Resource-Efficient Linear Optics Quantum Computation

via the Cluster State Approach

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Introduction

- *Photons* make excellent carriers of quantum information and *single qubit operations* can be achieved with linear optical elements – (polarising) beam splitters and phase shifters.
- *Two-qubit gates* cannot be achieved deterministically by linear optics alone.
- Need strong non-linear materials or implement *non-deterministic* gates via photo-detection.

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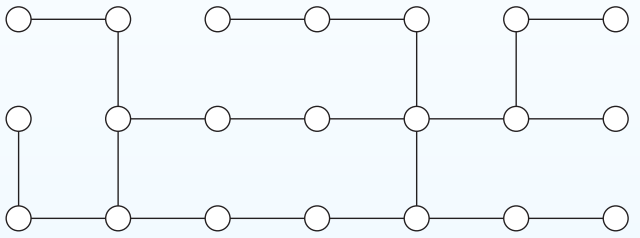
- Need strong non-linear materials or implement *non-deterministic* gates via photo-detection.
- Schemes[a] for *scalable* (near)-deterministic gates are complicated and need a large amount of resources (entangled modes, feed-forward detection) to implement even the simplest gate.
- Here we describe a new scheme which employs the measurement-based *cluster state quantum computation* approach and achieves significant gains in resource efficiency.

[a] E.g. Knill-Laflamme-Milburn (KLM), Nature (2001).

Cluster States

A cluster state[a] is an entangled multi-qubit state which may be represented by a graph.



- **Vertices** represent **qubits** prepared in state[b] $|+\rangle = |0\rangle + |1\rangle$.
- **Edges** represent the application of the entangling quantum **CPHASE** gate

$$|0\rangle^a\langle 0|\mathbb{1}^b + |1\rangle^a\langle 1|\sigma_z^b$$

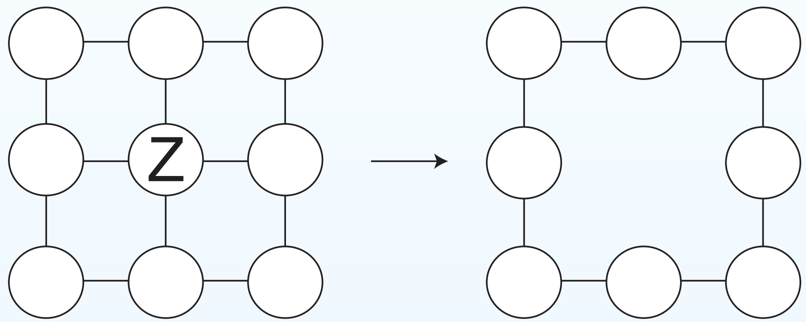
  between the connected qubits.
- We will refer to the graph edges as **bonds**.
- Known extra Pauli's on any cluster qubit can be accounted for.

[a] Briegel and Raussendorf, PRL, **86**, 910 (2001)

[b] Normalisation factors will be omitted.

Some Properties of Cluster States

- We review here a few properties of cluster states, which we shall use later on.
- A computational basis (σ_z) measurement on a cluster qubit removes the qubit from the cluster breaking all bonds.



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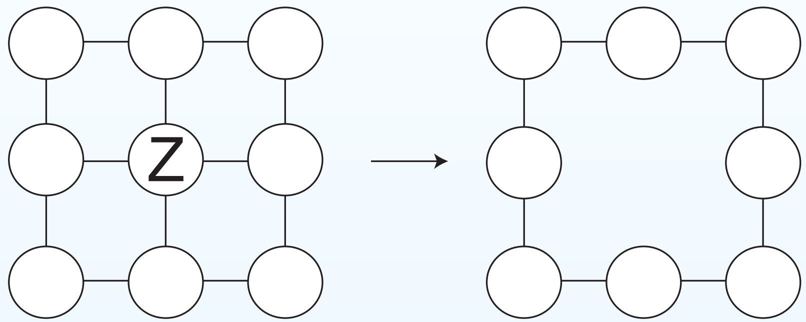
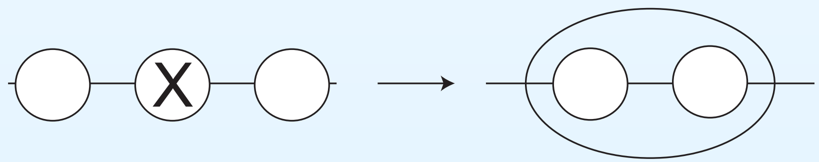


- **Redundant encoding** is to encode a logical qubit in several qubits. The 2-qubit redundant encoding is $|0\rangle_{\text{log.}} \equiv |0\rangle|0\rangle$, $|1\rangle_{\text{log.}} \equiv |1\rangle|1\rangle$.
- A σ_x measurement $\{(|0\rangle + |1\rangle), (|0\rangle - |1\rangle)\}$ on a qubit in a *linear* cluster combines the neighbouring qubits into a *single logical qubit* (redundantly encoded).

