Universal Approach to Dynamical Control of Decay and Decoherence

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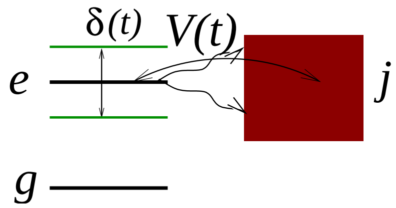
Dynamical control of decay and decoherence: Universal formula

A. G. Kofman and G. Kurizki, Nature **405**, 546 (2000), PRL **87**, 270405 (2001).



Weak coupling to environment: $\hat{V}_s = \Sigma_j\, \mu_{ej}|e\rangle\langle j| + \text{h.c.}$.
Amplitude/phase modulation/perturbation: $\hat{V}(t) = \epsilon(t)\hat{V}_s$.
Exact (reversible) evolution:

$$\dot{\alpha} \equiv \frac{d}{dt}\langle e|\Psi(t)\rangle = -\int_0^t dt' \epsilon^*(t)\epsilon(t')\Phi(t-t')e^{i\omega_a(t-t')}\alpha(t'),$$

$\Phi(t-t') = \Sigma_j\, |\mu_{ej}|^2 e^{-i\omega_j(t-t')}$ (**reservoir memory** function),
$\alpha(t)$ decays **slower** than $\Phi(t) \Longrightarrow \alpha(t') \approx \alpha(t)$.
$\Longrightarrow$ **Coherent** or random $\epsilon(t)$ obeys **universal modified decay rate:**

$$R(t) = 2\pi \int_{-\infty}^{\infty} d\omega G(\omega + \omega_{\mathrm{a}})F_t(\omega).$$

Overlap of reservoir coupling spectrum

$$G(\omega) = \pi^{-1}\mathrm{Re}\int_0^{\infty} dt e^{i\omega t}\Phi(t) \rightarrow \rho(\omega)|\mu(\omega)|^2$$

and the spectral intensity of modulation

$$F_t(\omega) = |\epsilon_t(\omega)|^2.$$

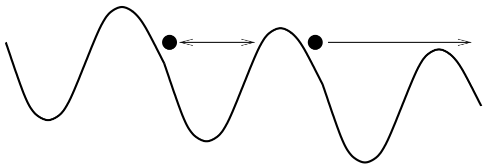
Overlap of $G(\omega)$ and $F_t(\omega)$ determines either suppressed or enhanced coupling to environment: Quantum Zeno effect (QZE) or anti-zeno effect (AZE).

Tunneling - barrier modulation: "α-decay" control

Fischer, Gutierrez and Raizen, PRL **87**, 040402 ('01):

Optical potential

"Washboard" potential on – τ_1, off – $\tau_0 \gg \tau_1$.

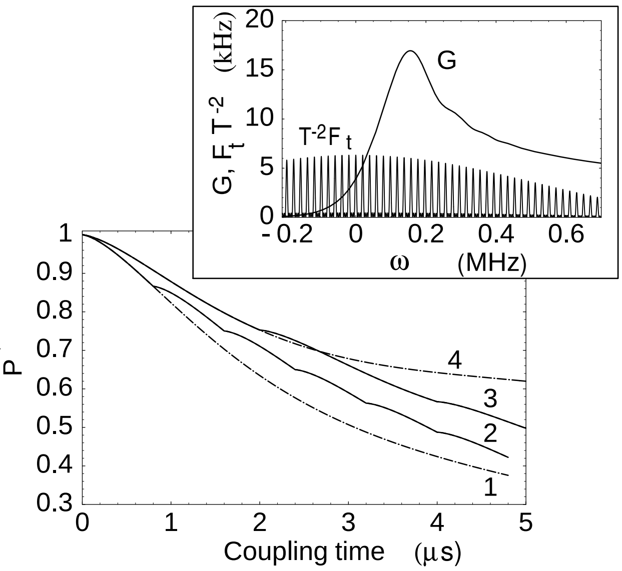


$G(\omega)$ does not change over $2\pi/\tau_0 \Longrightarrow$

$F_t(\omega) \sim$ measurement-induced broadening. $\nu \sim 1/\tau_1$: MHz.

QZE conditions: $\nu \gg \Gamma_R \gtrsim 1/\tau_c$.

AZE conditions: $1/\tau_c \ll \nu \ll \Gamma_R$



1, 4 – no modulation. 2 – QZE (compared to curve 1) $\tau_1 = 0.8\ \mu$s. 3 – AZE (compared to curve 4) $\tau_1 = 2\ \mu$s, $\tau_0 \simeq 50\ \mu$s.

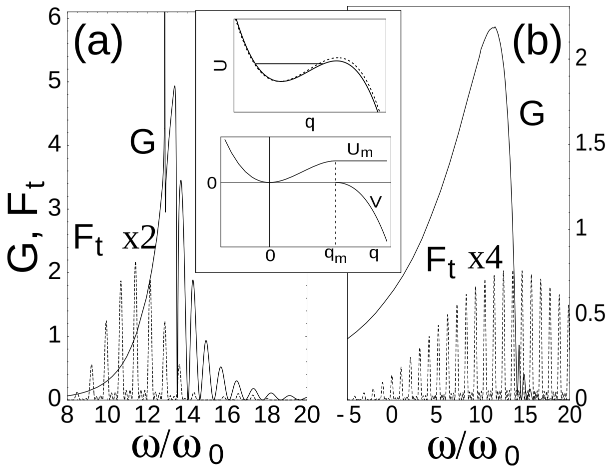
Lattice tilt (acceleration) – 15 km/s^2,

Na atoms barrier $\omega_g \sim 100$ kHz.

