Exploiting quantum control for quantum computation

Marker atoms and molecular interactions in optical lattices

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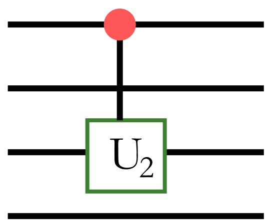
What we want to implement

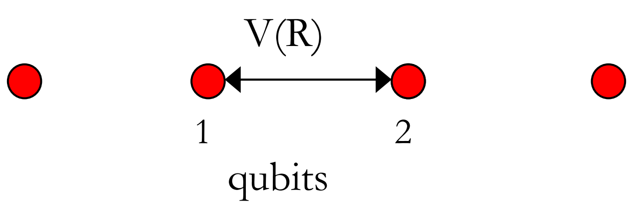
- implement entanglement of two qubits



example:
phase gate

$$
\begin{aligned}
|00\rangle &\rightarrow |00\rangle \\
|01\rangle &\rightarrow |01\rangle \\
|10\rangle &\rightarrow |10\rangle \\
|11\rangle &\rightarrow e^{i\phi}|11\rangle
\end{aligned}
$$

- Controlled two-body interaction



We must design a Hamiltonian

$$H = \Delta E(t)\,|1\rangle_1\langle 1| \otimes |1\rangle_2\langle 1|$$

so that

$$|1\rangle_1 \otimes |1\rangle_2 \rightarrow e^{i\phi}|1\rangle_1 \otimes |1\rangle_2$$

Examples:
Cold collisions in moving potentials: optical lattices (Jaksch et al PRL 1999)
Cold collisions in switching potentials: magnetic microtraps (Calarco et al. PRA 2000)
Dipole-dipole interactions: Rydberg gate (Jaksch et al. PRL 2000)
Electrostratic interaction: ion microtraps (Cirac and Zoller Nature 2000; Calarco et al. PRA 2001)
Pauli-blocked excitons: spintronics in quantum dots (Calarco et al. PRA 2003)

Feshbach two-qubit quantum gates

- identifying difficulties with quantum gates based on qubit/spin-dependent lattice movements

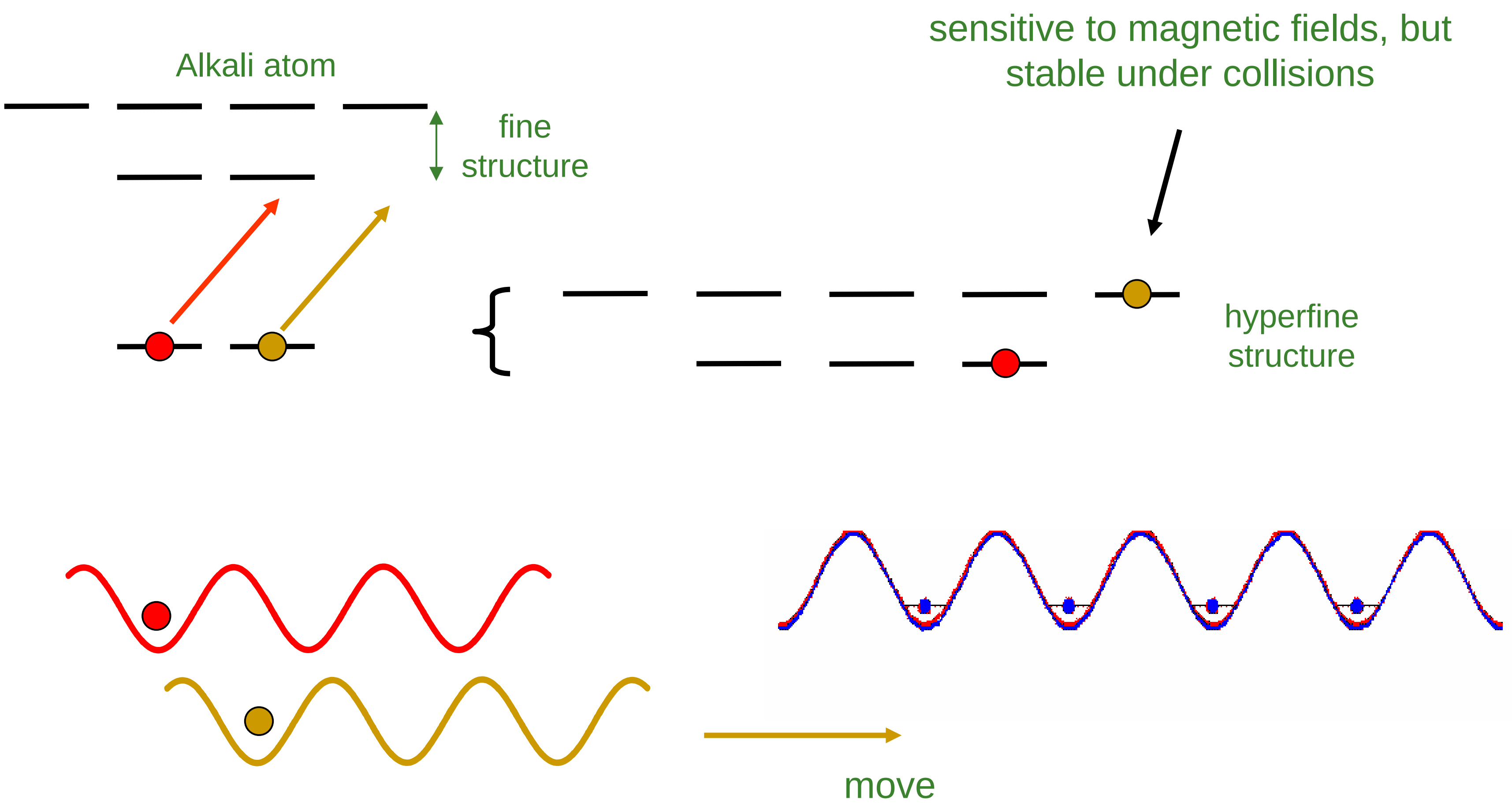
- improving on existing gate proposals

- a new Feshbach gate based on „adiabatic massage“

  - ✓ qubit as "clock transition": independent of magnetic fields
  - ✓ qubit-independent superlattice (instead of a spin-dependent lattice)
  - ✓ quantum gate based on state dependent Feshbach resonances
  - ✓ enhanced speed due to Feshbach resonance
  - ✓ relaxed addressability requirements

