Theoretical Methods (*)
and
Experimental Implementation (**)
of
Heat-Bath Algorithmic Cooling

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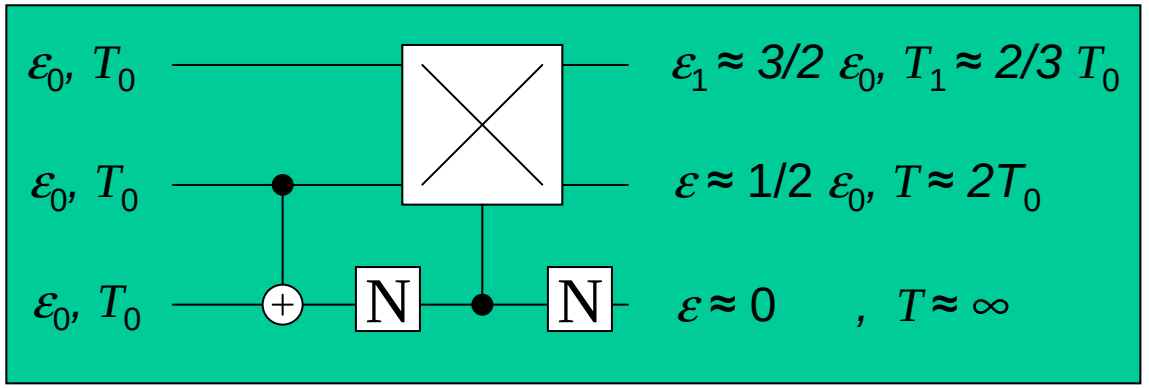
Abstract

Algorithmic Cooling is a generic term for techniques leading to the purification of qubit registers whose initial state is not pure. This term encompasses polarisation transfer techniques of NMR (pre-dating both QC and NMR-based QC) and also in-place quantum compression schemes such as those of Schulman and Vazirani. The entropy conservation law imposes, however, severe restrictions on the efficiency of such techniques, often referred to as the Shannon Bound. In addition, the necessity that all transformations applied be unitary (i.e. inner-product preserving), imposes an even tighter bound, referred to as the Sørensen Bound.

A new kind of algorithmic cooling which bypasses these bounds by pumping out excess entropy into the environment (i.e. heat bath) has been proposed in 1999 by Boykin, Mor, Roychowdhury, Vatan, and Vrijen and has been referred to as "non-adiabatic" or "heat-bath" algorithmic cooling. This poster presents an improved non-adiabatic cooling algorithm, more efficient in terms of the number of required qubits, which achieves an exponential increase in polarisation with a linear number of qubits. Furthermore, we report on the first proof-of-concept non-adiabatic cooling experiment ever, performed in April 2002 at the Université de Montréal on a 3-spin molecule.

Adiabatic Algorithmic Cooling

- Basic idea
  - Transfer of entropy/polarisation within register
  - Use only reversible/unitary/adiabatic transformation
  - "Cools" certain qubits while heating others
- How?
  - Basic building block (3-qubit): a Basic Compression Subroutine (BCS),



  - Also, there exist scalable algorithms performing in-place quantum compression on a *n* qubit register (e.g. Schulman-Vazirani)