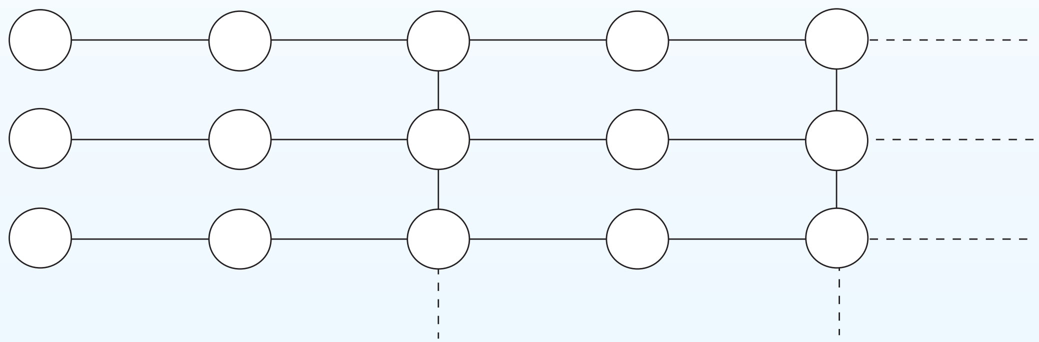


Single logical cluster qubit encoded redundantly in two physical qubits

Cluster State Quantum Computation

On a cluster state with sufficient size and bond layout, an arbitrary quantum network can be simulated by *adaptive single-qubit measurements* alone[a]. The following cluster state layout[b]

(2 x 1) rectangular grid pattern:



[a] Raussendorf and Briegel, PRL 86, 5188; Raussendorf, Browne and Briegel, PRA 68, 022312

[b] Nielsen, accepted PRL (2004). See also Childs, Leung and Nielsen, quant-ph/0404132.

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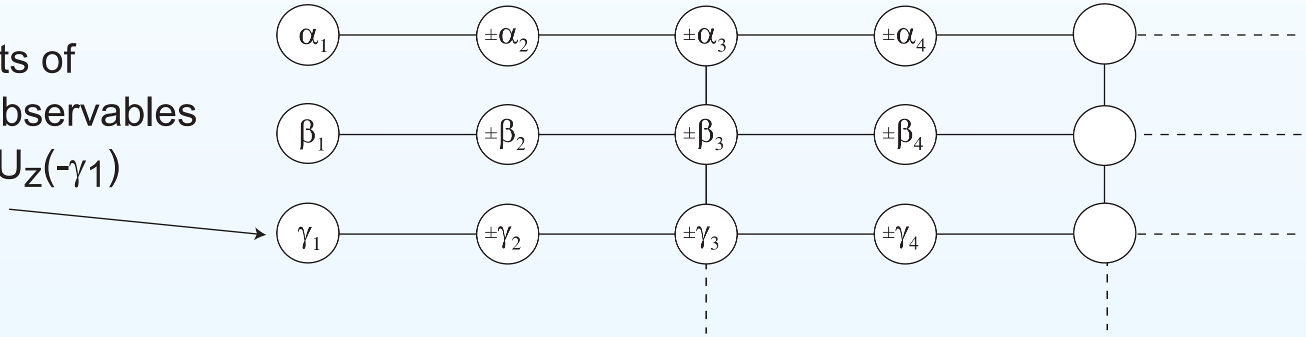
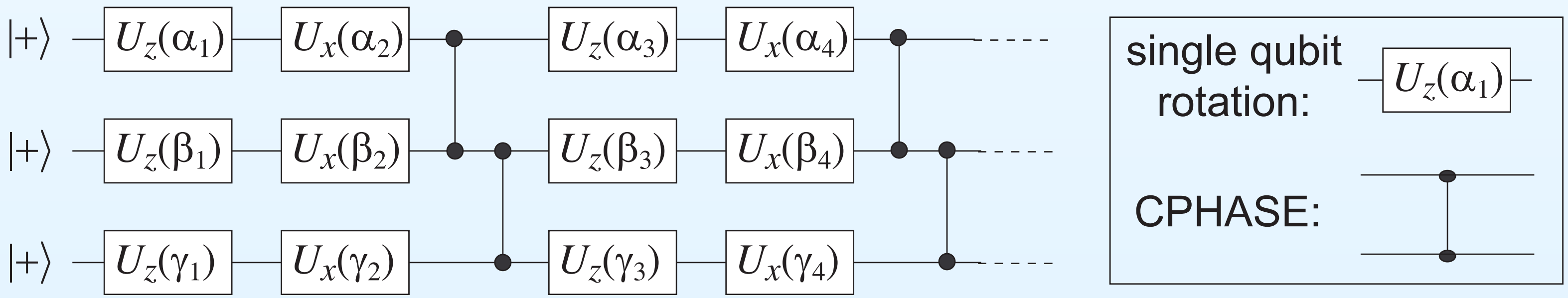
Measurements of
single qubit observables
e.g. $U_z(\gamma_1)\sigma_x U_z(-\gamma_1)$



with the above measurements, simulates the quantum network:



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Outline of our scheme

- Qubits will be single photon polarisation $|0\rangle \equiv |H\rangle$, $|1\rangle \equiv |V\rangle$.
- Polarisation measurement in arbitrary bases is trivial. The main part of our scheme is the cluster state generation.
- Instead of using CPHASE gates between qubits, we (probabilistically) *fuse* clusters[a].

[a] The same idea underlies the “valence bond model” of Verstraete and Cirac, quant-ph/0311130

[b] Can be generated e.g. via non-linear processes or linear optics and feed-forward.