Present situation

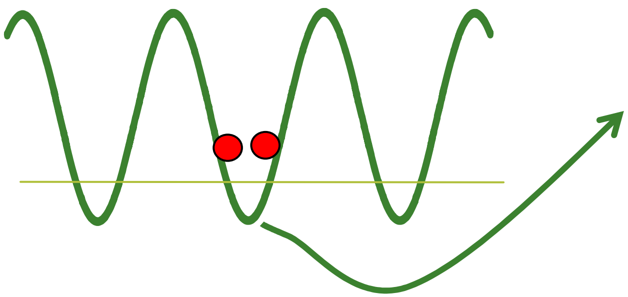
[Jaksch et al. 1999, Bloch et al. experiment 2003]

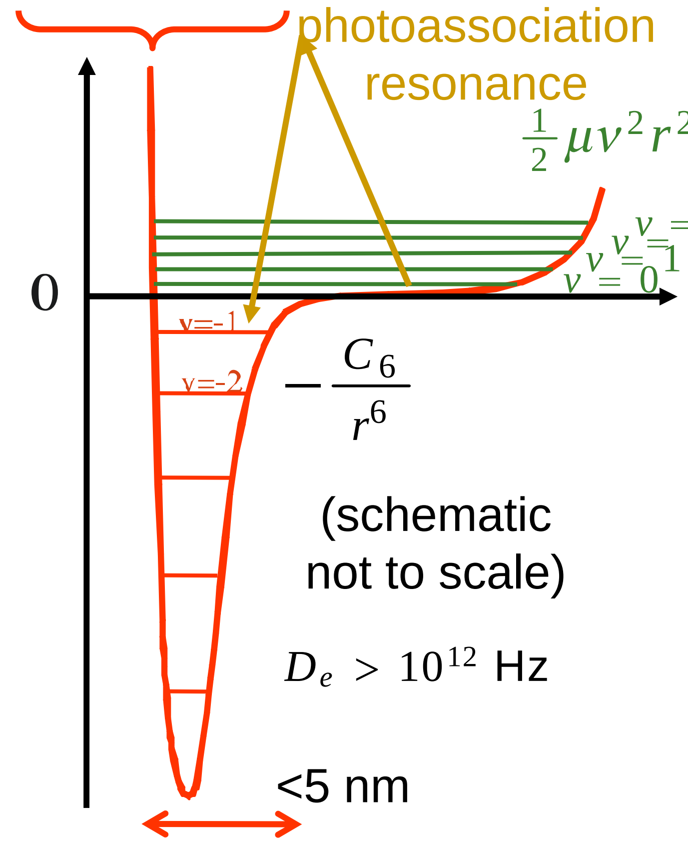


Molecular resonances (optical or magnetic)

$$\left[-\frac{\hbar^2}{2(2m)}\nabla_R^2 + \frac{1}{2}(2m)\nu^2 R^2\right]\psi_{cm}(R) = E_{cm}\psi_{cm}(R)$$

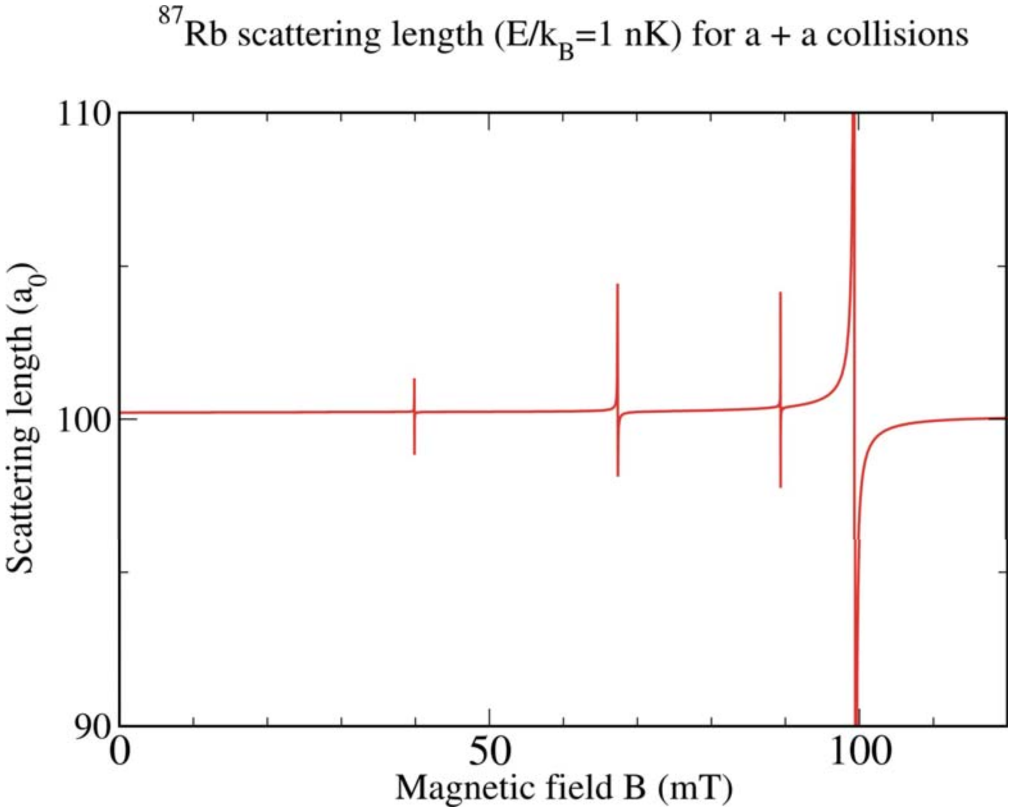
$$\left[-\frac{\hbar^2}{2\mu}\nabla_r^2 + \frac{1}{2}\mu\nu^2 r^2 + V(r)\right]\psi(r) = E\psi(r)$$