Josephson junction with bias-current (GHz) modulation: "Washboard" potential control.

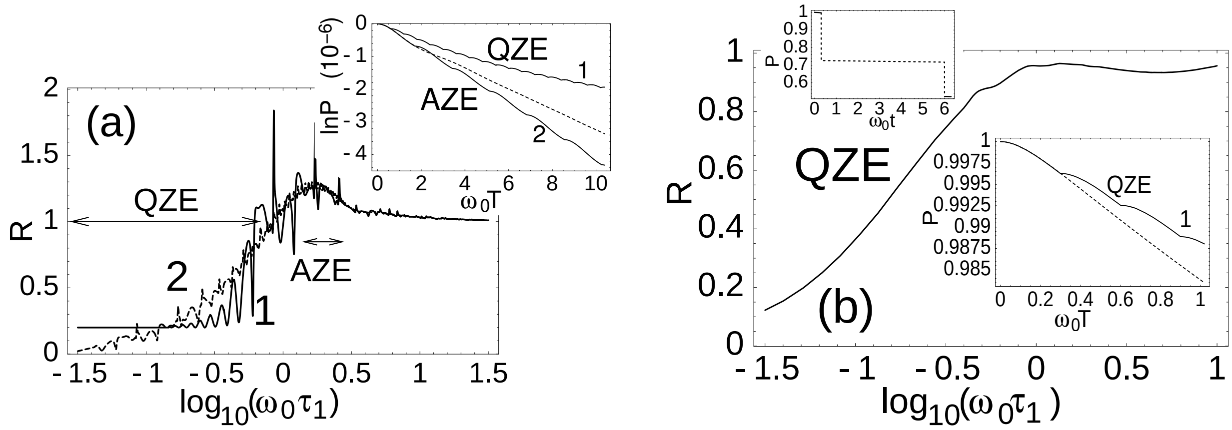
Barone, Kurizki, Kofman, PRL **92**, 200403 (2004).

(a) $G_{n=12}(\omega)$ and $F_{t=4\tau_0}(\omega)$ with $\tau_1 = 1/\omega_0 \sim 0.1$ ns, $\tau_0 = 5\tau_1$ (ω_0 – fundamental frequency in the well).
(b) $G_{n=15}(\omega)$, $\tau_1 = 0.3/\omega_0$, and $F_{t=4\tau_0}(\omega)$.



$$R_n \approx \frac{2\pi\tau_1}{\tau_0} \sum_{k=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{k\pi\tau_1}{\tau_0}\right) G\left(\omega_n + \frac{2k\pi}{\tau_0}\right).$$

$$\text{QZE:}\quad \underbrace{1/\tau_1}_{F_t \text{ width}} \gg \underbrace{\omega_m}_{G \text{ width}} \to R(\tau_1) \ll \underbrace{R_0 = 2\pi G(\omega_a)}_{\text{Golden Rule}}.$$



(a) R (in units of Golden-Rule rate R_{GR}) for $n = 12$, as a function of interruption time τ_1 (in units of $1/\omega_0$) for $\tau_0 = 5\tau_1$ (curve 1) and $\tau_0 = 50\tau_1$ (curve 2).
(b) for $n = 15$, R exhibits QZE behavior. Upper inset—P vs. total time t, showing impulsive jumps: $I_b = 0.9928 \pm 2 \times 10^{-4} I_c$.

Dynamic (coherent) control of qubit decoherence

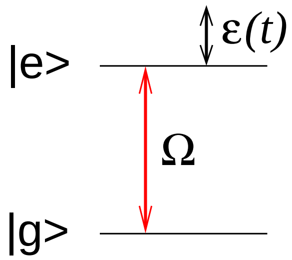
A. G. Kofman and G. Kurizki, PRL 87, 270405 (2001)

a) **Resonant field** can dynamically reduce proper dephasing ($\mathcal{E}(t)$ fluctuations).

$\omega_e = \bar{\omega}_e + \epsilon(t)$, $T_2^{-1} =< \epsilon^2 > \tau_c$.

τ_c- correlation time

$\Omega \gg 1/\tau_c \rightarrow T_2' \gtrsim T_2(\Omega\tau_c)^2$.



CW dynamical decoupling simpler than an echo ("bang-bang") pulse sequence. For spectrally-biased fluctuations: usual "bang-bang" fails, tailor $\Omega(t)$.

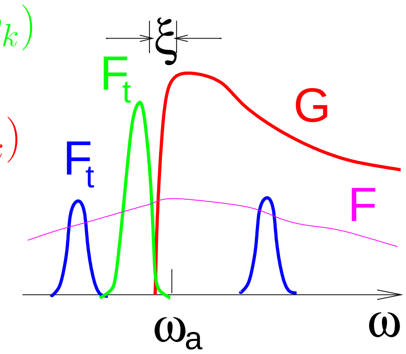
b) **Phase modulation (PM):** control of vibrational decay.

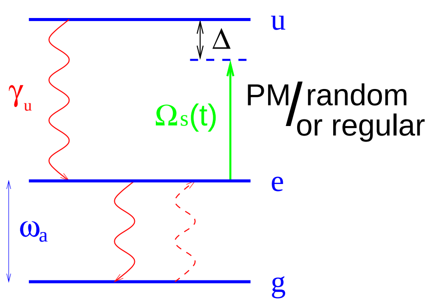
AC Stark modulation: $\delta(t) \simeq \Omega_s^2(t)/\Delta$.

