Unifying & Simplifying Measurement-based Quantum Computation Schemes

Debbie Leung, Caltech

quant-ph/0404082,0404132
Joint works with Panos Aliferis, Andrew Childs, & Michael Nielsen

Hashing ideas from Charles Bennett, Hans Briegel, Dan Browne, Isaac Chuang, Daniel Gottesman, Robert Raussendorf, Xinlan Zhou

Universal QC schemes using only simple measurements:

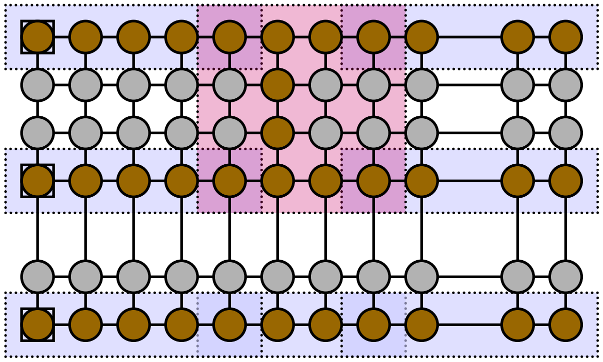
Universal QC schemes using only simple measurements:

1) One-way Quantum Computer "1WQC" (Raussendorf & Briegel 00)

2) Teleportation-based Quantum Computation "TQC" (Nielsen 01, L)

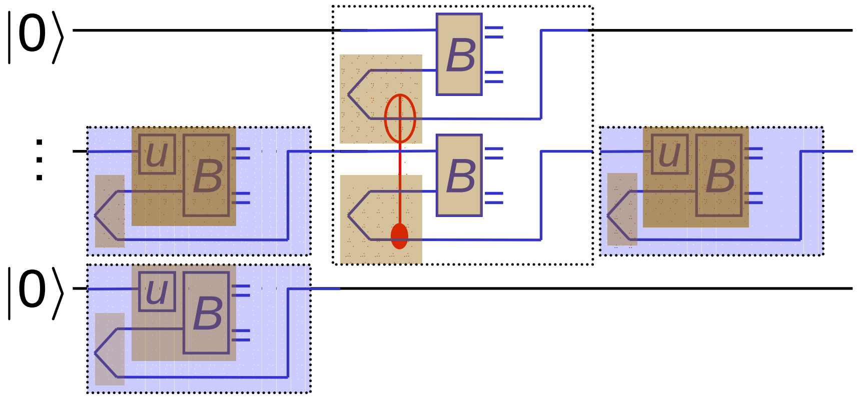
1WQC:

- Universal entangled initial state
- 1-qubit measurements



TQC:

- Any initial state (e.g. $|00\cdots0\rangle$)
- 1&2-qubit measurements



strawberry ice-cream & strawberry smoothy

Qn: are 1WQC & TQC related & can they be simplified?

Here: derive simplified versions of both using

"1-bit-teleportation" (Zhou, L, Chuang 00)

(simplified version of Gottesman & Chuang 99)

Rest of talk:

0. Define simulation
1. Review 1-bit-teleportation
2. Derive intermediate simulation circuits (using much more than measurements) for a universal set of gates
3. Derive measurement-only schemes

