance. The growth rate of the Bragg scatter as well as the location of the resonance depends on the nonlinearity of the incoming wave. A downshift/upshift of resonance is observed for reflection/transmission and in order to explain this we develop a new third order theory for bichromatic waves.

CHIANG C. MEI
Massachusetts Institute of Technology

Localization of Surface Waves by Random Variations in Depth

Spatial attenuation of sea waves can be caused by radiation of incoherently scattered waves, in addition to bottom friction. We describe recent theories of water waves scattered by a long stretch of randomly rough seabed, where the root-mean-square height of the roughness is moderately small compared to the sea depth. Two separate theories will be reported for wavelength comparable to and much smaller than the water depth. By using two-scale expansions and Green's functions, evolution equations for the wave amplitudes are derived. In finite depth, narrow-banded, nearly monochromatic waves are found to be governed by a nonlinear Schrödinger equation, modified by an additional complex damping term. For periodic waves in shallow water, energy exchange among different harmonics is accompanied by radiation damping. For a transient wave pulse in shallow water, disorder gives rise to a diffusion equation with additional terms representing effects on phase velocity and dispersion. A variety of numerical examples will be discussed.

W. KENDALL MELVILLE
Scripps Institution of Oceanography

The Ocean Surface in the High-Wind Regime

Much of our knowledge of the ocean surface and the surface wave field is based on observations, models and theories that cover the low to moderate wind speed regime, say winds of less than 10-15 m/s. Most theories of ocean surface waves are based on the idealization of irrotational linear free waves; that is, waves that are propagating in an ideal fluid with no leading-order forcing acting on the surface and no departures from the classical free-surface boundary conditions. Such waves satisfy a linear dispersion relationship with gravity and surface tension as restoring forces. Departures from this state are introduced as weak perturbations, including the effects of nonlinearity, wave growth due to the wind, and wave decay due to viscosity, turbulence, or (weak in the mean) intermittent breaking. The advantage of this approach is that it is anchored in the linear dispersion relationship, with all the simplicity that it affords as a basis for perturbation expansions, and all the detailed understanding of the linear kinematics and dynamics. There is an extensive literature on surface waves that follows this approach and it has been the foundation of numerical wind-wave prediction schemes. However, there is growing empirical evidence that under high winds or strongly-forced conditions, in which the time scales for surface-wave growth may be comparable to the wave period, the dynamics of the surface may change significantly, and new approaches must be found. In this talk, I will review some of the evidence for qualitative changes in the ocean surface in high winds, discuss some of the phenomena that may have to be included in improved models and theories, and present initial results of a model that couples a strongly-forced well-mixed shallow surface layer to a deeper lower layer (Fedorov and Melville, 2004). The introduction of new scales in the problem leads to new surface-wave modes that may include quasi-periodic breaking.

PAUL MILEWSKI
University of Wisconsin

Hydraulic Jumps and Fluid Mixing in Two-Layer Shallow Water

The shallow water, or hydraulic, limit for wave breaking in a single fluid layer is well understood: conservation of mass and momentum completely define the shock (or hydraulic jump), and conservation of energy gives, a posteriori, the rate at which "internal" energy (mostly in the form of small scale turbulence) is generated. For the case of two layers of miscible fluids, this same shallow water limit is much harder, and, in particular, requires an additional postulate for the mixing rate at the shock. One can imagine that the energy dissipated at the shock can now flow both into small scale turbulence and into mixing the fluid. We discuss three possibilities for deriving the additional constraint on the problem, motivated by: 1. Kinematics, 2. Energy, 3. Entropy maximization. These yield surprisingly similar shock conditions and result in physically reasonable mixing rates.

A. R. OSBORNE
Universita' di Torino

Periodic Inverse Scattering Transform: Very Fast Numerical Simulations and Analysis of Laboratory Deep-Water Wave Trains

The Inverse Scattering Transform with periodic boundary conditions solves nonlinear integrable wave equations (Korteweg-deVries, nonlinear Schroedinger, etc.) using the Riemann theta function. Two additional applications of the method are discussed in this talk: (1) Very fast numerical simulations of nonlinear wave equations and (2) the analysis of time series from the Trondheim wave tank (depth 10 m, width 10 m, length 300 m). The numerical simulations are made to the Dysthe equation (extended nonlinear Schroedinger equation in 2+1) for the propagation of deep-water wave trains. The analysis of time series of random wave trains is based upon the projection of the data onto the Riemann theta function and the subsequent interpretation of the analysis in terms of unstable modes of the NLS equation. Both of these applications of Riemann theta functions are useful for the study of rogue or freak waves in deep water.