$F_t(\omega) \sim \Sigma_k |\epsilon_k|^2 \delta(\omega - \omega_k)$

$R \approx 2\pi \sum_k |\epsilon_k|^2 G(\omega_a + \omega_k)$

Phase jumps by ϕ at $\tau, 2\tau, \ldots$.

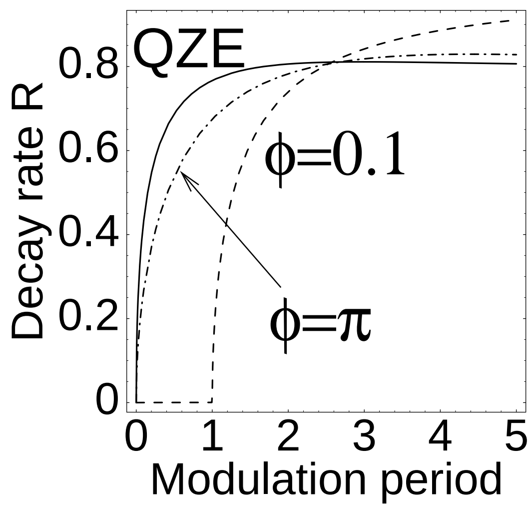




Periodic PM with $\phi \ll 1$ – most effective near band edge.

Random PM (QZE) – ineffective.

Periodic PM with $\phi = \pi$ (Agarwal, Scully, Walther,2001)- most effective for lorentzian bands.



Dynamical control of qubit decoherence at finite T

A. Kofman and G. Kurizki, PRL (2004)

Zwanzig's method used to write most general Master Eq. for driven/modulated systems, coupled to bath B, without RWA:

$$\dot{\rho} = -\frac{i}{\hbar}[H_S(t), \rho] + \int_0^t dt' \{ \overbrace{\Phi_T(t,t')}^{\text{B memory func.}} [\tilde{\mathcal{S}}(t',t)\rho \overbrace{\mathcal{S}(t)}^{\text{B-S coupling}} - \mathcal{S}(t)\tilde{\mathcal{S}}(t',t)\rho] + \text{H.c}\}.$$

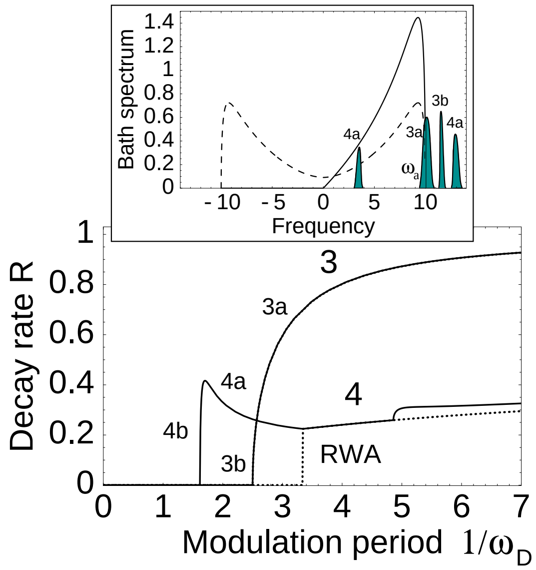
Quasiperiodic modulation of $\mathcal{S}(t) \propto \epsilon(t) = \sum_k \epsilon_k e^{i\omega_k t}$ $(k = 0, \pm 1, \ldots)$,

$$R_{e(g)}(t \to \infty) = 2\pi \int_{-\infty}^{\infty} d\omega F(\omega) G_T(\pm\omega) = 2\pi \sum_k |\epsilon_k|^2 G_T(\pm(\omega_a + \omega_k)) \qquad (1)$$

$$G_T(-\omega) = e^{-\beta\omega} G_T(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} \Phi_T(t) e^{-i\omega t} dt \qquad (2)$$

.

Fast modulation, high ω_k: Non-RWA $g \to e$ transitions even at $T = 0$!



Solid: $G_0(\omega)$; dashed: $G_S(\omega) = [G_T(\omega) + G_T(-\omega)]/2$, $\beta = 10/\omega_D$; dark: $F(\omega)$. $\omega_a = 0.94\omega_D$. 3: $\phi = -0.15$; 4: $\phi = \pi$. Curve 3 is optimal. Dotted: RWA.

Qubit decoherence control: Conclusions

A. How to control state decay into energy continuum/reservoir? Perturb system on quasi-reversible memory time scale.

B. Our simple **universal** formula results in general criteria for dynamical control of decay, decoherence, and quantum information (QI)/fidelity loss.

C. We considered in detail various systems: tunneling in optical lattices, Josephson junctions, entangled photon states.

D. Coherent (unitary) modulation of the coupling to the reservoir (continuum) can be designed for much more effective suppression of decoherence/QI loss than QZE.

E. We account for thermal and antiresonant (non-RWA) effects: reservoir-induced excitation of the system at $T = 0(!)$ in the presence of phase modulation.

F. Radiative decay requires different control: Subradiant two-atom interference or sudden phase jumps near continuum edge.