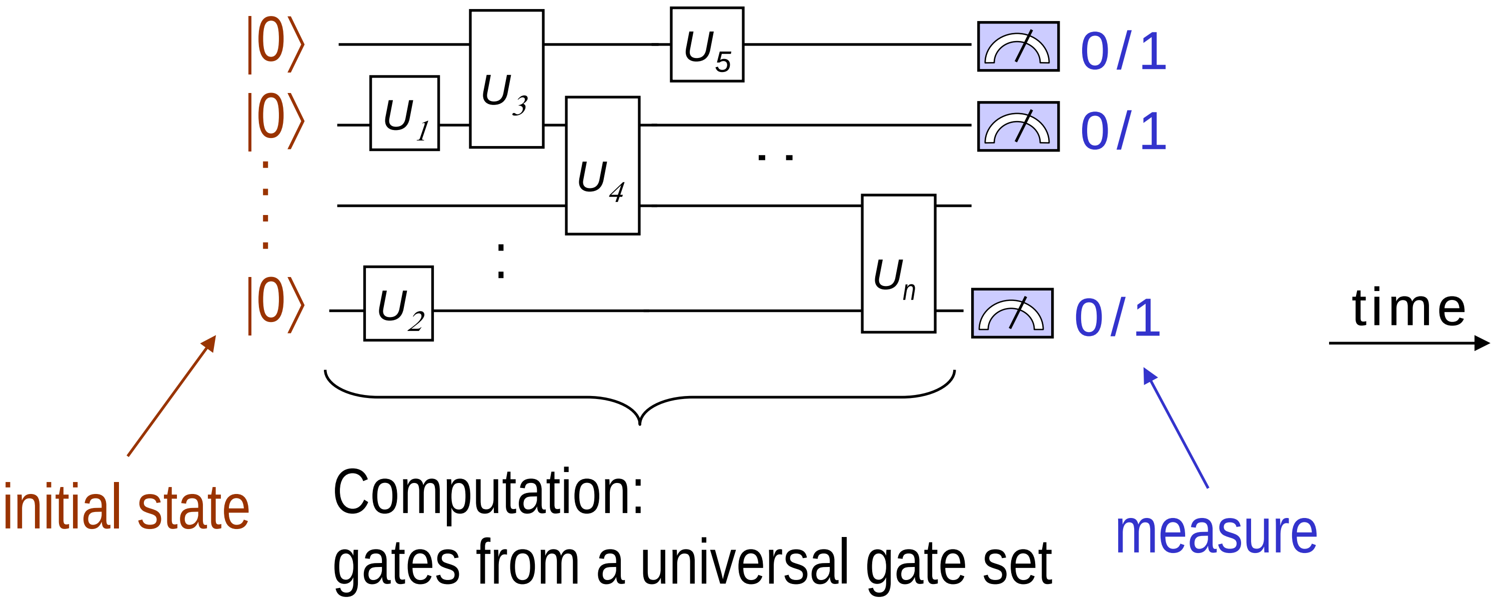
milk

strawberry

freeze & mix
or
mix & freeze

Standard model for universal quantum computation :

DiVincenzo 95



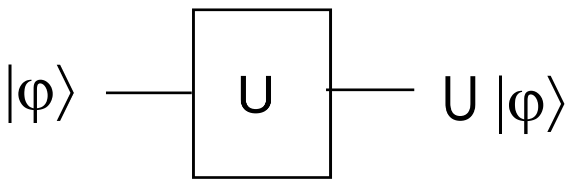
Simulation of components *up to known "leftist" Paulis*

e.g. U

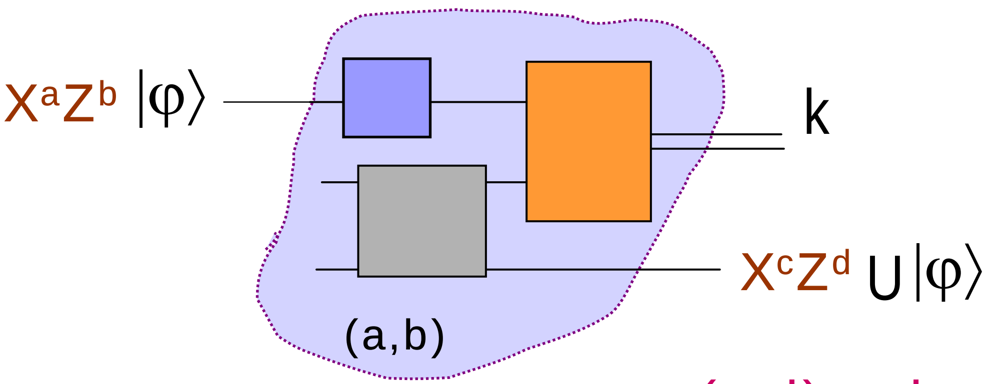
$\forall\, |\varphi\rangle$ (input to U), $\forall\, X^a Z^b$ (arbitrary known Pauli operator)

X,Z: Pauli operators, $a,b,c,d \in \{0,1\}$

Wanted



Simulation



(c,d) only depends on (a,b,k)

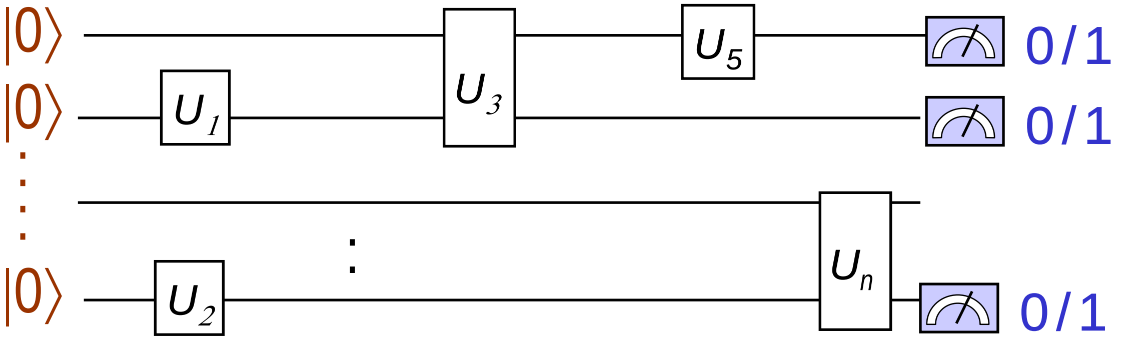
e.g. U simulates itself

$\Leftrightarrow \forall_{|\varphi\rangle,a,b}\ U X^a Z^b |\varphi\rangle = X^c Z^d U |\varphi\rangle$