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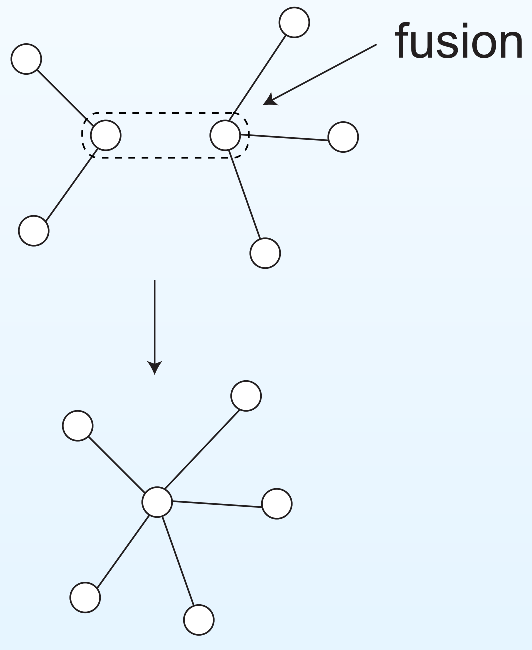
- This replaces two qubits with a single one while *retaining* all cluster state bonds on each qubit.



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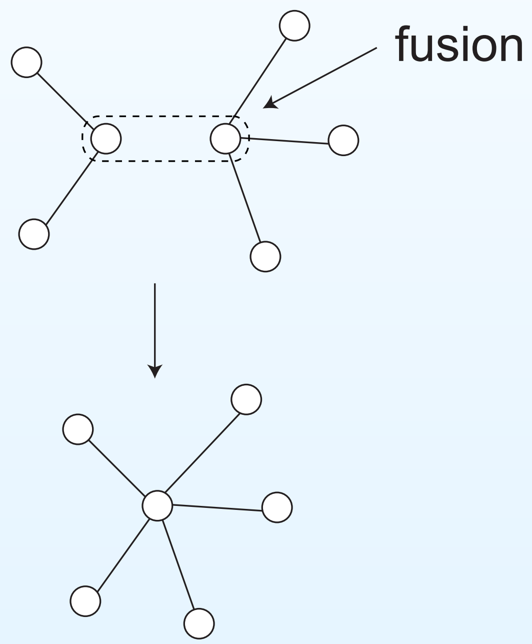
- The initial resource will be **photon Bell-pairs**[b] $|H\rangle(|H\rangle + |V\rangle) + |V\rangle(|H\rangle - |V\rangle)$. These are 2-qubit cluster states.



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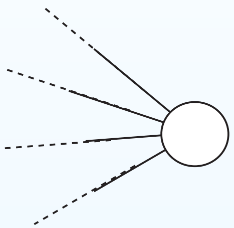
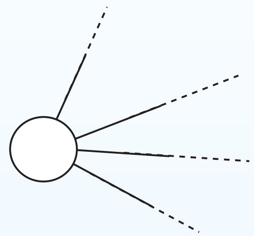
The fusion operation

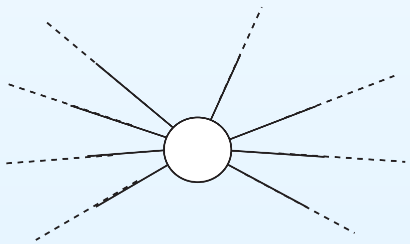
- The fusion can be seen explicitly if we write out each bond CPHASE:





$$\left(|0\rangle \prod_{i=1}^{m} \mathbb{1}^{(i)}|\psi\rangle + |1\rangle \prod_{i=1}^{m} \sigma_z^{(i)}|\psi\rangle\right) \otimes \left(|0\rangle \prod_{i=1}^{n} \mathbb{1}^{(i)}|\psi'\rangle + |1\rangle \prod_{i=1}^{n} \sigma_z^{(i)}|\psi'\rangle\right)$$

which, after the fusion operation $|0\rangle\langle 00| + |1\rangle\langle 11|$, becomes