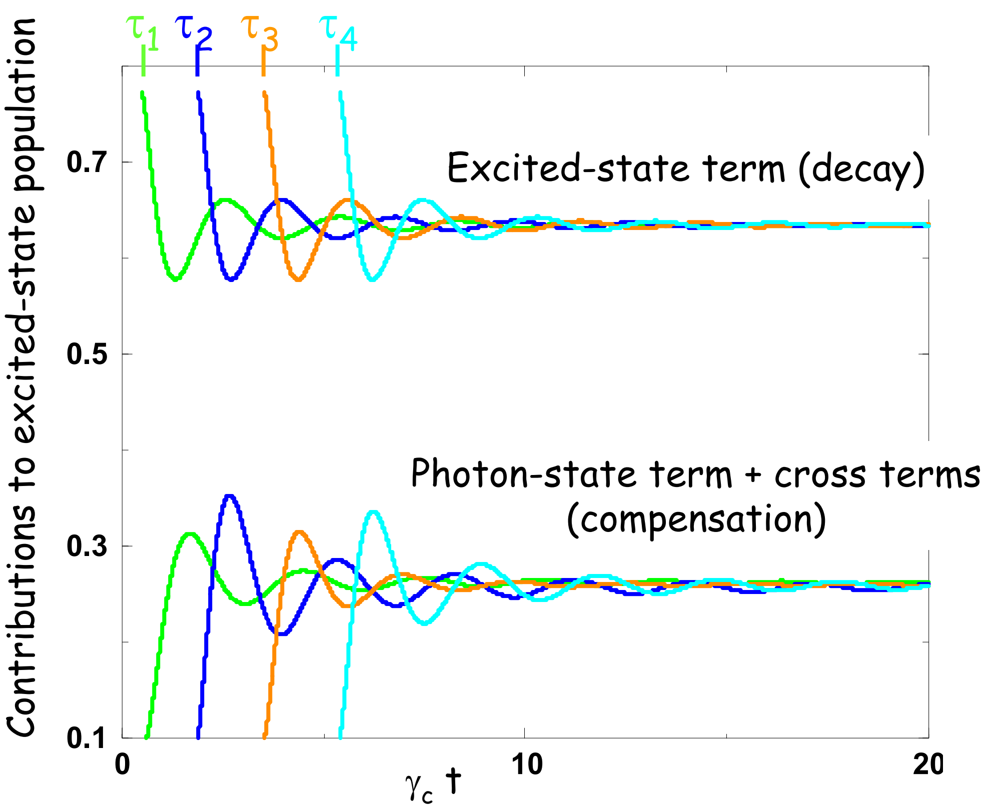
Sudden Change Dynamics

$$\alpha_{dyn}(t) = \alpha_A^{stat}(\tau)\, \alpha_B^{stat}(t-\tau) + \int_0^\infty \beta_{\omega,A}^{stat}(\tau)\, \beta_{\omega,B}^{stat}(t-\tau)\, \rho(\omega)\, d\omega, \quad t \geq \tau$$

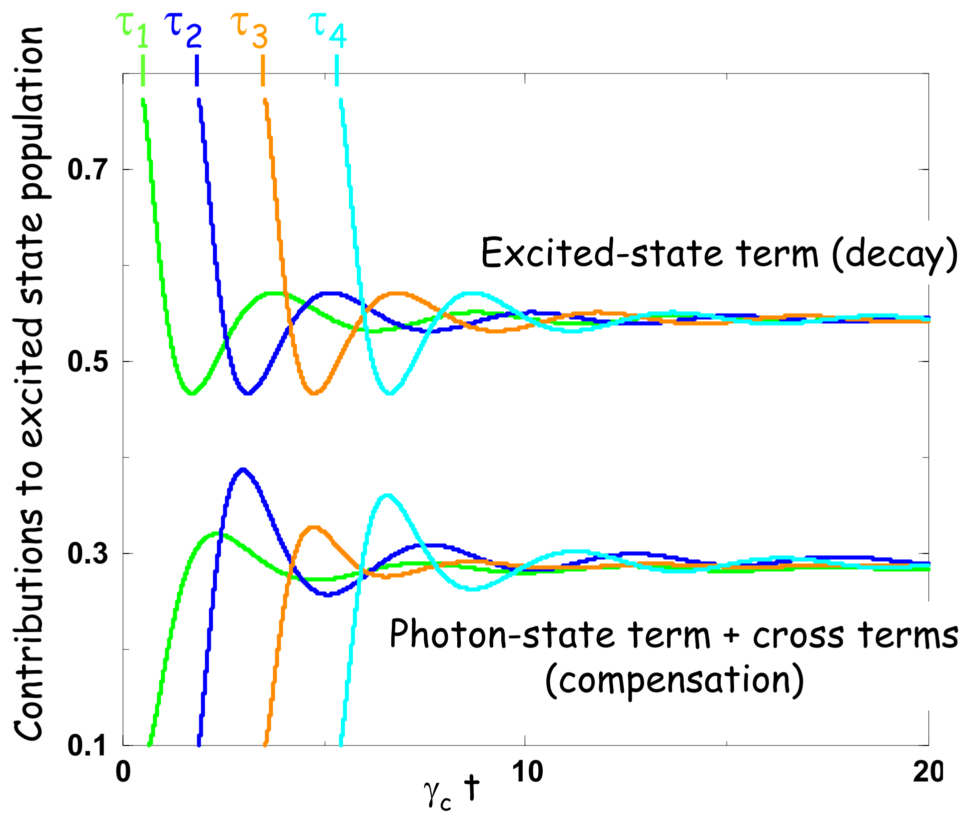
$\alpha_{A/B}^{stat}$ = excitation amplitude