➔ Ultimate limit: ***Entropy must be conserved***

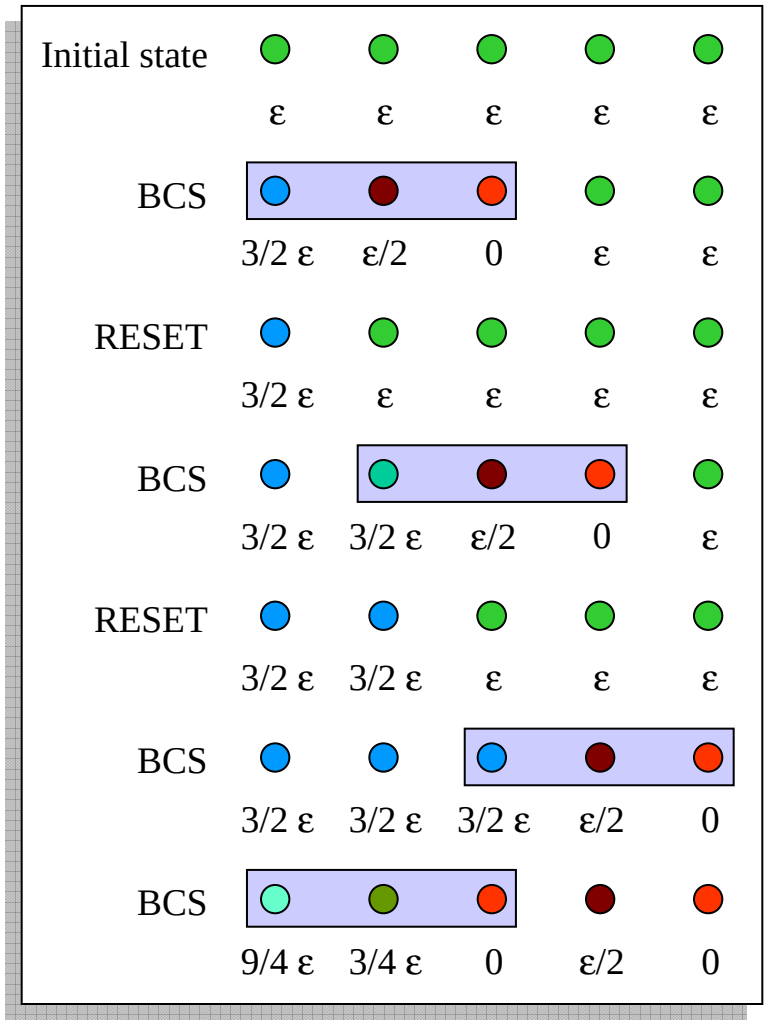
Non-Adiabatic Algorithmic Cooling

- Basic idea (Boykin, Mor, Roychowdhury, Vatan, Vrijen)
  - Pump out entropy of hot qubits by interaction with the environmental heat bath:
    - Use natural relaxation mechanisms to do this, i.e. "wait"
    - ➔ Hot qubits will naturally return to "room" spin temperature (i.e. natural bias ε)
    - This step refered to as "thermalisation" or "reset"
- How? (Original algorithm)
  - Perform compression first by using repeated XOR
  - Push qubits expected to be cold to the left of the register
  - Re-thermalise bits "expected" to be hot
  - Continue cooling by alternating compression and thermalisation
- Requirements

  ➔ ***Hot qubits must be able to re-thermalise (cool) much faster than cold qubits re-thermalise (become hot)***

An Efficient Non-adiabatic Algorithm

```
procedure NAC (i: posn, j: int)
    if j = 0
        RESET(i);
        return;
        end if
    do NAC (i, j - 1);
    do NAC (i + 1, j - 1);
    do NAC (i + 2, j - 1);
    do BCS(i,i + 1,i + 2)
```



Algorithm Performance

- Algorithm has recursive structure
  - After j iterations we have attained bias level:

$$\varepsilon^{(j+1)} \cong \left(\frac{3}{2}\right)^{j} \varepsilon$$

(for small ε)

  - In terms of spin temperature:

$$T^{(j+1)} \cong \left(\frac{2}{3}\right)^{j} T_0$$

(for "high" temperatures)

- To cool m qubits to the j-th level we have:
  - Total space used:

$$s(m, j) = m + 2j$$

  - Number of resets (entropy pumped out) :

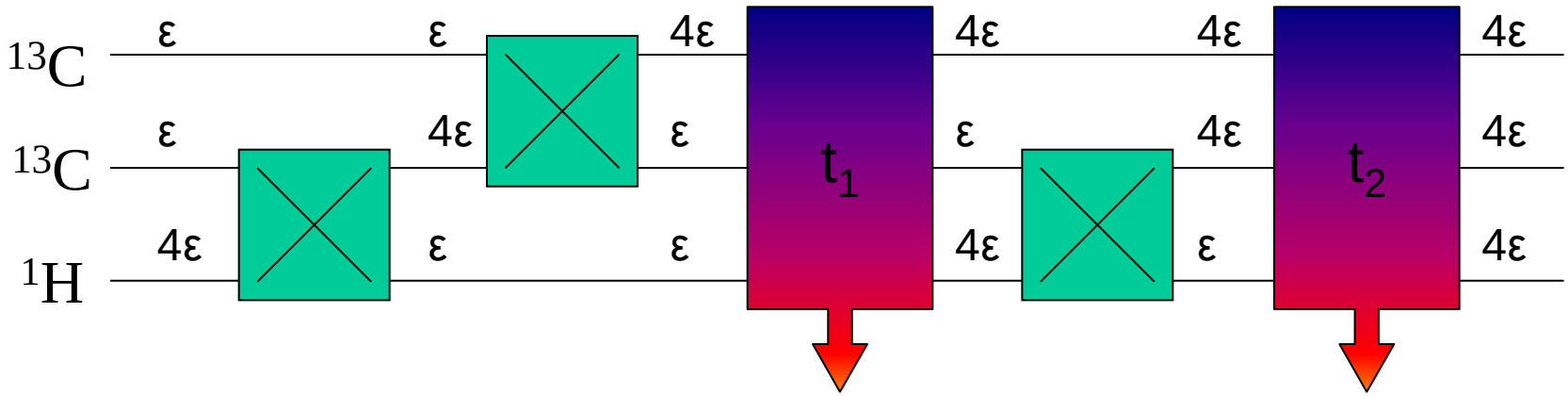
$$c(m, j) \cong m\frac{3^j}{2} - 1$$

  - Total time elapsed:

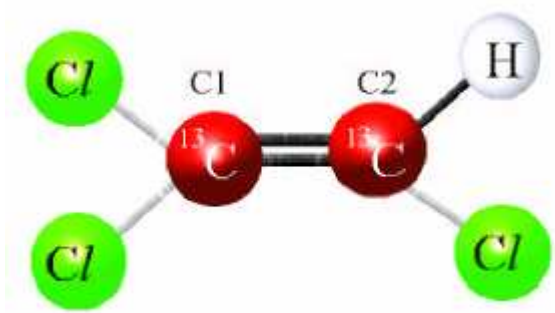
$$t(m, j) \cong m3^{j-1}$$

Experimental Non-adiabatic AC

Idealised algorithm

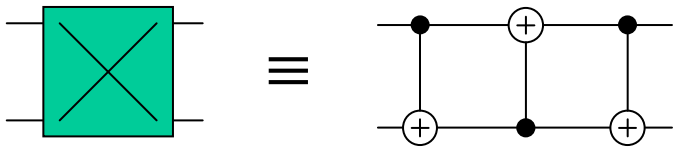


Trichloroethylene (TCE)



3-qubit register

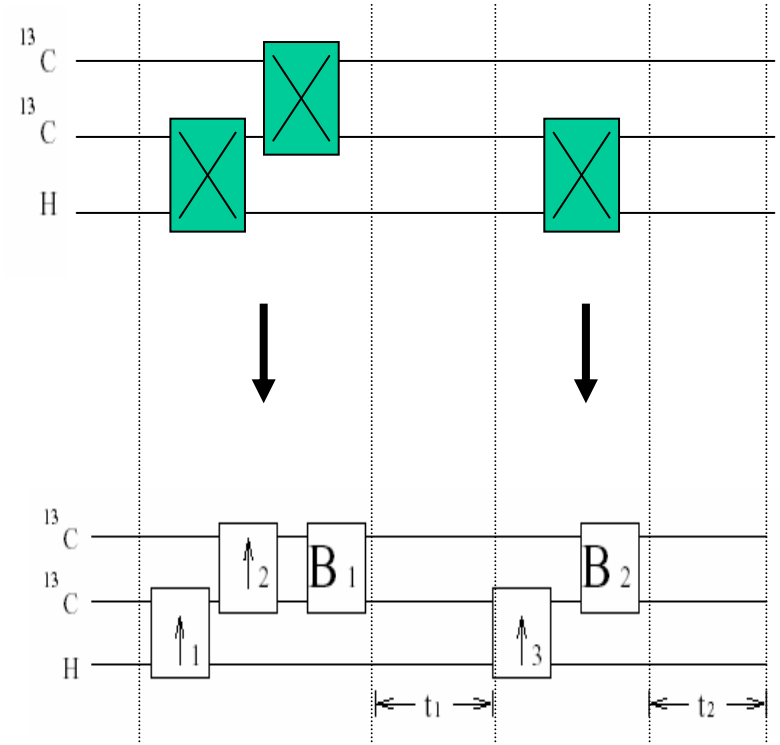
SWAP



Basic Building Block

Actual Experiment

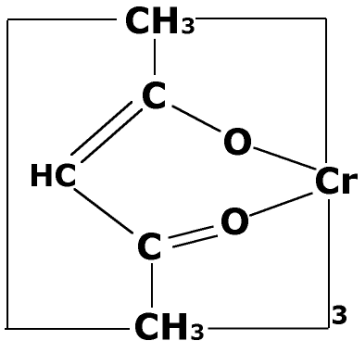
- "Dirty"-SWAP
  - implemented with INEPT-based sequences
  - Shorter sequence than perfect SWAP (less error)
  - Suitable for these particular initial states
- B-Boxes
  - Final state must be aligned with Z, in order for polarisation to "survive" decoherence (during waiting times t_1 and t_2).