$\Leftrightarrow$ U $\in$ Clifford group

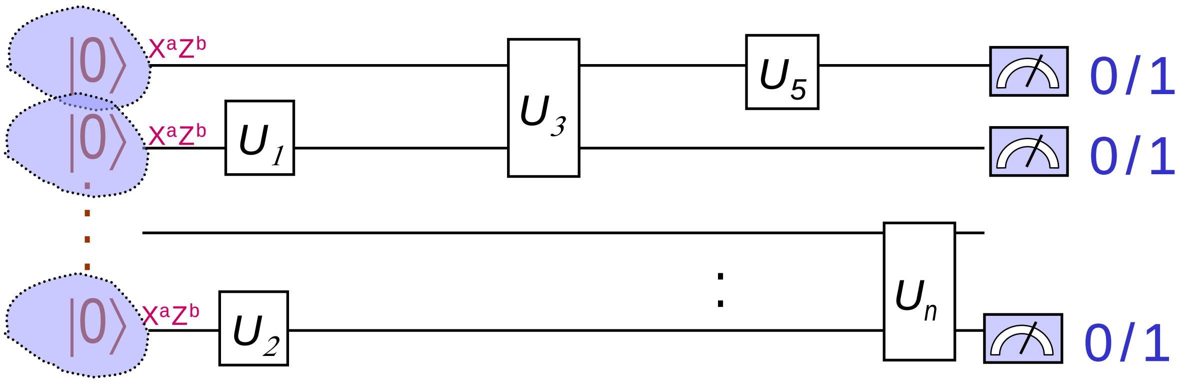
<u>Simulation of circuit</u> *up to known "leftist" Paulis*

Composing simulations to simulate any circuit :



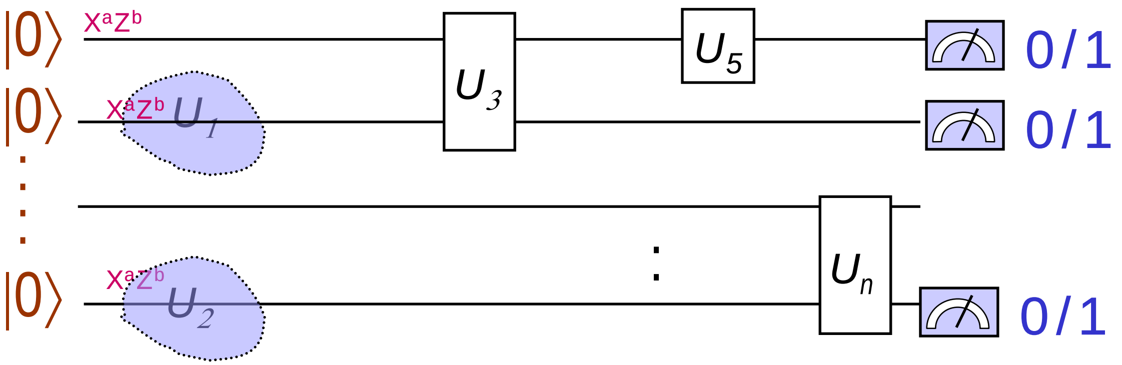
Simulation of circuit *up to known "leftist" Paulis*

Composing simulations to simulate any circuit :



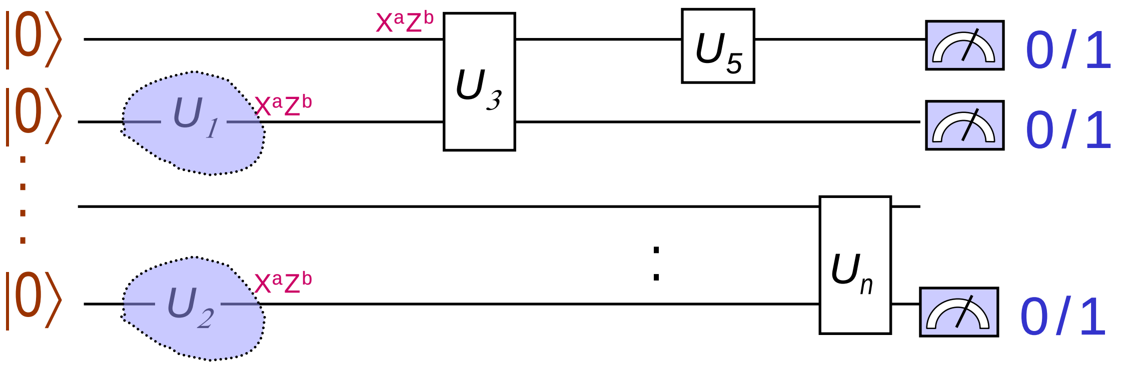
Simulation of circuit *up to known "leftist" Paulis*