$\beta_{\omega,A/B}^{stat}$ = mode ω amplitude

} at a fixed frequency ω_A (ω_B)



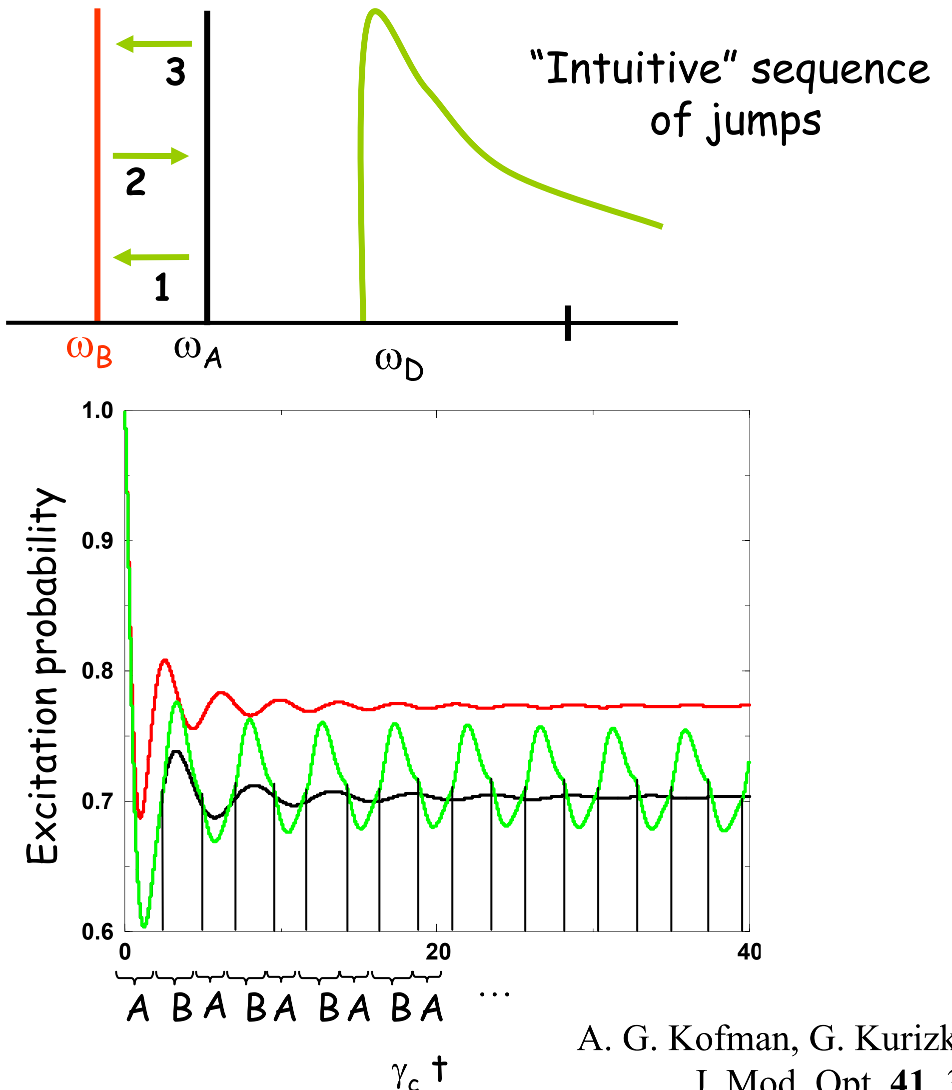
$\omega_A \rightarrow \omega_B$ increased detuning