Heat-engine "fine tuning"

- Key factor is T1-ratio (relaxation times)
  η = T1(C1)/T1(H)
- Natural ratio does not allow experiment to "break even"
- T1's manipulated by lacing sample with a paramagnetic impurity:
  - Impurity facilitates relaxation and decreases T1
  - H more affected by impurity, ratio increases

Chromium Acetyacetonate

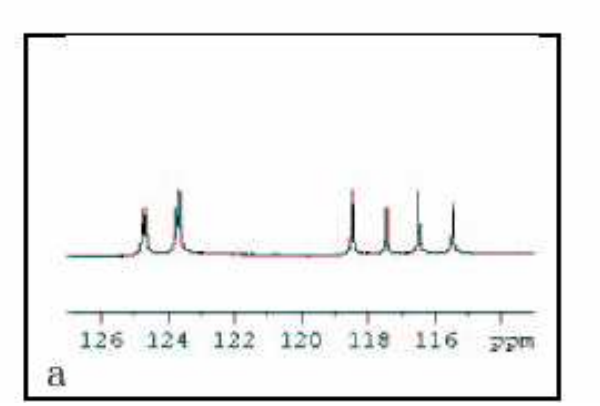


T1	Unlaced	Laced
C1	30.9 s	28.3 s
C2	27.5 s	16.0 s
H	5.46 s	1.88 s
η	**5.65**	**15.05**

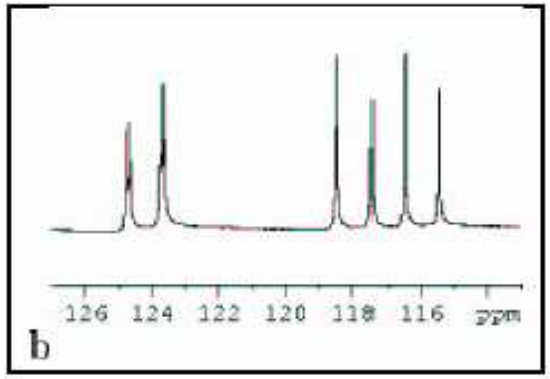
Experimental Results

^{13}C spectra

Before



After



- Peak area ✪ bias ε
  - *Final biases* ≈ 2.6 ε, 2.5 ε, 3.8 ε *(bias for* ^{1}H *not directly observed)*
  - *Entropy difference*:

$$H_f - H_0 \approx [(2.6^2+2.5^2+3.8^2) - (1^2+1^2+4^2)]\ \varepsilon^2/\ln 4$$

$$= 6.82\ \varepsilon^2 \text{ bits}$$

Conclusions

THEORY

- ☺ Non-adiabatic AC allows cooling beyond limits set by Shannon and Sørensen bounds
- The proposed algorithm allows:
  - ☺ ☺ Exponential increase in bias, which only requires linear number of qubits
  - ☺ Allows full polarisation with "few" qubits (~60 vs. 100 billion qubits)
- 😐 Requires a very good T1-ratio (η)
- ☹ Is thermodynamically unefficient

EXPERIMENT

- ☺ First (known) experiment to break Shannon bound on spin temperatures
- 😐 Results obtained marginal:
  - Low decoherence time (T2,T2*) makes lossy transfer sequences
  - efforts continue to improve efficiency (Technion & U de Montreal)
- Further research:
  - Generalise to other molecules (other than QC applications)
  - Find other methods of increasing η

References

1. Fernandez. J.M., Lloyd, S., Mor, T., Roychowdhury, V. "*Algorithmic Cooling of Spins: A Practicable Method for Increasing Polarization*", manuscript in preparation.
2. Brassard, G., Fernandez, J.M., Laflamme, R., Mor, T., Weinstein, Y., "*Improved Method for POlarization Transfer Exploiting Nuclear Thermalization (IMPOTENT)*", manuscript in preparation.
3. Fernandez, J.M., Mor T., Weinstein, Y., "*Paramagnetic Materials and Practical Algorithmic Cooling for NMR Quantum Computing*", 2003 SPIE Aerosense conference
4. Boykin, P.O, Mor T., Roychowdhury, V., Vatan, F., Vrijen, R. "*Algorithmic Cooling and Scalable NMR Quantum Computers*," PNAS, 2002, vol. 99, no. 6, pp. 3388-3393.
5. Schulman, L., Vazirani, U., "*Scalable NMR Quantum Computation*," 1999, STOC 99
6. Sørensen, O.W. "*Polarization transfer experiments in high-resolution NMR spectrosctopy*", Prog. NMR. Spec., 1989, v21, 504-569