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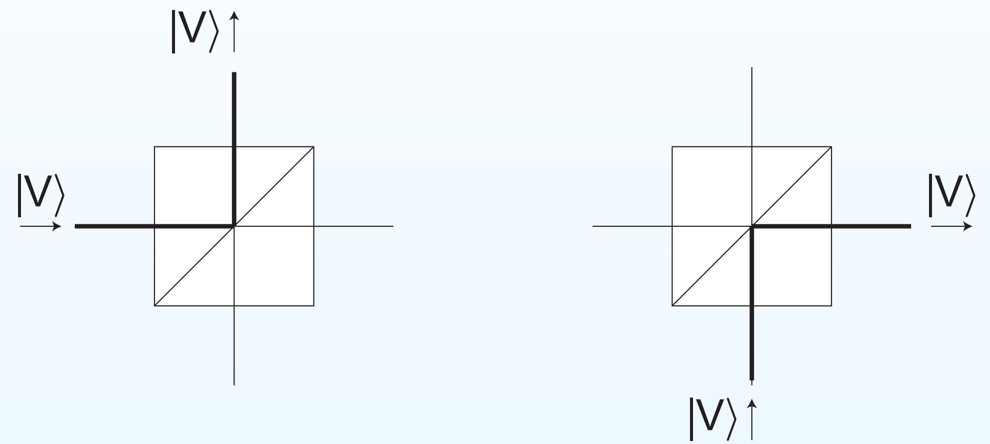
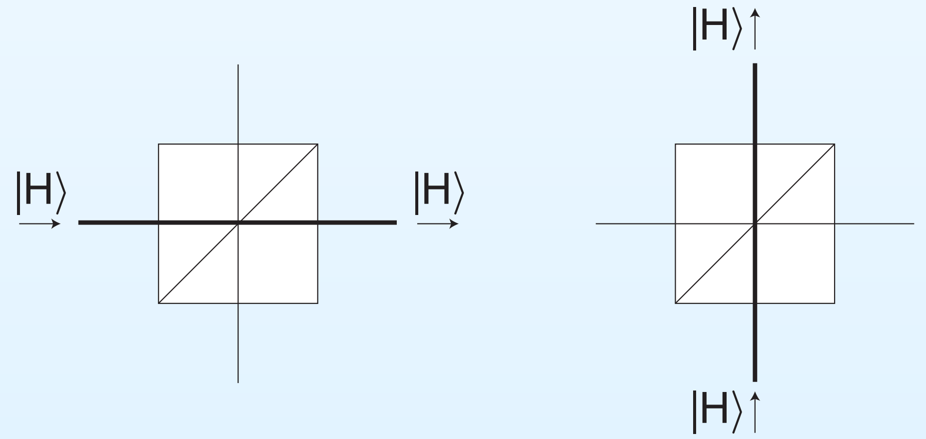
Polarising Beam Splitter - (PBS)

The key component for realising the fusion operation is the PBS.

Vertically polarised light is **reflected**

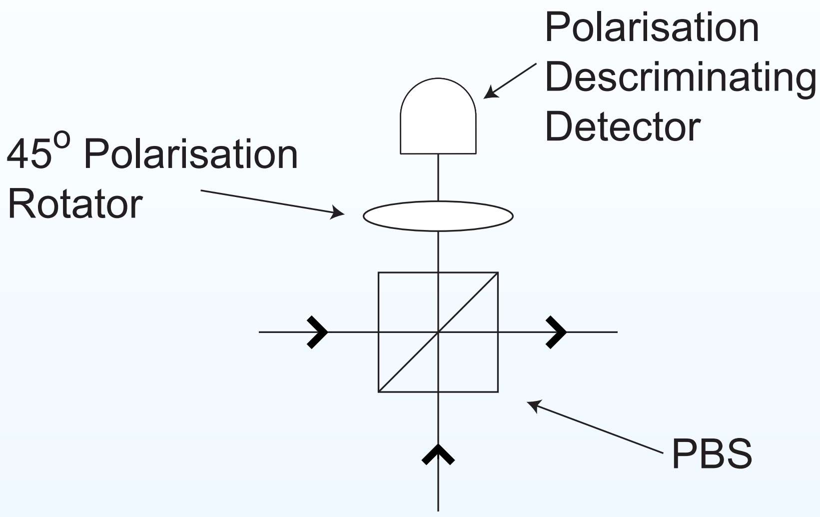


Horizontally polarised light is **transmitted**



Building Linear Clusters - Type-I Fusion

- The fusion operation can be realised *non-deterministically* using the illustrated setup:[a]
- With a photon incident in each port, there are 4 possible outcomes, each with probability 25%.



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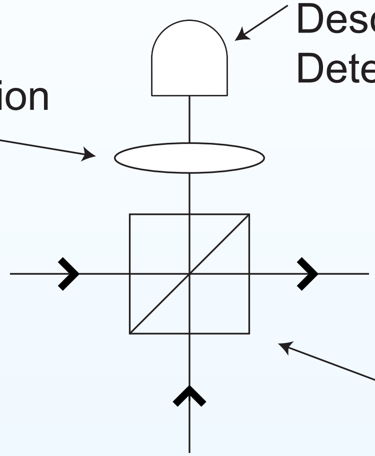

- **Two** outcomes give us the desired fusion operators $|0\rangle\langle 00| + |1\rangle\langle 11|$ or $|0\rangle\langle 00| - |1\rangle\langle 11|$, (one and only one photon, H or V). The second of these adds an extra σ_z but this is naturally accounted for. Thus, overall success probability is 50%.

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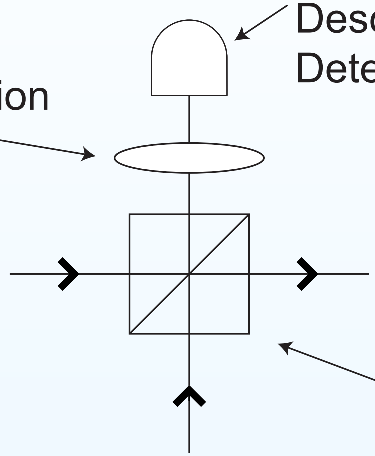

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- If 0 or 2 photons are detected, this is a *failure*, equivalent to measuring both qubits in the (σ_z) computational basis. The qubits are thus both cut from their respective clusters.