Born Oppenheimer potentials including trap

Feshbach resonances



Scattering length close to a resonance

$$A_\beta(B) = A_{bg}\left(1 - \frac{\Delta_\beta}{B - B_\beta^{\mathrm{res}}}\right)$$

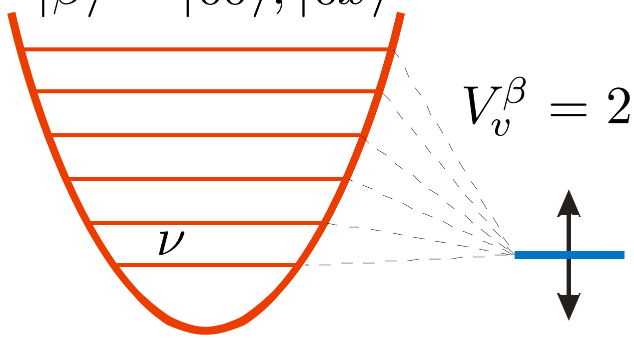
Coupled-channel CI model

[Mies et al. 2000]

State

$$\Psi_\beta(t) = |\beta\rangle \sum_v \phi_v(R) c_v(t) + |n_\beta(t)\rangle \phi^{\mathrm{res}}(R) a_\beta(t)$$

$$|\beta\rangle = |00\rangle, |0x\rangle$$



$$V_v^\beta = 2\hbar\nu \sqrt{\sqrt{4v+3}\, a_{bg} \delta_\beta / \pi}$$

$$\delta_\beta \equiv \Delta_\beta s_\beta^{\mathrm{res}} / (\hbar\nu)$$

$$a_{bg} \equiv A_{bg} \sqrt{m\nu/\hbar}$$

$$|n_\beta(t)\rangle$$

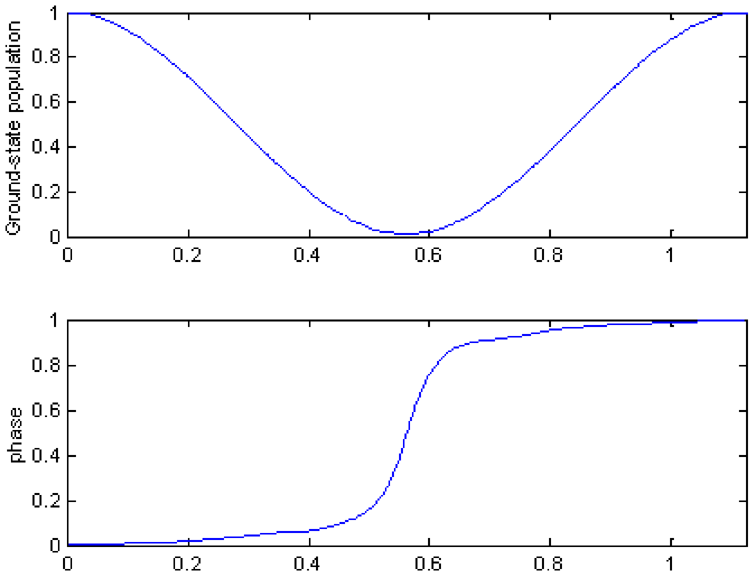
$$\varepsilon_\beta^{\mathrm{res}}(B) = s_\beta^{\mathrm{res}}(B - B_\beta^{\mathrm{res}})$$

Equations of motion

$$\begin{aligned} i\hbar \dot{a}_\beta &= \varepsilon_\beta^{\mathrm{res}}(B) + \sum_v V_v^\beta c_v / \sqrt{2} \\ i\hbar \dot{c}_v &= (v + 1/2)\hbar\nu c_v + V_v^\beta a_\beta / \sqrt{2} \end{aligned}$$

Feshbach switching in a spherical trap

- Start with the (100 G) Feshbach resonance state 10 trap units above threshold
- Switch it suddenly close to threshold
- Wait ~ 1 trap time (~ μs assuming MHz trap)
- Switch back
- A phase π is accumulated
- Fidelity: 0.9996
- No state dependence required, however difficult atom separation with state-independent potential



Quantum optimal control in a nutshell

- Evolve an initial guess according to control *u*

$$\left|\dot{\psi}(t)\right\rangle = -\frac{i}{\hbar} H(u,t)\left|\psi(t)\right\rangle$$

- Project onto the goal state and evolve back

$$\left|\chi(T)\right\rangle \equiv \left|\psi_0\right\rangle \left\langle\psi_0|\psi(T)\right\rangle$$

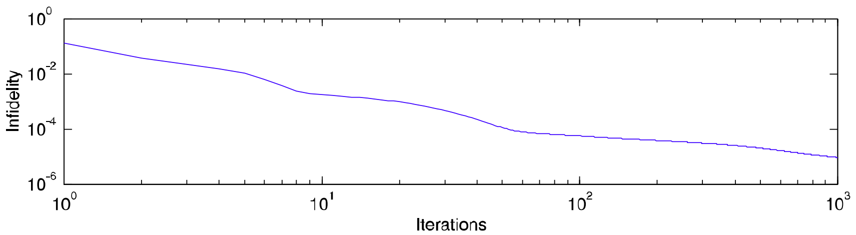
- Evolve forward again while updating the control pulse

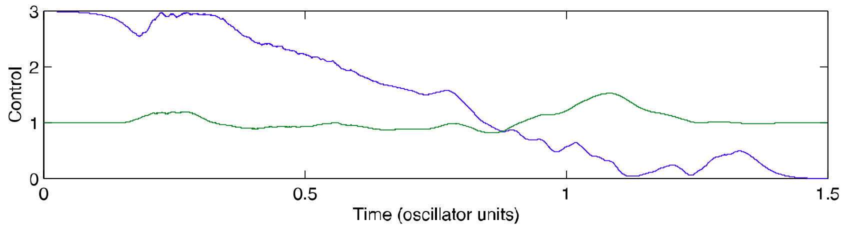
$$u_{n+1}(t) = u_n(t) + \frac{2}{\lambda(t)} \Im \left\langle\chi(t)\right| \frac{\partial H}{\partial u} \left|\psi(t)\right\rangle$$