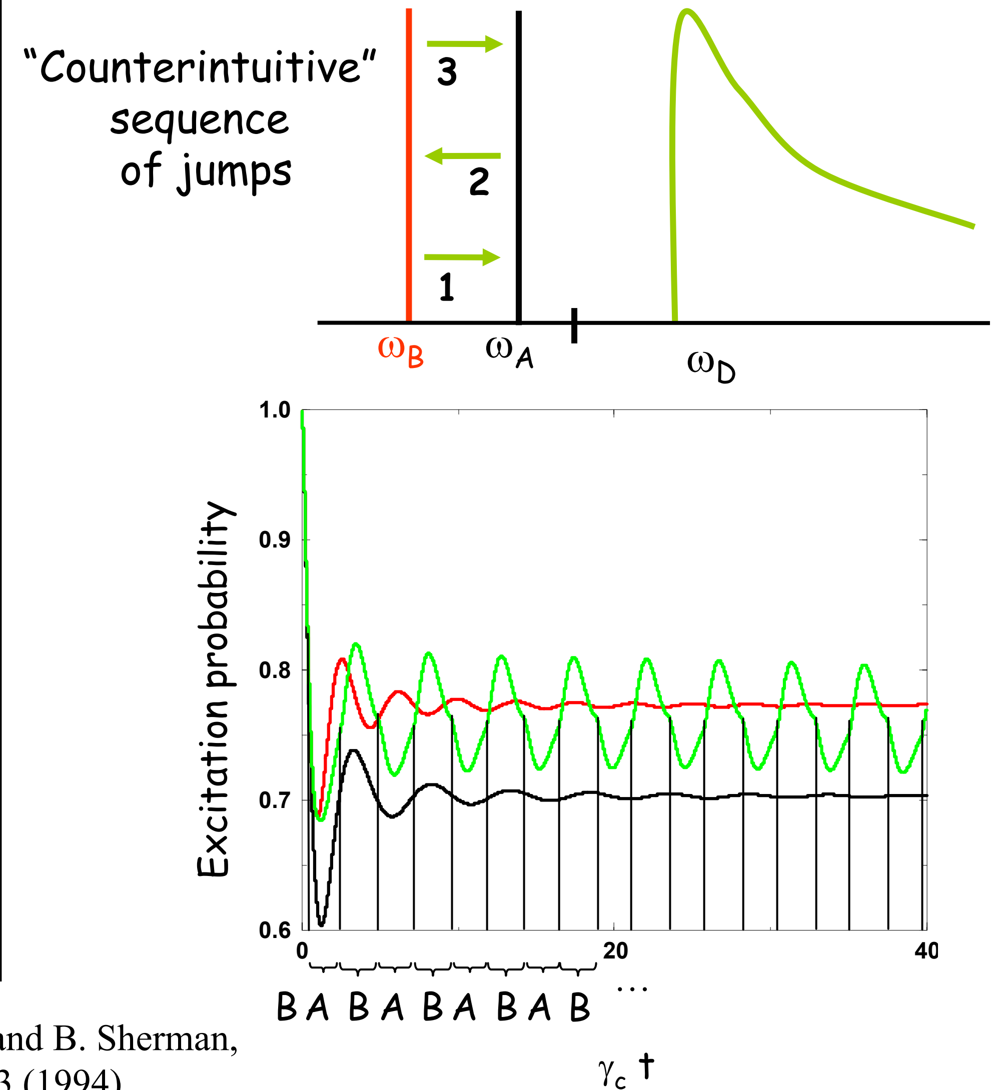


$\omega_B \rightarrow \omega_A$ reduced detuning

Nonadiabatic Dynamical Protection from Decoherence in PBGs: Periodic Frequency Jumps