HARVEY SEGUR
University of Colorado

Stabilizing the Benjamin-Feir Instability

The Benjamin-Feir instability is a modulational instability in which a uniform train of oscillatory waves of finite amplitude loses energy to a small perturbation of waves with nearly the same frequency and direction. The concept is well established in water waves, in plasmas, and in optics. Even so, we show that any amount of dissipation (of a certain type), no matter how small, stabilizes this instability. We confirm our analytical predictions with laboratory experiments on waves in deep water. Finally, we reexamine previous experimental studies of an instability that we claim does not exist in the presence of dissipation.

V. I. SHRIRA
Keele University,UK

Direct Numerical Simulation of Nonlinear Evolution of Random Wave Fields

The study is concerned with the problem of direct numerical simulation of nonlinear evolution of random wave fields and is aimed at establishing the range of applicability of the classical kinetic description as well as the underlying statistical hypotheses. The problem is more challenging than DNS of turbulence, since to address this problem the study should involve integration of basic equations of hydrodynamics for a very large number of interacting modes over very long time spans with very high accuracy. At present DNS of the basic equations with desired accuracy over the required time scales is impractical if not impossible.

We assume weak nonlinearity of the wave field and develop an algorithm based upon integration of the integro-differential Zakharov equation. We apply it to a number of "model" problems where the required accuracy of simulation can be ensured. The study is focussed on modelling of evolution of finite number localized wave packets. The ensemble averaged results are compared with the solutions of the kinetic equation for water waves. It is found that an effective way of modelling the packets is to "construct" them out of small number of discrete harmonics. Within a certain range of parameters neither the size of the clusters nor the number and positions of the constituting harmonics affect the evolution of the statistical characteristics of the wave field. It is shown that the near-resonant interactions play the key role in the nonlinear evolution, while modelling of a wave field by any number of strictly resonant harmonics leads to a qualitatively inadequate description of the field evolution. We compared the DNS predictions based on employing clusters for a wide range of field configurations. When the standard kinetic description is applicable, then the DNS predictions coincide with the solutions of the kinetic equations with a good accuracy. Application of the cluster approach for modelling evolution of the continuous wave fields and general issues concerned with the DNS are also discussed.

MICHAEL STIASSNIE
Technion-Israel Institute of Technology, Haifa Israel

On the Interaction of Four Water-Waves

The mathematical and statistical properties of the evolution of a system of four interacting surface gravity waves are investigated in detail. Any deterministic quartet of waves is shown to evolve recurrently, but the ensemble averages taken over many realizations with random initial conditions reach constant asymptotic values. The characteristic time-scale for which such asymptotic values are approached is extremely large when randomness is introduced through the initial phases. The characteristic time-scale becomes of an order comparable to that of the recurrence periods when beside the random initial phases, the initial amplitudes are taken to be Rayleigh-distributed. The ensemble-averaged results in the second case resemble, to a certain extent, those derived from the kinetic equation.

JOHN STRAIN
UC Berkeley

Fast Semi-Lagrangian Computations with Complex Interfaces

Models of physical phenomena such as crystal growth or water waves generally involve complex moving interfaces, with velocities determined by interfacial geometry and material physics. Numerical methods for such models tend to be customized. As a consequence, they must be redesigned whenever the model changes.

We present a general computational algorithm for evolving complex interfaces which treats the velocity as a black box, thus avoiding model-dependent issues. The interface is implicitly updated via an explicit second-order semi-Lagrangian advection formula which converts moving interfaces to a contouring problem. Spatial and temporal resolutions are decoupled, permitting grid-free adaptive refinement of the interface geometry. A 2D modular implementation computes highly accurate solutions to geometric moving interface problems involving merging, anisotropy, faceting, curvature, dynamic topology and nonlocal interactions. The implementation couples with a Laplace solver based on classical potential theory, to provide fast accurate solutions of the Ostwald ripening model for grain growth by volume diffusion.