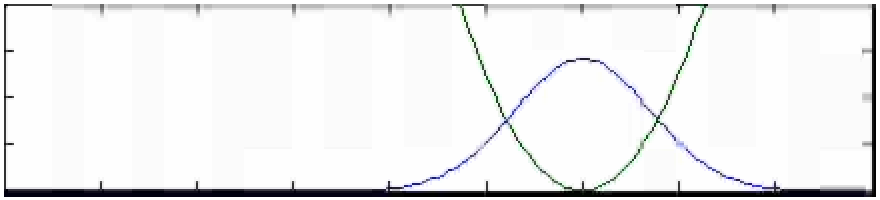
- Repeat until approaching the goal

Nonadiabatic transport in a lattice

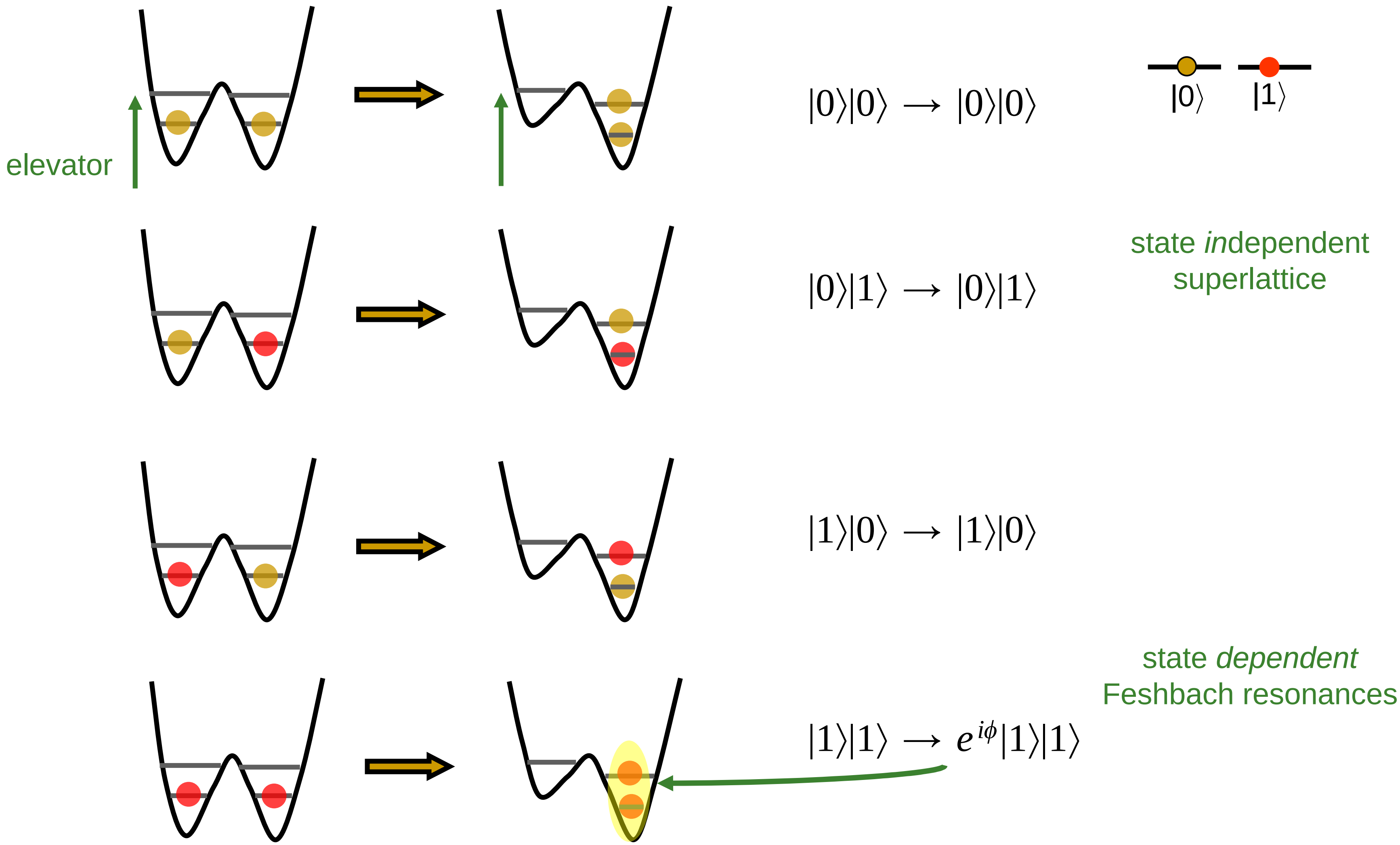
- State dependence: lattice displacement
- Non-adiabatic transport in optical lattice: simulation with realistic potential shape
- Fidelity 0.99999 in 1.5 trap times through optimal control theory