Composing simulations to simulate any circuit :



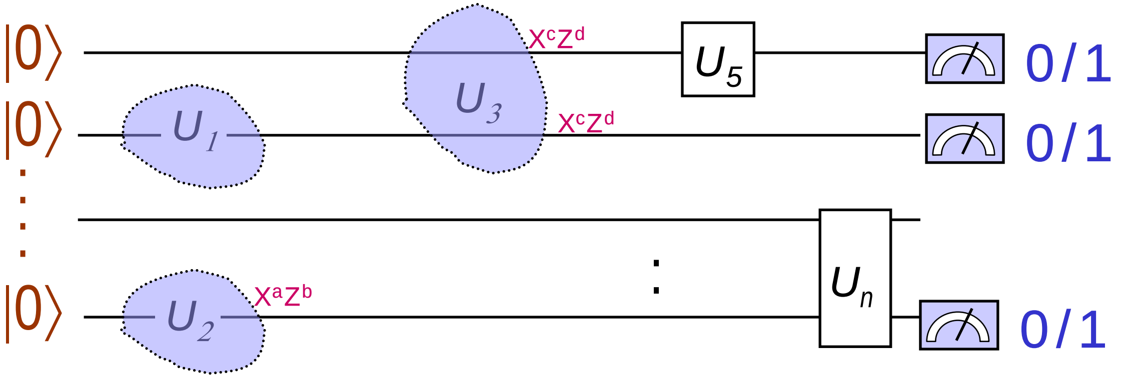
Simulation of circuit *up to known "leftist" Paulis*

Composing simulations to simulate any circuit :



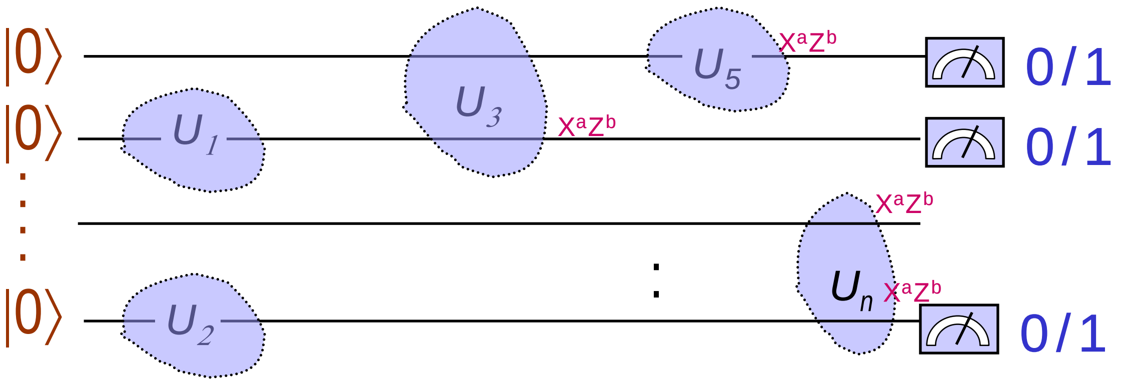
Simulation of circuit *up to known "leftist" Paulis*

Composing simulations to simulate any circuit :



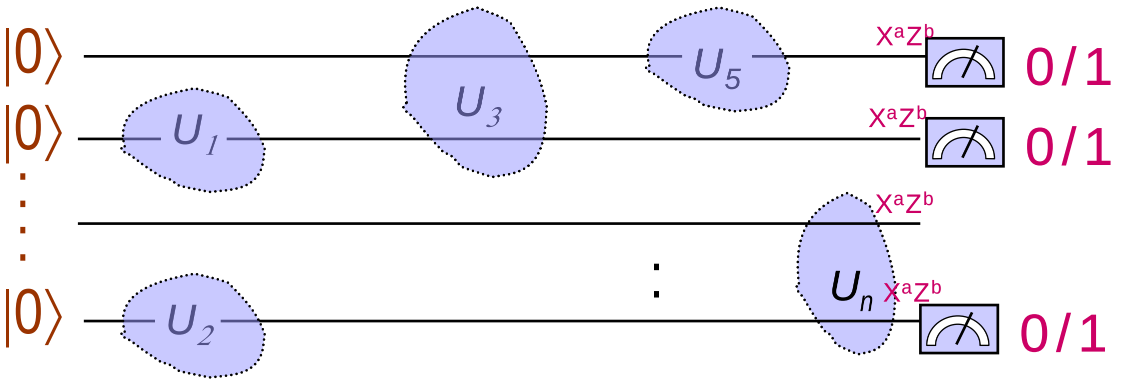
Simulation of circuit *up to known "leftist" Paulis*