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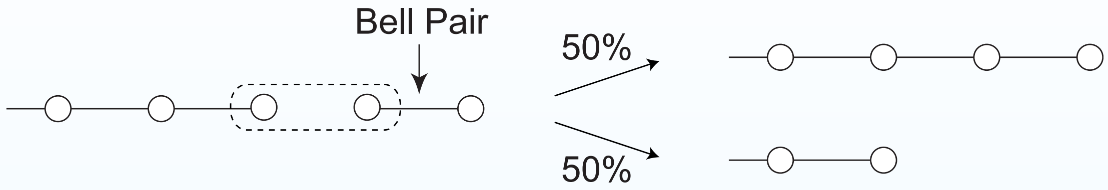
Linear Clusters

- Adding Bell Pairs to a linear cluster:



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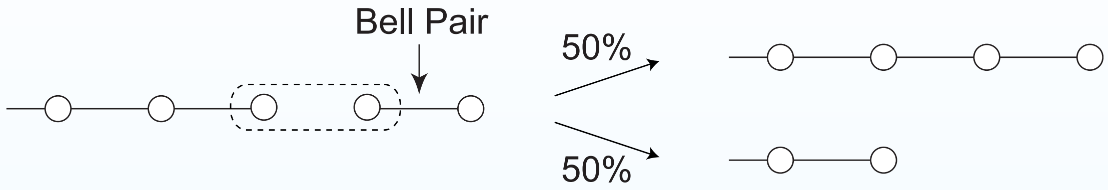

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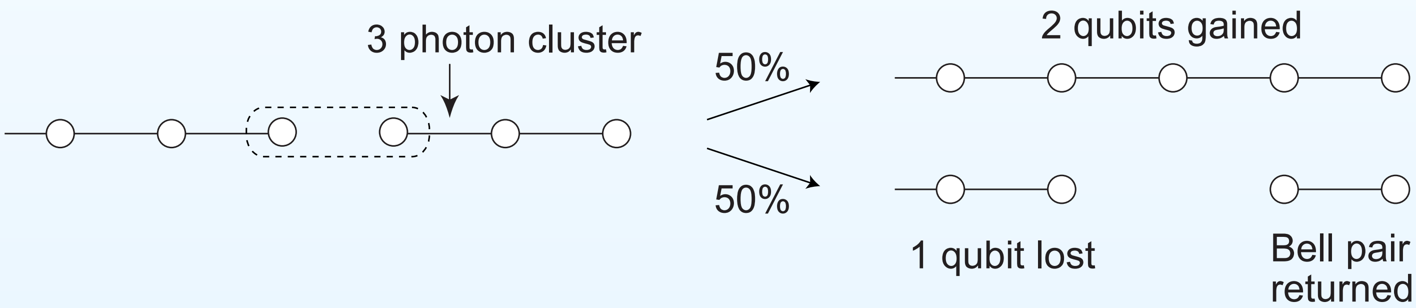

- On average, cluster length increases by 1/2 qubit. Therefore, to add one qubit to the cluster you need $2 \times 4 - 1 = 7$ Bell Pairs.

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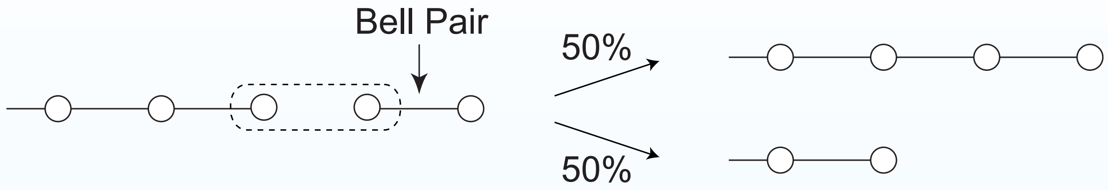

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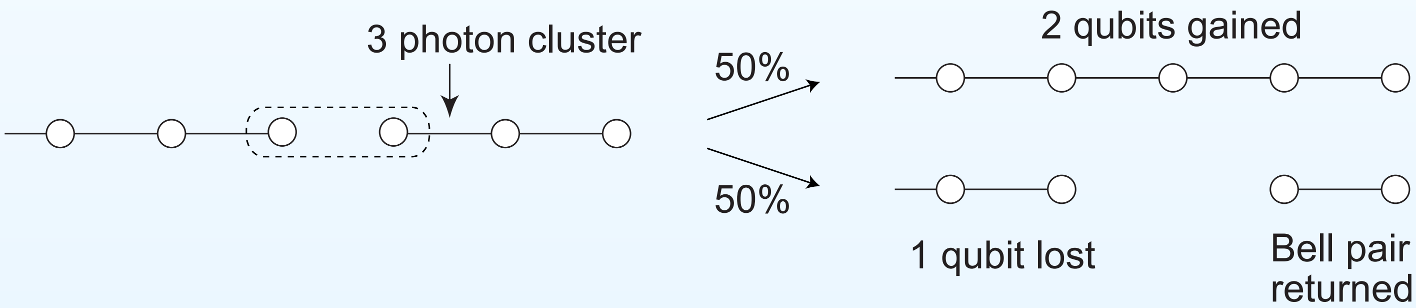

- On average, cluster length increases by 1/2 qubit. Therefore, to add one qubit to the cluster you need $2 \times 4 - 1 = 7$ Bell Pairs.
- The best protocol we have found uses 5-photon clusters as building blocks. This gives a rate: 6.5 Bell pairs per added qubit.