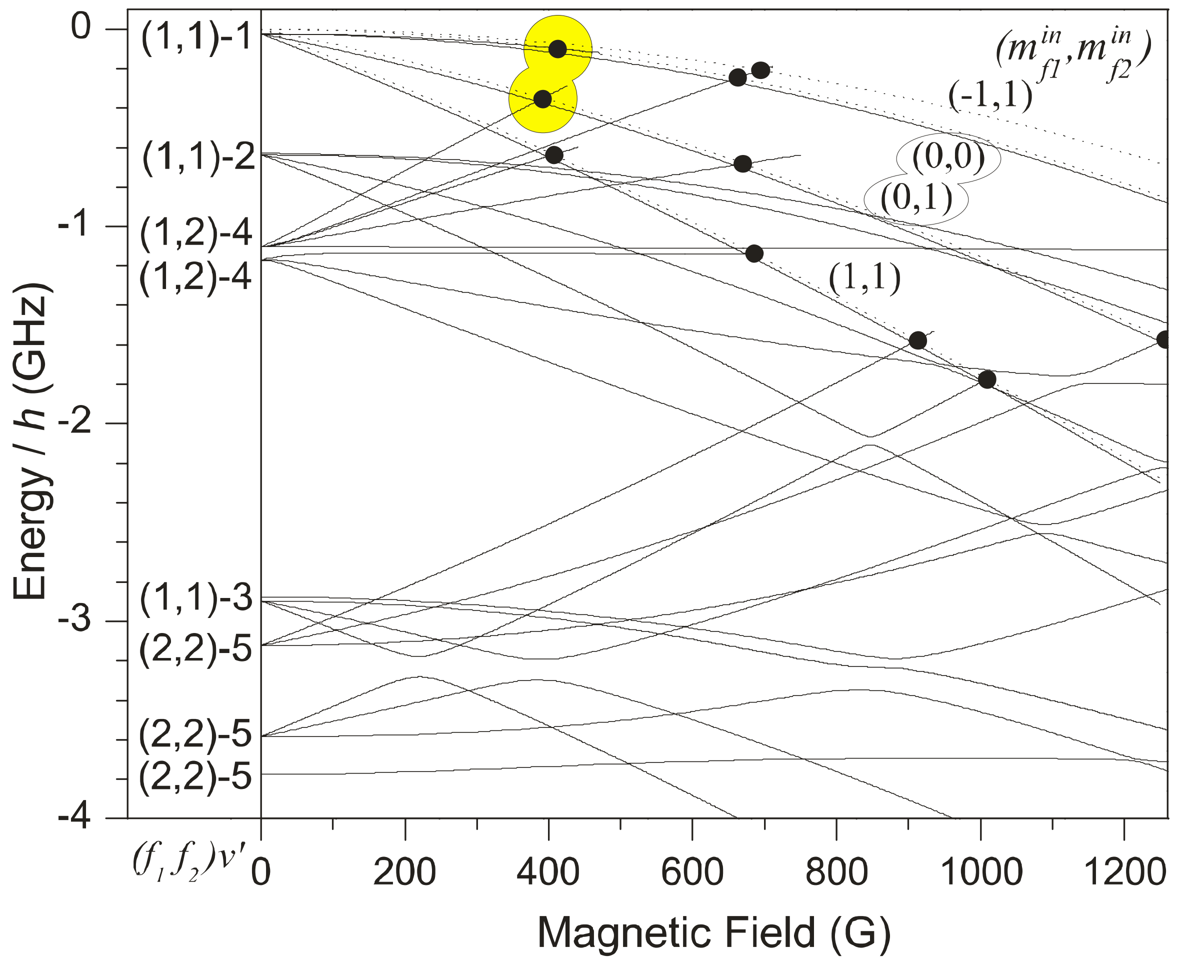
"Adiabatic-massage" phase gate



Looking for resonances

[Marte et al. 2002]

Feshbach resonances happen when some molecular state crosses the dissociation threshold for a particular entrance channel

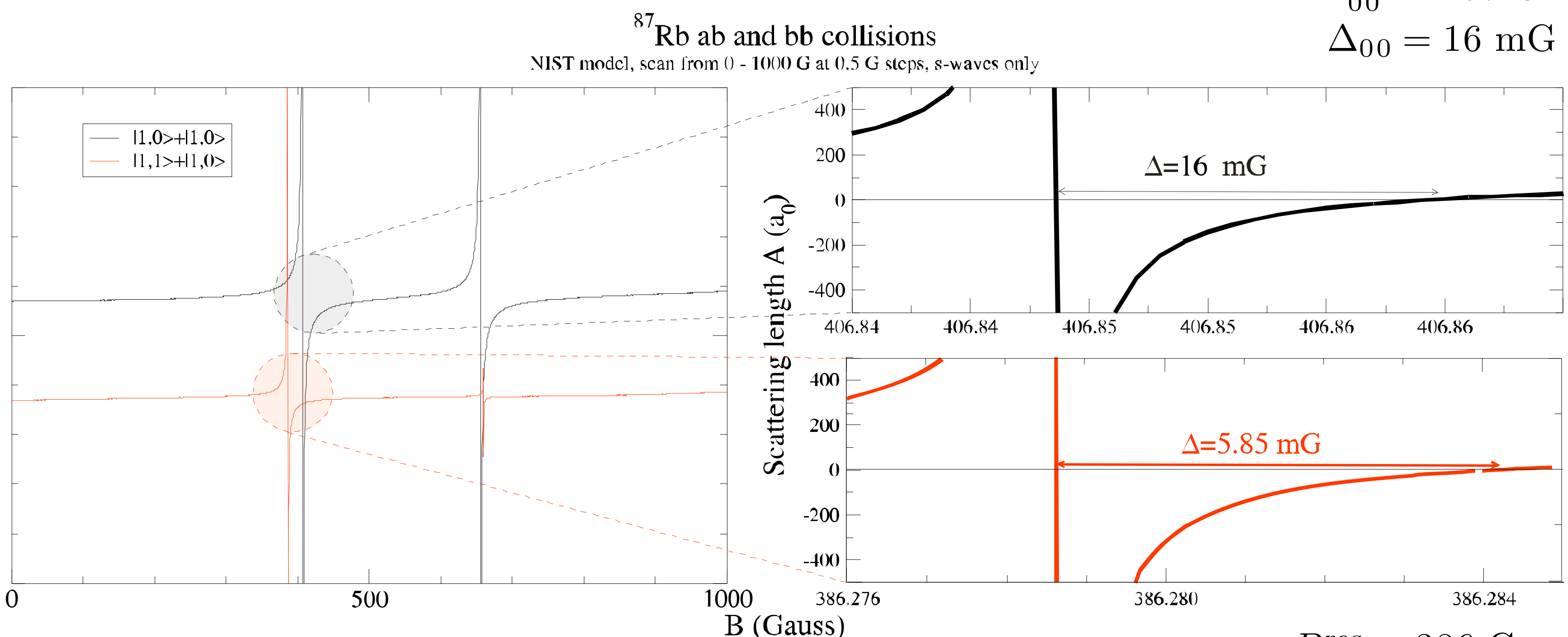


Identifying qubit states

- Logical states: ^{87}Rb clock states $|0\rangle \equiv |F=1, m_F=0\rangle \quad |1\rangle \equiv |F=2, m_F=0\rangle$
- Auxiliary state $|x\rangle \equiv |F=1, m_F=1\rangle$