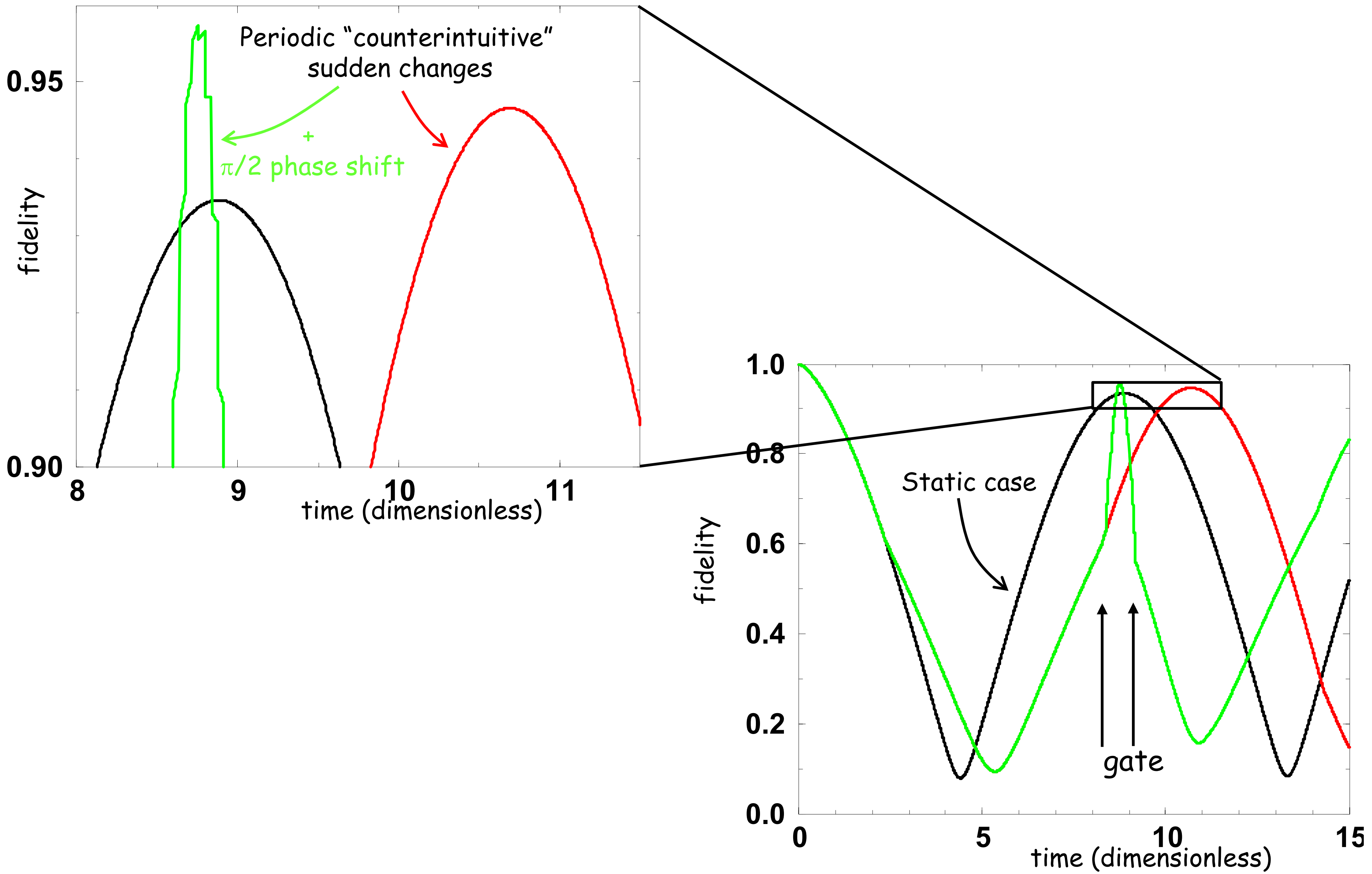
Sophie Pellegrin & Gershon Kurizki





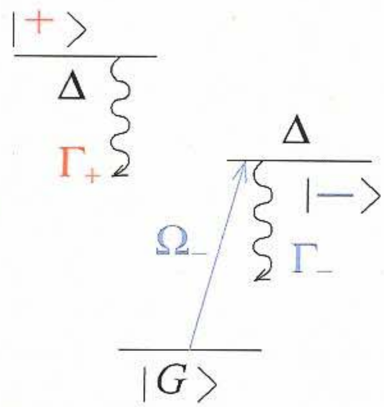
A. G. Kofman, G. Kurizki and B. Sherman, J. Mod. Opt. **41**, 353 (1994).

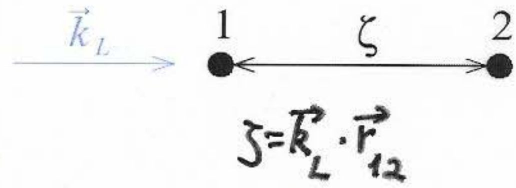
Fidelity and phase gates



Dipole-dipole interacting diatom qubit

D. Petrosyan and G. Kurizki, PRL **89**, 207902 (2002)





Eigenstates of the system:

$|G\rangle = |g_1 g_2\rangle,\ |E\rangle = |e_1 e_2\rangle,\ |\pm\rangle = \frac{1}{\sqrt{2}}(|e_1 g_2\rangle \pm |g_1 e_2\rangle)$

$$\text{For } \zeta \ll 1 \quad \Rightarrow \quad \Delta \approx \frac{3\gamma}{4\zeta^3} \gg \gamma, \ \ \Gamma_- \approx \frac{\gamma\zeta^2}{5} \ll \gamma,$$
$$\Gamma_+ \approx 2\gamma, \ \ \Gamma_E \approx 2\gamma$$

$|G\rangle$ **and** $|-\rangle$ **are the qubit states.**

$$\Omega_- \simeq \frac{\Omega(\vec{k}_L \cdot \vec{r}_{12})}{\sqrt{2}} = \frac{\Omega\zeta}{\sqrt{2}}, \quad \Omega_+ = \sqrt{2}\Omega \quad (\zeta \ll 1)$$

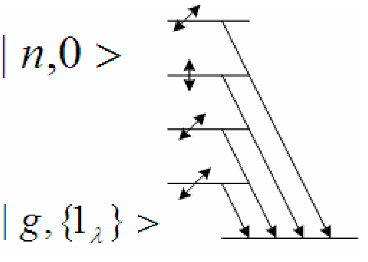
Single-qubit gate operation—rotation:

$$T_{\text{flip}} = \frac{\pi}{2\Omega_-} \quad \Rightarrow \quad P_{\text{decay}} = \Gamma_- T_{\text{flip}} = \frac{\pi\gamma\zeta}{5\sqrt{2}\Omega}$$

Take: $\zeta \simeq 0.02,\ \Omega/\gamma \simeq 30 \Rightarrow$
$P_{\text{decay}} \simeq 3 \times 10^{-4},\ \Delta \approx 10^5\gamma,\ \gamma T_{\text{flip}} \simeq 3.7$

Dynamical Control of Multiparticle/Multilevel Systems

G. Gordon, G. Kurizki, A. Kofman



- Common ground state and n excited states, energies $\omega_{n.}$
- Collection of reservoirs: <u>partial</u> or <u>no-cross correlations</u>.
- Modulating AC Stark shifts via $\varepsilon_n(t)$.

Matrix equation for the excited-states vector $\underline{\alpha}=\{\alpha_n\}$

$$\partial_t\underline{\alpha}(t)=-i\Omega\underline{\alpha}(t) -\int_0^t dt' K(t,t')\Phi(t-t')e^{i\omega_n t-i\omega_{n'} t'}\underline{\alpha}(t')$$

Rabi matrix: $\Omega_{nn'} = \vec{\mu}_{nn'} \cdot \vec{\mathrm{E}}_n(t)$ computing/ entanglement

Modulation matrix: $K_{nn'}(t,t')=\varepsilon^*_n(t)\varepsilon_{n`}(t')$, $\mathbf{F}_t(\omega)$: Spectral Density of $\mathbf{K}(t,t')$

Relaxation matrix: $G_{nn'}(\omega)=\sum_\lambda g_{n\lambda}{}^* g_{n'\lambda}\delta(\omega-\omega_\lambda)$, $g_{n\lambda}=\mu_{ng}\cos\theta_{n\lambda}$

cross-correlations: $\cos\theta_{n\lambda}\cos\theta_{n`\lambda}$

$$\alpha(t)=e^{-\mathbf{R}(t)}\alpha(0)$$

$$R_{nn'}(t) = 2\pi e^{i(\omega_n-\omega_{n'})t}\int_{-\infty}^{\infty} d\omega G_{nn'}(\omega+\omega_{n'})F_{t,nn'}(\omega)$$

Minimize $|R_{nn'}(t)|$, by choosing appropriate $\varepsilon_n(t)$

Create quasi "decoherence-free subspaces" although $\{G_{nn'}\} \neq 0$

Compare:
Zanardi&Rasetti
Lidar&Whaley

$$\underbrace{N_{control}}_{\text{no. of control parameters}} > \underbrace{\frac{n(n+1)}{2}}_{\text{no. of eqs.}}$$

Coherent quasi-periodic modulation

$$\varepsilon_n(t)=\sum_k \kappa_{n,k}\, e^{-i\nu_{n,k}t}$$

$$\sum_k |\kappa_{n,k}|^2 =1$$

2nk degrees of freedom with n constraints.