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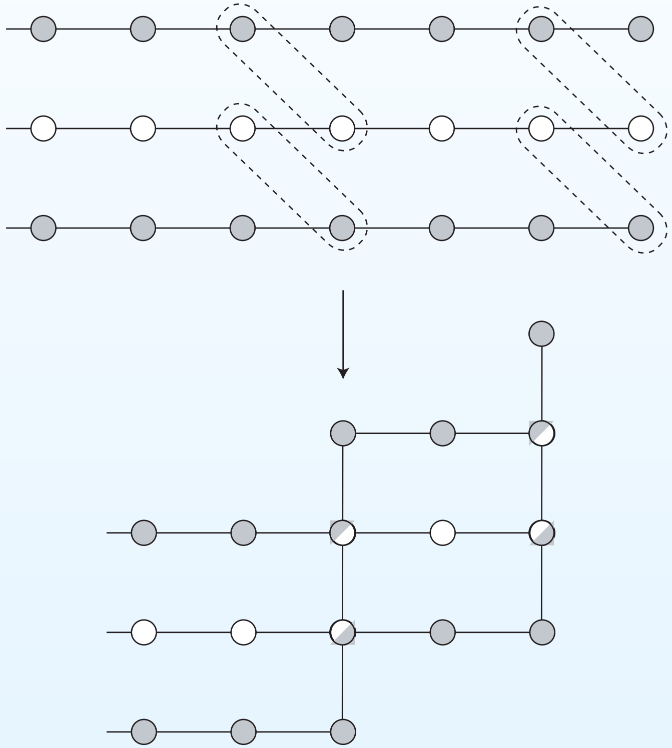
Joining Clusters into the 2-D Pattern

- We now need to join these linear clusters into the desired 3-dimensional layout.

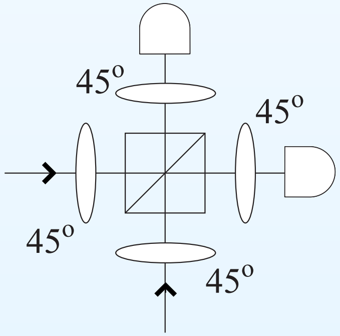


- With a deterministic fusion, this pattern of fusions produces the cluster state layout required.

- However, our Type-I fusion is only successful half the time. Failure is equivalent to σ_z measurement, which would break up hard won existing bonds!

Joining Clusters - Type-II Fusion

- Recall that a σ_x measurement on a cluster state does not cut the qubits bonds, but merges neighbouring qubits into a single redundantly encoded qubit.
- If we modify our fusion operation by introducing extra 45° rotations to each qubit, the failure outcomes will be σ_x measurements.
- However, the "success" projection is then no longer diagonal in the computational basis and does not perform the required fusion.



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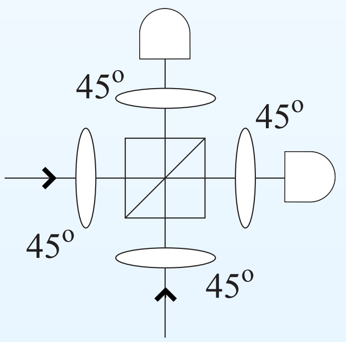
- However, the "success" projection is then no longer diagonal in the computational basis and does not perform the required fusion.
- We get round this by measuring *both* outputs.
- This leads to projections onto states $|{+}{+}\rangle + |{-}{-}\rangle = |00\rangle + |11\rangle$ or $|{+}{+}\rangle - |{-}{-}\rangle = |01\rangle + |10\rangle$.

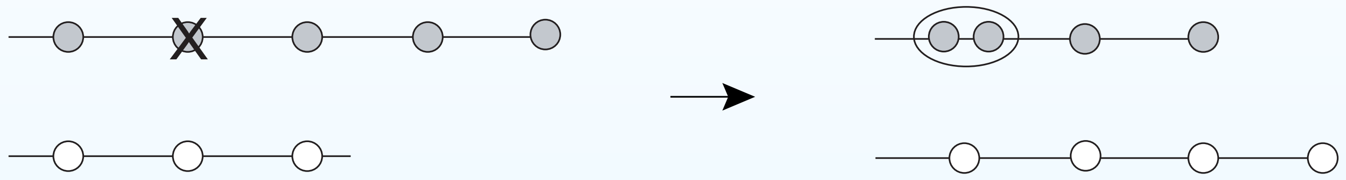


- If one of the qubits this is applied to is *redundantly encoded* this gives us the desired fusion! We call this a Type-II fusion.
- Again, the success probability of this step is 50%.

Joining Clusters - Type-II Fusion

- We now have a recipe to make the inter-cluster fusions.