$B_{00}^{\text{res}} = 407\text{ G}$

$\Delta_{00} = 16\text{ mG}$

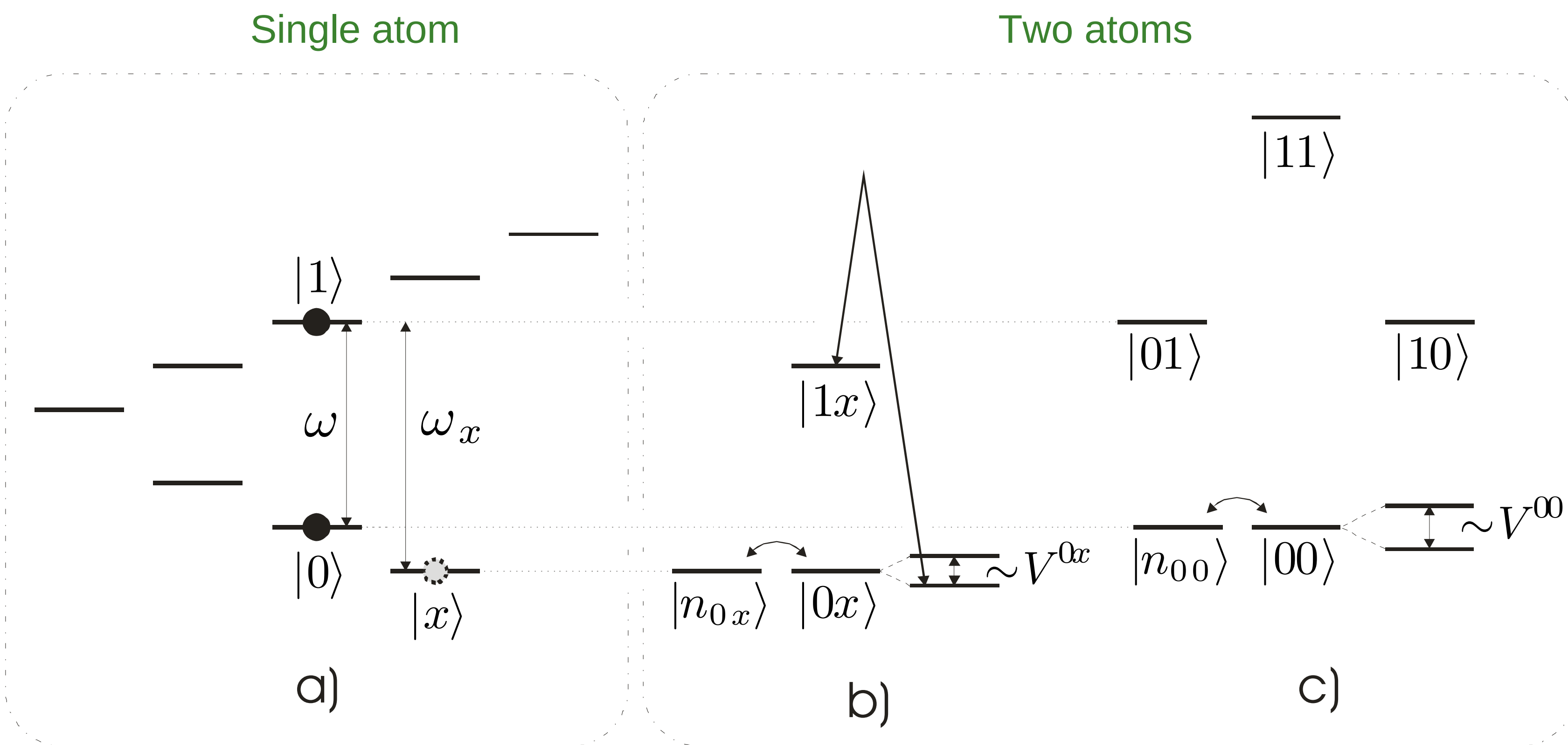


$B_{0x}^{\text{res}} = 386\text{ G}$

$\Delta_{0x} = 5.85\text{ mG}$

$s^{\text{res}}/h = 3.0\text{ MHz/G}$

Level scheme



Single-qubit rotation via auxiliary state

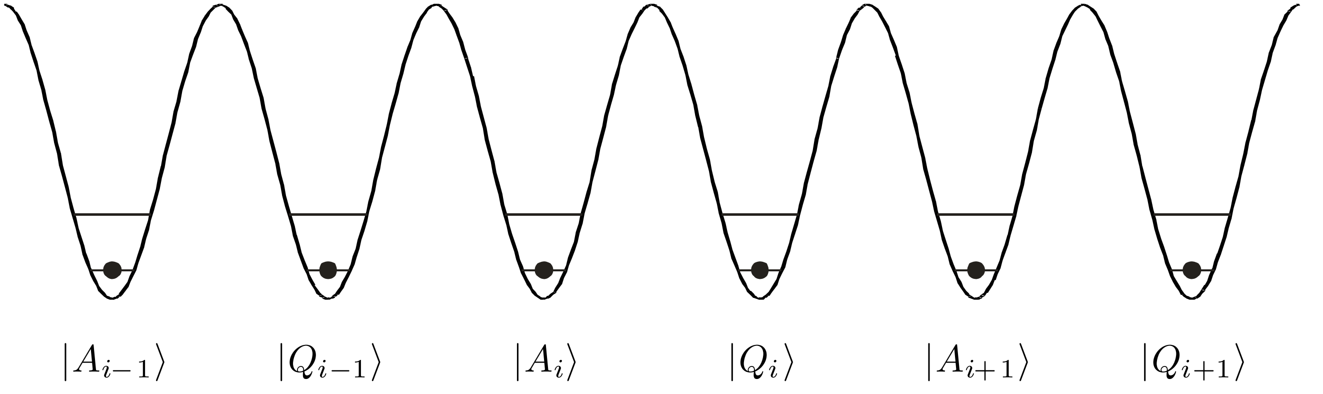
Two-qubit gate via Feshbach interaction

Massaging the potential

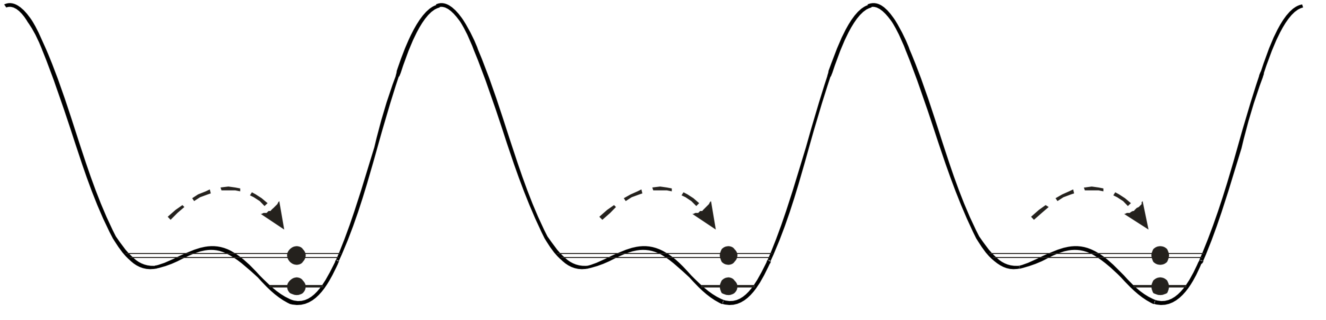