WALTER STRAUSS
Brown University

Periodic Water Waves with Vorticity

Consider classical inviscid water waves with non-trivial vorticity, with a free surface under the influence of gravity over a flat bottom. There exist many global continua of such 2D periodic traveling waves. The governing equations are Euler's and the pressure is constant on the free surface. There is a global continuum of such solutions for every choice of speed c, spatial period L, relative mass flux p_0 and vorticity function γ that satisfy a certain explicit inequality. The inequality is satisfied for any c, L and γ provided p_0 is sufficiently small. The waves are symmetric around each crest, the period is the distance between successive crests, and the profiles are monotone between crest and trough. Each continuum extends from flat waves all the way to waves that have stagnation points. Thus there exist many rotational periodic traveling waves of large amplitude. These results have been obtained as joint work with Adrian Constantin. Almost all the previous work on this problem, notably by Stokes, Nekrasov, Levi-Civita, Keady, Norbury, Amick and Toland, has been restricted to irrotational flows.

ESTEBAN G. TABAK
New York University

Breaking Waves and Shear Instability in Two-Layer Flows

We study a model in which two scenarios for eventual mixing of stratified flows (shear-instability and internal breaking waves) are, in principle, possible. In this shallow water two-layer flow model it is found that unforced flows cannot reach the threshold of shear-instability without breaking first. Mathematically, for 2X2 autonomous systems of mixed type, a criterium is found deciding whether the elliptic domain is reachable from hyperbolic initial conditions.

PAUL H. TAYLOR
University of Oxford

Directional Spreading and Non-Linear Wave Focussing on Deep Water

Observations of waves on the open sea are consistent with weakly non-linear modifications to a linear random process, at least most of the time. The occurrence of large waves can then be attributed to the random alignment of many small independent components. This random focussing by frequency and directional dispersion can be represented by numerical or physical experiments on focussed wave groups.

Both fully non-linear and non-linear Schrodinger based simulations of focussed wave groups show the marked effect of directional spreading. A 1-D group contracts along the mean wave direction and becomes considerably taller than predicted by linear theory. The non-linear dynamics are active for many wave periods and cumulative, leading to solitons and recurrence at least in the NLS equation. For groups with realistic directional spreading, there is still significant contraction along the mean wave direction but the group also becomes considerably more long-crested. Some extra crest elevation may result but this is much smaller than in 1-D. Although the non-linear dynamics occur over a much shorter period, the effects are still significant and the spectral content well after focus is markedly different to that before. Although not quantitatively accurate, the NLS equation and its solutions are useful in interpreting the effects of wave directionality and we suggest the use of an NLS conserved quantity $[Ax^2] - 2[Ay^2] - 2ko^4[A^4]$ as comparable to a Benjamin-Feir index for directionally spread groups.

MARSHALL P. TULIN
Santa Barbara, California

Some Recent New Results in Water Waves

In this Lecture we present some recent results of a mathematical nature, not well known, on the mechanics of water waves. Three separate subjects are considered.

In the first, the transport of energy in uniform and modulated water waves of sinusoidal form has been considered and it has been shown that each of the three forms of energy constituting the waves (i.e. kinetic, gravity potential, and surface tension energy) travel at different speeds, and that the group velocity, cg, is the energy weighted average of these speeds, depth and time averaged in the case of kinetic energy. It has also been shown that the time averaged kinetic energy travels at every depth horizontally at a speed equal (deep water) or faster than the wave itself. In contrast, the propagation speed along the surface of the gravity potential energy is null, while the surface tension energy travels forward along the wave surface everywhere at twice the wave velocity. Finally, the propagation of both sinusoidal waves and wave group envelopes are shown to be made possible by the vertical transport of kinetic energy to, or in the case of capillaries, from the free surface, where it provides the balance in surface energy just necessary to allow the propagation of the wave.

In the second, the mathematical criterion for the inception of the breaking process in a modulated sinusoidal wave in water of arbitrary uniform depth is precisely derived, and shown to be identical to the simple condition earlier (1994) determined from numerical experiments on the evolution of modulated waves (Yao, Wang, and Tulin):

"that upon passing through the peak of a modulation group, when the orbital velocity at the wave crest, u(crest), exceeds the wave group velocity, cg, then the wave crest and trough both rise, the front face steepens, the wave crest sharpens, and eventually a jet forms at the crest, leading finally to splashing and a breakdown of the wave."

The present mathematical demonstration of this failure of progressive motion reveals that the modulated sinusoidal wave in water of any depth is incompatible with kinematical requirements while passing through the peak of a modulation, once, u(crest) ¿ cg. Experimental verification of this result is also shown, based on stereo video measurements of surface velocities in modulating breaking waves in a large wave tank.