Composing simulations to simulate any circuit :



Simulation of circuit *up to known "leftist" Paulis*

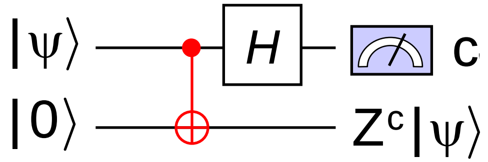
Composing simulations to simulate any circuit :



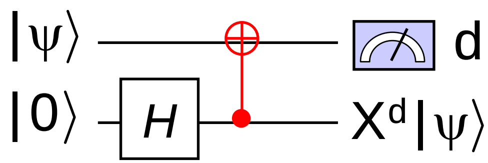
Propagate errors without affecting the computation. Final measurement outcomes are flipped in a known (harmless) way.

1-bit teleportation

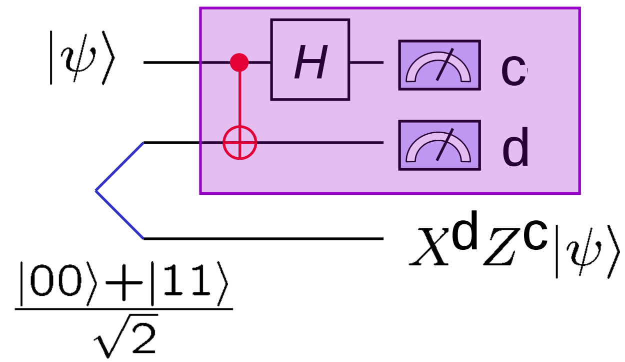
Z-Telepo (ZT)



X-rtation (XT)



Teleportation without correction



NB. All simulate “I”.

Recall:

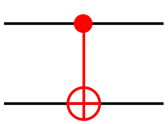
Pauli’s:

I, X, Z

Hadamard:

H

CNOT:

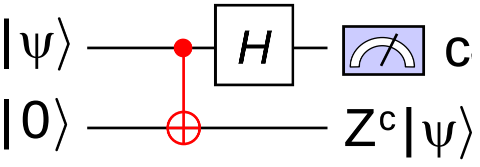


Simulating 1-qubit gates & controlled-Z with mixed resources.

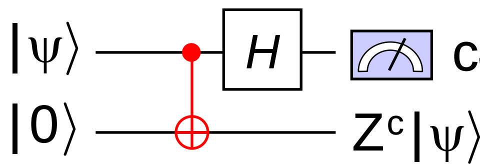
Goal: perform Z rotation $e^{i\theta Z}$

Z-Telep (ZT)

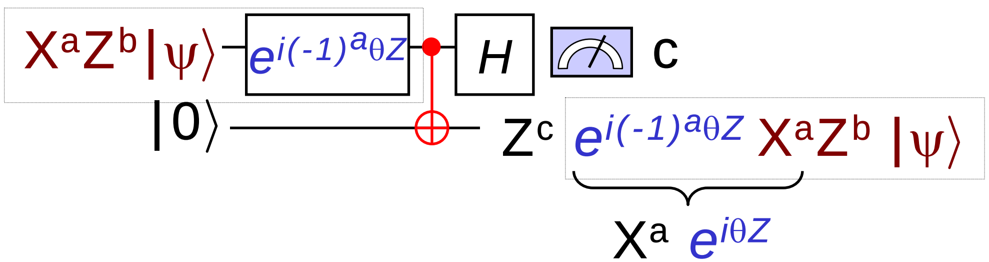
Goal: perform Z rotation $e^{i\theta Z}$



Z-Telep (ZT)

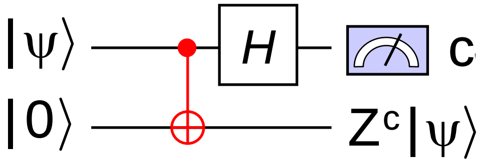


Goal: perform Z rotation $e^{i\theta Z}$

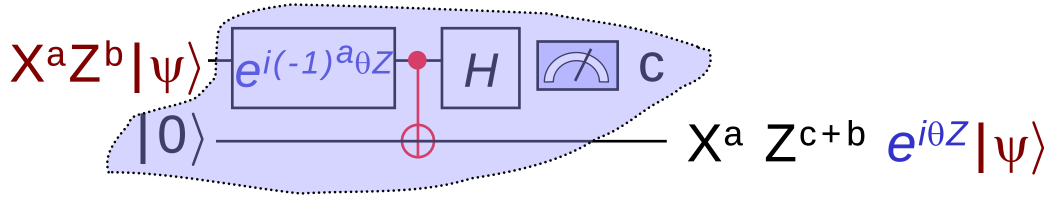


Input state = $e^{i(-1)^a \theta Z} X^a Z^b |\psi\rangle$

Z-Telep (ZT)