- Example: Two-color optical lattice

$$U(\mathbf{x},\phi) \;=\; U_\perp(y,z) + U_0\left[\left(1+\frac{u_1+u_2}{2}\right)\cos^2(2kx) + \sqrt{u_1^2+u_2^2}\cos^2\left(kx+\frac{\sigma}{2}\arctan\frac{u_2}{u_1}+\frac{p\pi}{2}\right)\right]$$

- Auxiliary and Qubit atomic sites

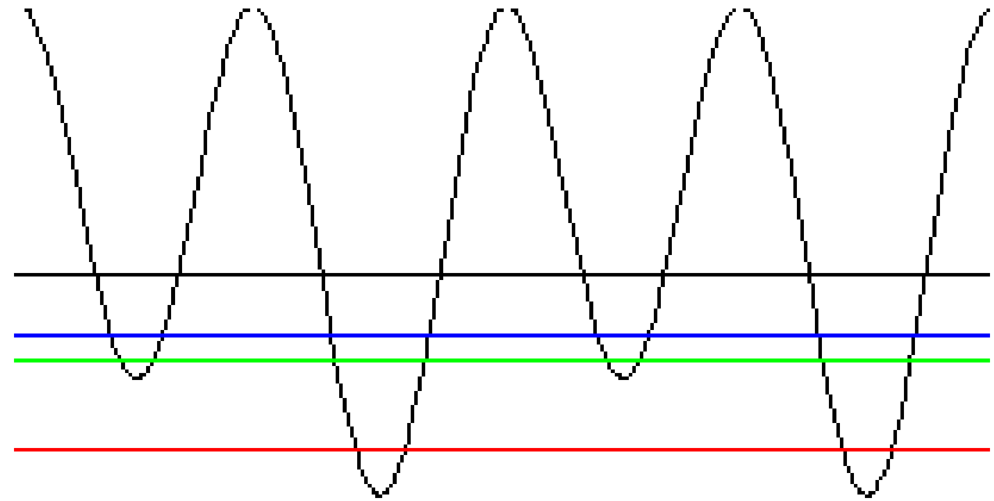


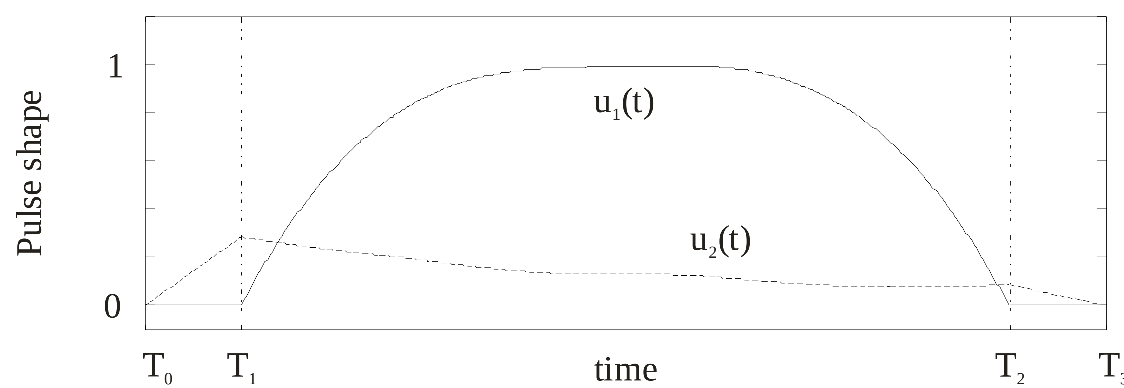
- Adiabatic transfer by barrier lowering