Apply a σ_x measurement to prepare redundantly encoded qubit



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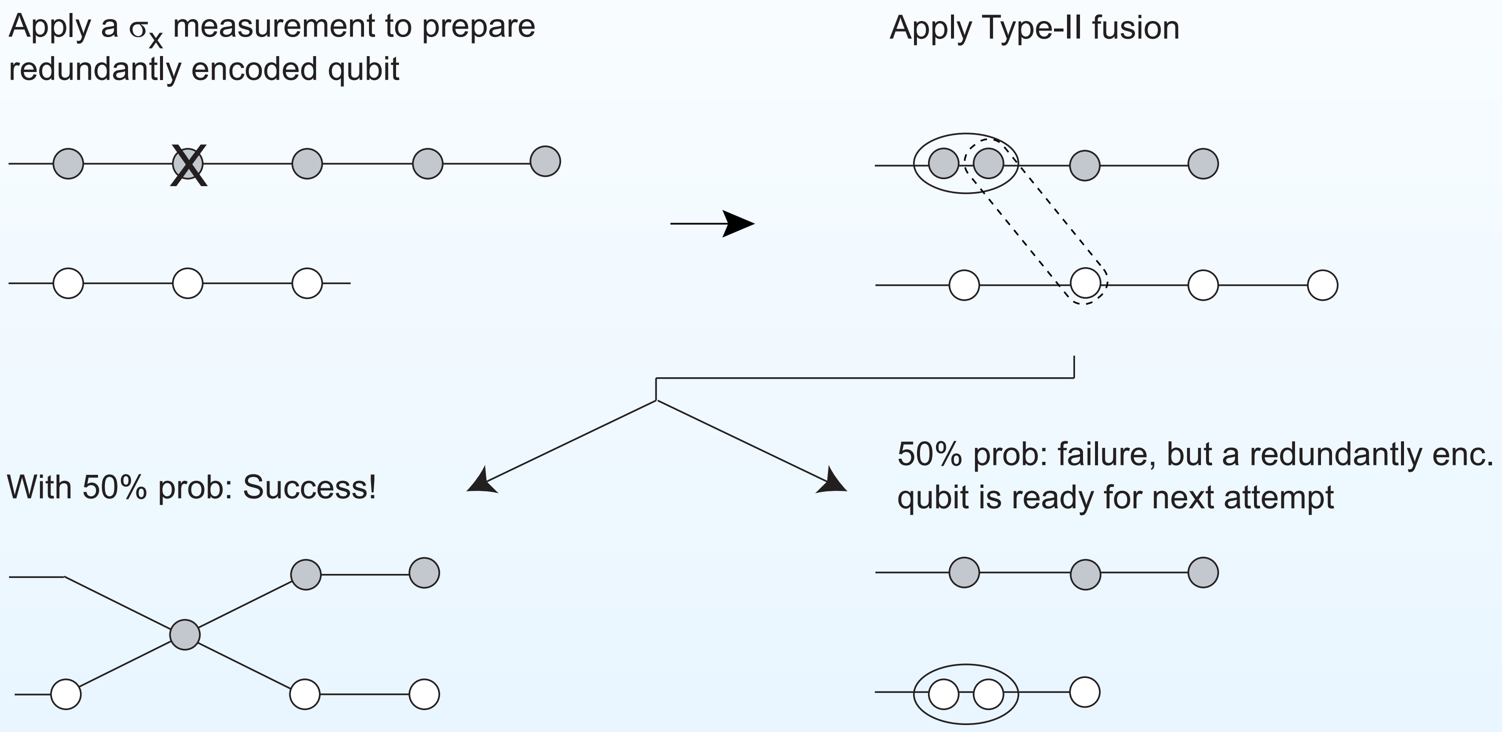

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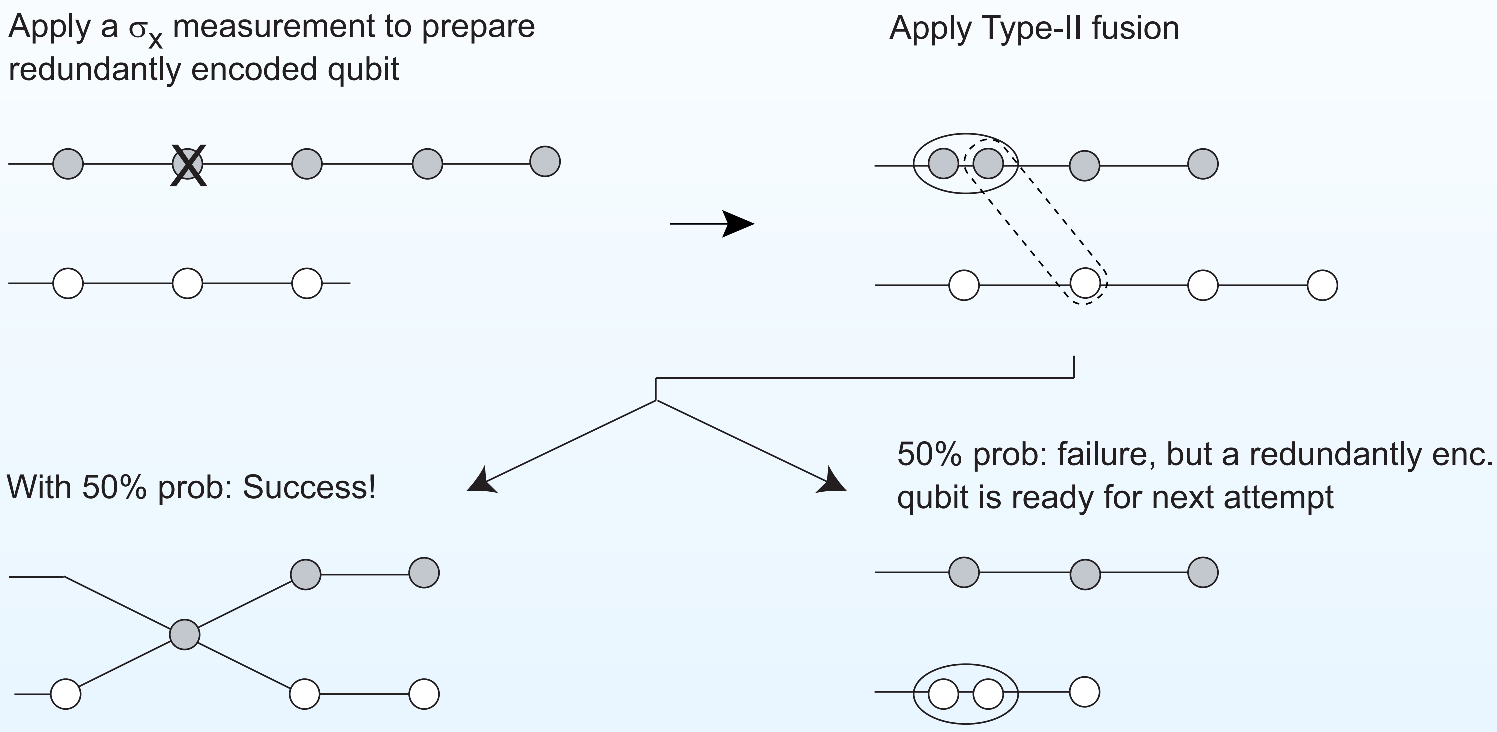
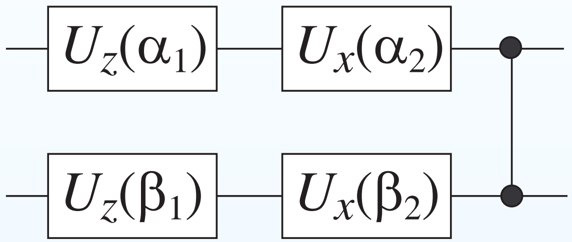