For a given set of $\nu_{n,k}$

Search for $\kappa_{n,k}$ such that $\sum_{nn`}|R_{nn`}(t)|\to 0$

Long time modulation: QZE or AZE

$$R_{nn'}(t)=2\pi t\delta_{n,n'}\sum_k|\kappa_{k,n}|^2G_{nn}(\omega_n+\nu_{nk})$$

Ultrashort time modulation: Full reversibility

$$R_{nn'}(t)=t^2e^{i(\omega_n-\omega_{n'})t}\int d\omega G_{nn'}(\omega)\sum_{kl}\kappa_{n,k}\kappa_{n',l}$$

No modulation: Golden rule

$$R^{ref}_{nn`}=2\pi\, t\, e^{i(\omega_n-\omega_{n`})t}\, G_{nn'}(\omega_{n`})$$

Numerical examples

Relaxation matrix:

$G_{nn'}(\omega)=\cos\theta_n \cos\theta_{n`} e^{-\omega^2/2\Gamma^2}$

Modulation frequencies:

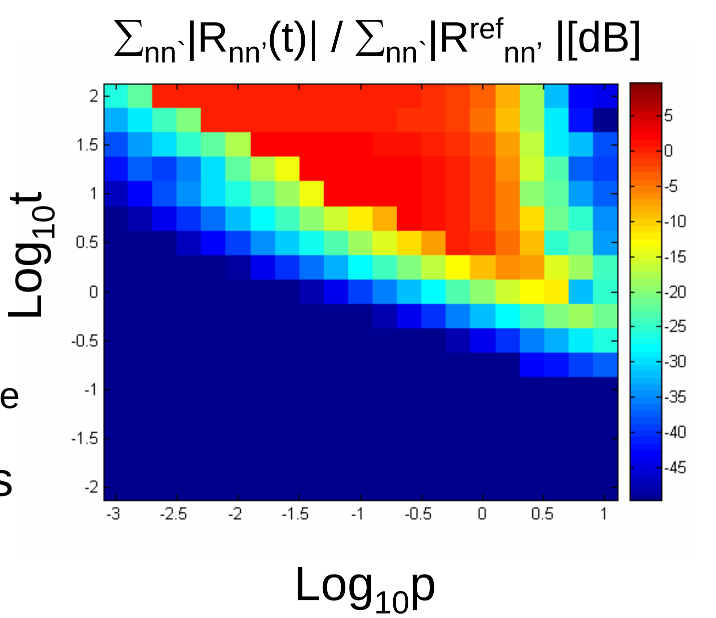
- $\nu_{1,1}=-p\Gamma$ Γ– reservoir width
- $\nu_{1,2}=p\Gamma$
- $\nu_{2,1}=-p\Gamma+\Delta$
- $\nu_{2,2}=p\Gamma+\Delta$

n=2

Long time modulations
$t \gg 1/\Delta, 1/\Gamma$ QZE or AZE

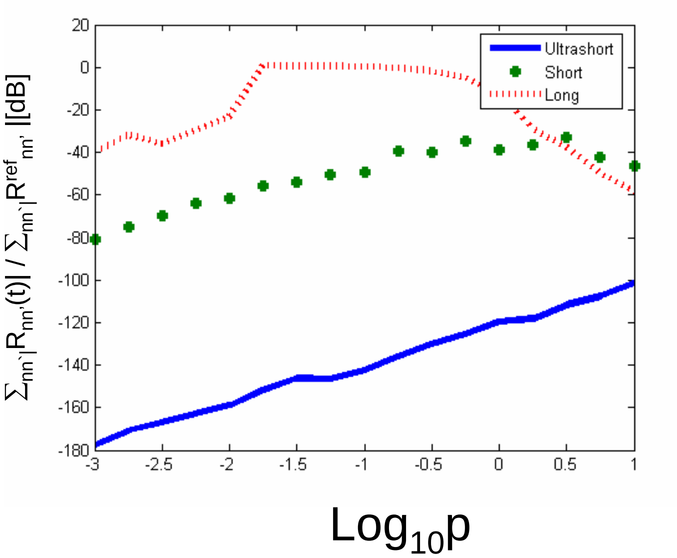
Short time modulations
$t \sim 1/\Delta, 1/\Gamma$ Channels interference

Ultrashort time modulations
$t \ll 1/\Delta, 1/\Gamma$ Full reversibility & $|R_{nn'}|$suppression



n=4

Preliminary results



Conclusions

- Efficient control of multi-qubit decoherence (also chaos) by multiple pulsed AC Stark shifts. Pulse engineering replaces ancilla.
- Works for both local and correlated reservoirs.
- Decoherence suppressed for AC Stark shifts <u>within</u> bath spectrum due to short-time multichannel interference.
- Unified dynamical theory of driven multipartite coupling to arbitrary environments & unitary control of their irreversibility & classicality.