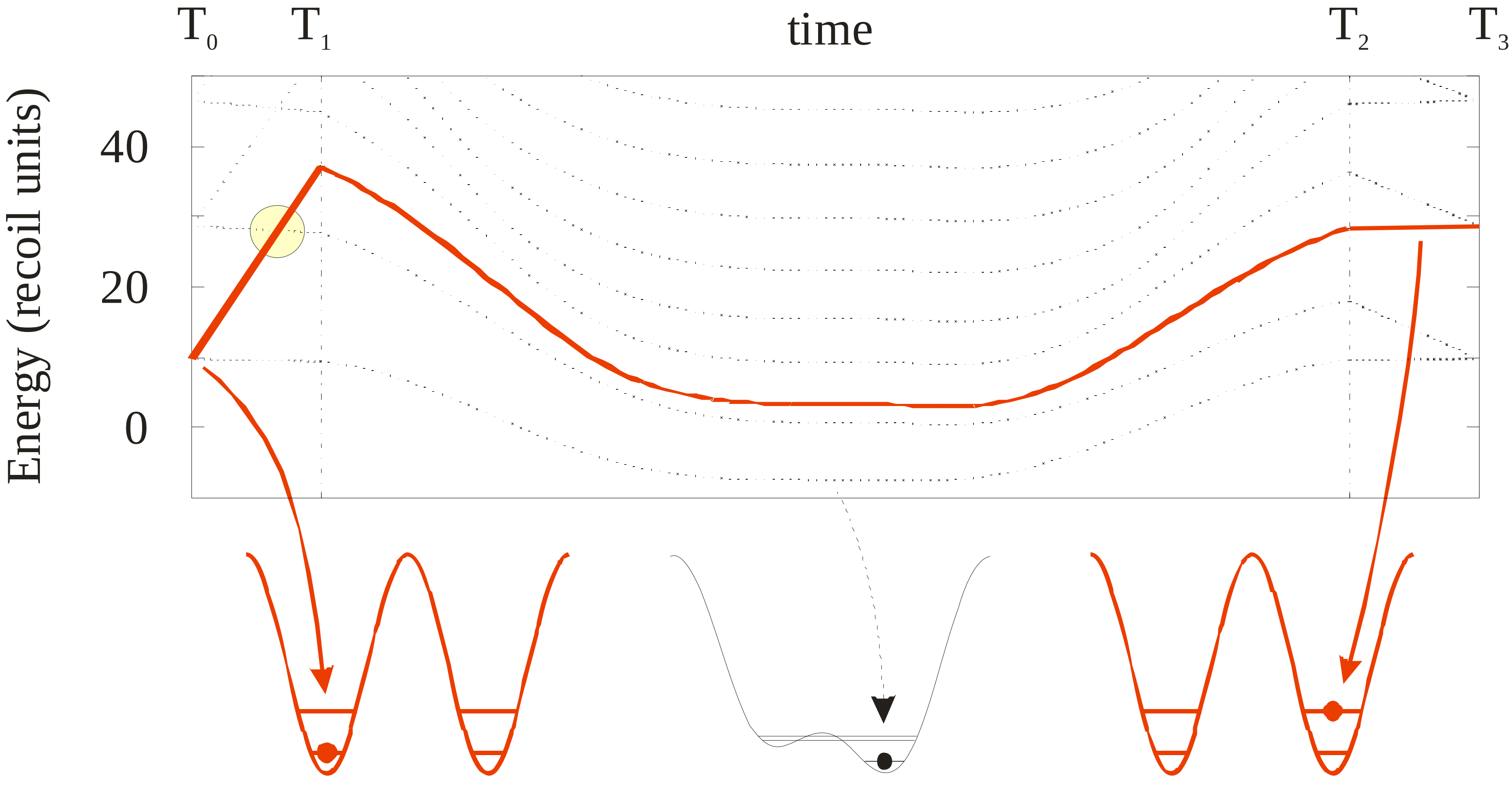
Transport in a two-color optical lattice

- Raise abruptly one every two wells
- Slowly lower the barrier and raise it again
- Transport infidelity ~ 10^{-3} in 10 trap times with a pulse optimized “by hands”





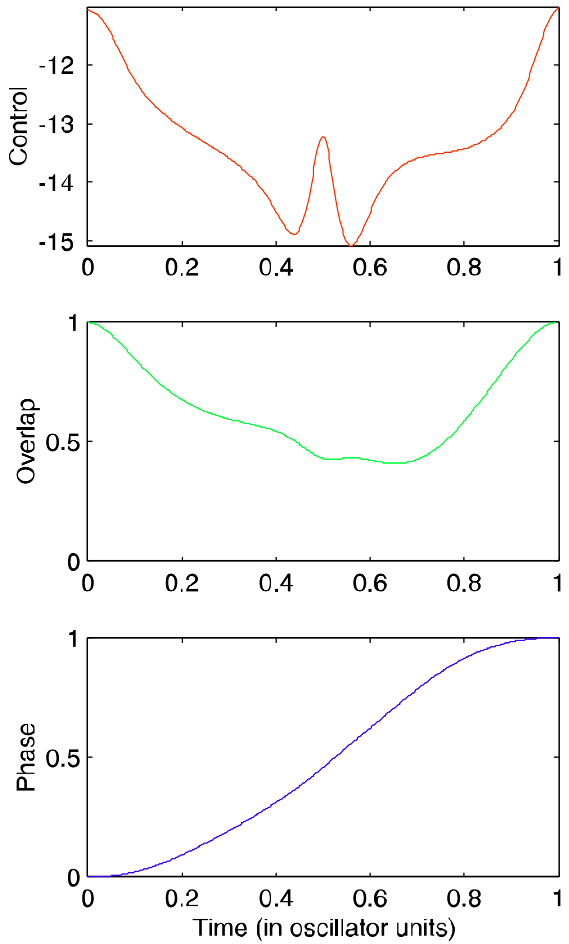
Connection diagram



The left ground-state atom gets transferred to the right first excited state, while the right atom remains in its ground state

Phase gate via optimal control

- Ramping the resonance state across threshold
- The state |00> is selectively affected
- A phase π is accumulated over one oscillation period
- Infidelity ~10^{-5} in a 20kHz trap with anisotropy factor 10



Summary

- Gate "markers"
  - Single-qubit: $|x\rangle_A$
  - Two-qubit: $|0\rangle_A, |1\rangle_A$
- Preparation of task-specific patterns
  - Relaxing single-qubit addressability requirements
- Implementation
  - Optical lattices
  - Atom chips
    - Magnetic
    - Optical (microlenses; "mirror lattice" on a chip?)
- Quantum optimal control is useful for
  - fast atom transport
  - efficient entangling operations