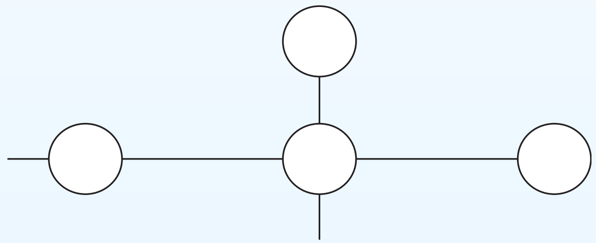
- Note that failures only "use up" cluster qubits to the right of the fusion.
- Thus, failures *cannot* propagate back through the cluster as in the Type-I fusion and other schemes.

Quantifying Resource Requirements

- In our cluster state measurement pattern, there are the same number of:
  - simulated 2-qubit gates:



  - T-shaped units in the cluster state:



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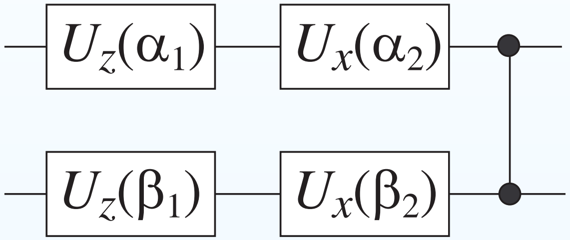

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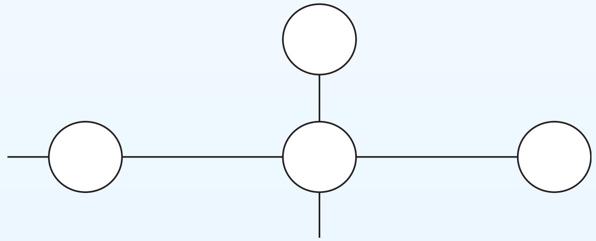

  This means the resources required to build the T-shape are a measure of the resources per two-qubit gate.

- If we use the method above, the construction of a T-shape consumes on average 8 bonds from the linear clusters used.
- Thus, per general 2-qubit gate the resource requirements are:

$$8 \times 6.5 = 52 \text{ Bell Pairs.}$$

Other Schemes: Rough Comparison of Resources

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Approx. entanglement resources required per (general) 2-qubit gate:

Knill-Laflamme-Milburn (KLM), Nature 401, 46 (2001)	(for 92.5% gate success prob.) ∼100-photon "KLM state"[a]
Yoran and Reznik, PRL 91, 037903 (2003). (Measurement based "chain state" q.c., uses KLM gates)	∼23 12-photon "KLM states"
Nielsen, accepted PRL (2004) (Cluster state scheme using KLM gates)	∼54 8-photon "KLM states"
Our scheme, quant-ph/0405157	52 2-photon Bell states

[a] Note that the KLM resource states require a complicated linear optical network conditional on several / many measurements for their generation.

Summary

- We have presented a scheme for *linear optics quantum computation* based on the cluster state approach, that is very resource efficient compared to other schemes.
- For the shorter-term, the work provides a recipe for the generation of interesting new entangled states.
- The procedures at the heart of the scheme have *already been implemented* experimentally.
- The scheme has other advantages. For example, the absence of concatenated beam-splitters, unavoidable in other schemes, makes the mode-matching requirements much less strict.

Future Directions

- Cluster state layout can be optimised for specific algorithms – how much more gains in resource efficiency are possible?
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This work can be found in pre-print **quant-ph/0405157**.

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