Simulating a Z rotation $e^{i\theta Z}$



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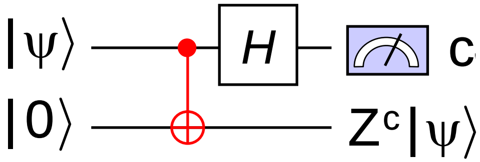
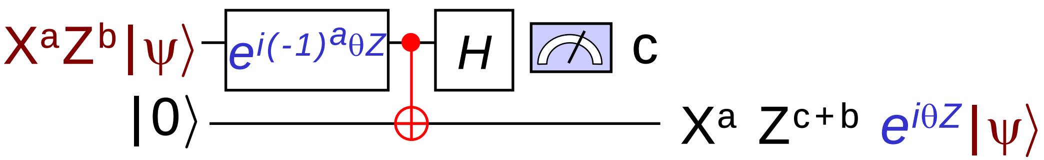
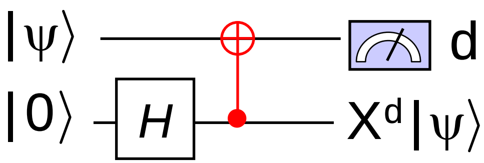


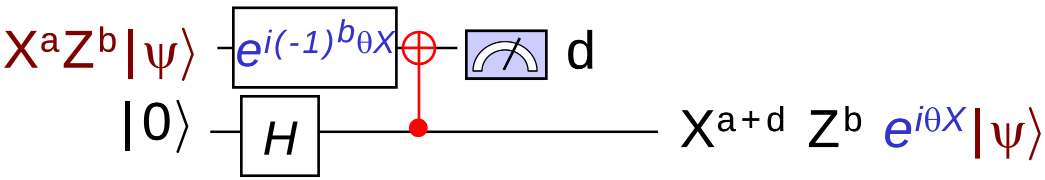
Simulating a Z rotation $e^{i\theta Z}$



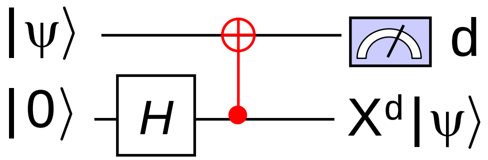
X-Telep (XT)



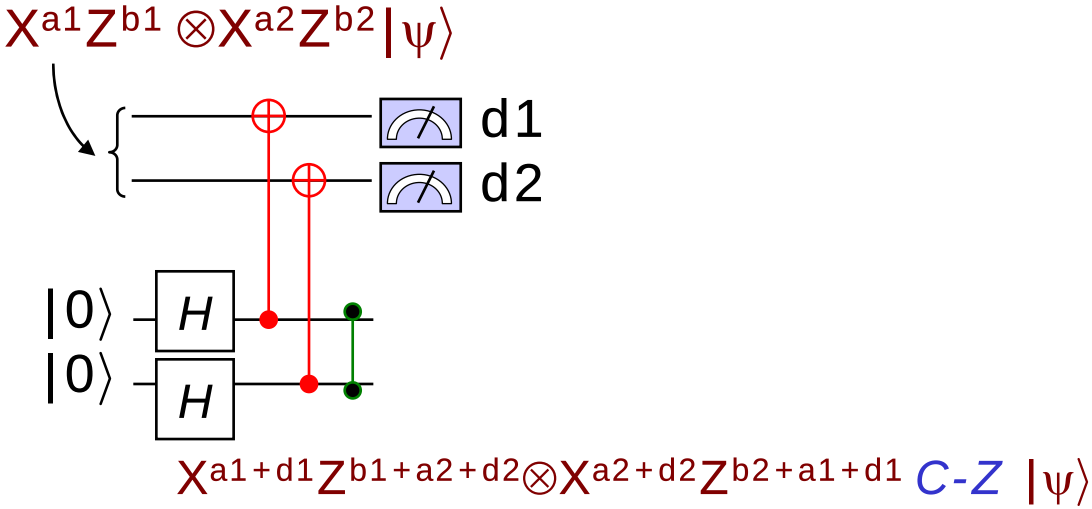
Simulating an X rotation $e^{i\theta X}$



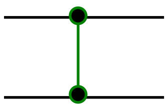
X-Telep (XT)



Simulating a C-Z



C-Z:

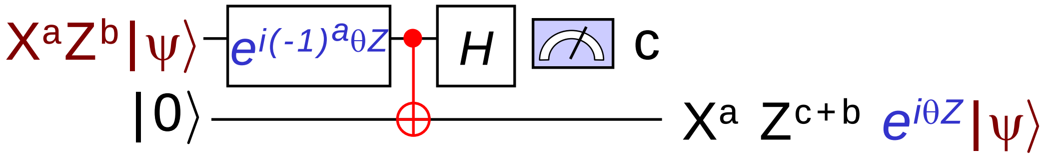


$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

From simulation with mixed resources to TQC --

QC by 1&2-qubit projective measurements only

Simulating a Z rotation $e^{i\theta Z}$



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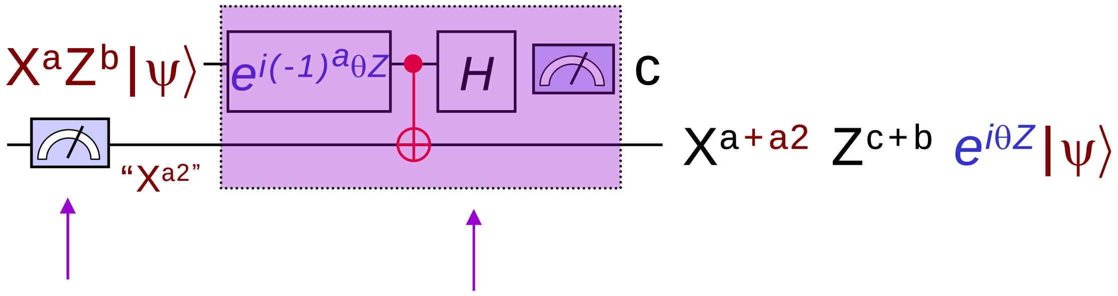
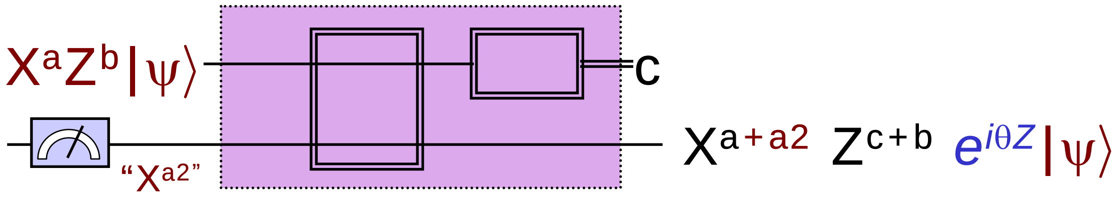


$|0\rangle$ up to X^{a2}

An incomplete 2-qubit measurement, followed by a complete measurement on the 1st qubit .

Simulating a Z rotation $e^{i\theta Z}$



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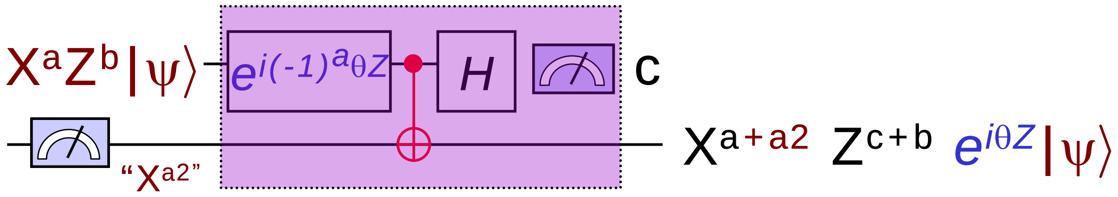
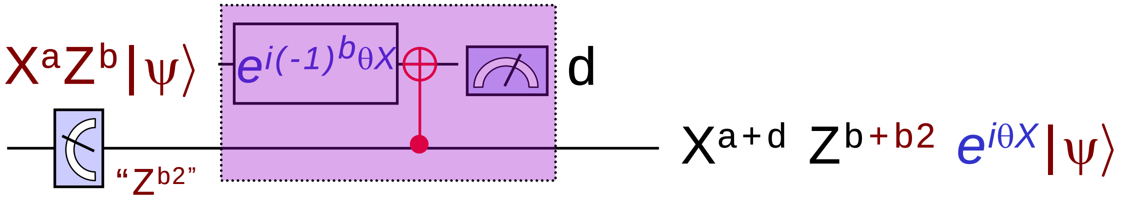
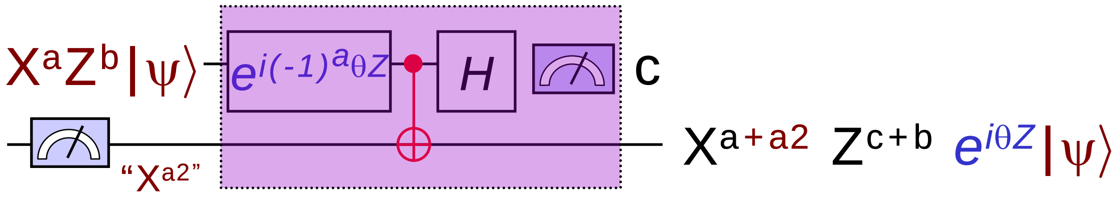