In the third, the necessary modification to the cubic NLS model of the wave evolution equations is shown which accounts properly for both the loss of energy and momentum in the organized wave motion due to breaking, and the importance of these terms is discussed.

HARRY YEH
Oregon State University

Long-Wave Runup on a Plane Beach

The exact analytical solution of fully nonlinear non-breaking shallow-water waves on a uniformly sloping beach was derived by Carrier and Greenspan in 1958. Because the derivation involves nonlinear and hodograph-type transformation, the Carrier-Greenspan solution is not in a convenient form to be converted to presentations in real time and space domains. For this reason, only a limited number of the applications have been reported. To improve this deficiency, we re-derive the Carrier-Greenspan equation (the form of a linear cylindrical wave equation) with different non-dimensionalization and different transformation. To solve the problem with arbitrary initial conditions, we apply the Fourier-Bessel transform, and inversion of the transform leads to an exact integral representation of the solution. However, this integral is not convenient for computation due to its highly oscillatory behavior of the integrand. The integral is further manipulated to yield the exact Green function representation, which involves the complete elliptic integral of the first kind. This solution form is convenient for numerical integration to obtain the solutions in the physical time and space domains. With this semi-analytic solution technique, several examples of tsunami runup and drawdown motions are presented. In particular, detailed shoreline motion, velocity field, and inundation depth on the shore are closely examined. It was found that the maximum flow velocity occurs at the moving shoreline and the maximum momentum flux occurs in the vicinity of the extreme drawdown location. The direction of both the flow velocity and the maximum momentum flux depend on the initial waveform: it is in the inshore direction when the initial waveform is predominantly depression; in the offshore direction, when the initial waves have a dominant elevation characteristic. Lastly, we present an envelop-curve of the maximum momentum flux in space, which might be used as a design criterion for the tsunami resistant building code.

Mathematical derivation presented in this talk was reported by Carrier, Wu and Yeh (2003 JFM 475).

DICK K.P. YUE
MIT

A Capability for Severe Weather Automated/Assisted Ship Handling (SWASH)

Concurrent advances in the large-scale deterministic simulation and prediction of nonlinear ocean wave-fields, wave sensing capabilities, large-amplitude ship motions predictions, control and optimization, and high-performance computing, have made it possible for the first time to radically increase the operational capability and survivability of ships in severe conditions. The goal is an integrated real-time deterministic predictive system that provides the optimal control commands and path selection for a single or networked vessels for a given mission and ocean environment. The main challenges to achieving this goal are associated with wave hydrodynamics — the sensing of the waves, and the deterministic reconstruction and forecasting of the wave-field based on the sensed data. In this talk, I will focus on the latter and the related developments that make the overall capability possible. I will also show a number of examples or demos of SWASH in realistic applications.

V. ZAKHAROV
University of Arizona

Free-Surface Hydrodynamics in Conformal Variables and the Origin of Freak Waves

Euler equations describing the potential flow of an incompressible ideal fluid with free surface in 1+1 dimensions can be studied efficiently by performing the conformal mapping to the lower half-plane. For the finite depth fluid the half-plane has to be replaced by a horizontal strip. The Euler equations in conformal variables are Hamiltonian, though the both symplectic and Poisson structures are not canonical.

In essential degree, the surface dynamics is defined by motion of singularities of the Jacobian for the conformal mapping posed in the upper half-plane. The Jacobian's zeros generate commuting motion constants of the system. It does not mean that the system becomes integrable. Integrability takes place only in two limiting cases: when Jacobian has a "narrow" cut and when the surface is "almost flat". In these cases the system is reduced to the complex one, then to the real Hopf equation. In a general case, the cut is a structurally stable type of singularity. The equations imposed on the spectral densities on a cut could be written in the closed form.

Equations of free-surface hydrodynamics in conformal variables are convenient for numerical simulation. To solve them, one can use either a spectral code or a direct numerical calculation of the Hilbert transform.

Using the conformal variables we performed the numerical simulation of a nonlinear stage of the Stokes wave modulational instability. We showed that the development of the instability for waves of a moderate amplitude ($k_a \simeq 0.15$) leads to formation of sporadic freak waves with the amplitude exceeding the Stokes limit ($k_a \simeq 0.5$). This leads us to the hypothesis that freak waves are a result of modulational instability.