Simulating an X rotation $e^{i\theta X}$



Simulating a Z rotation $e^{i\theta Z}$



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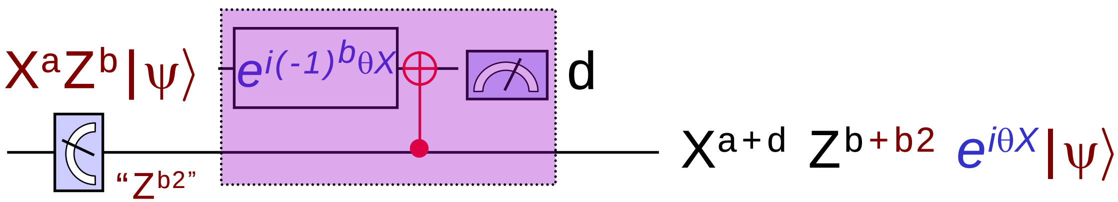
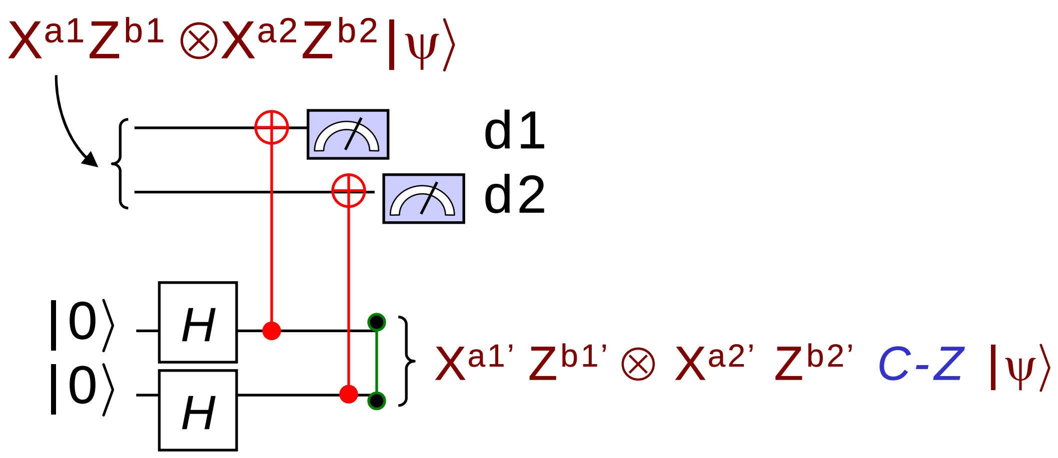
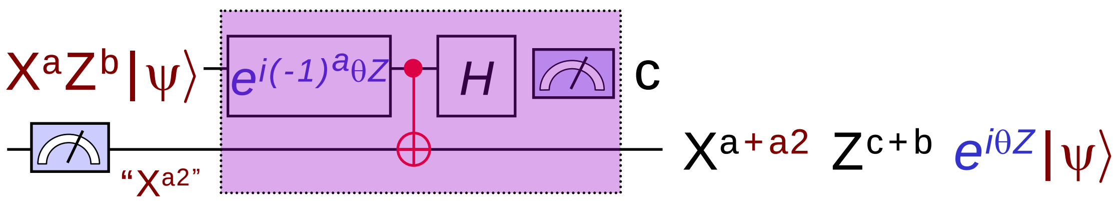


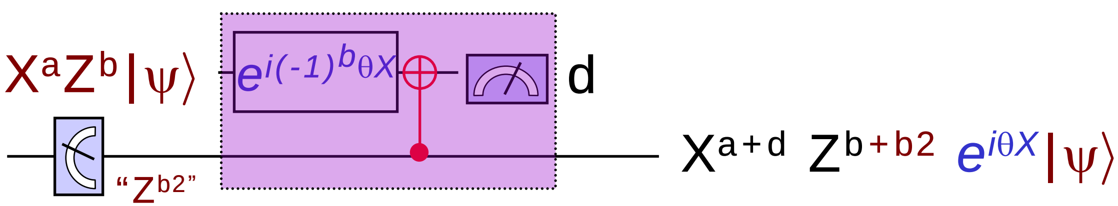
Simulating a C-Z



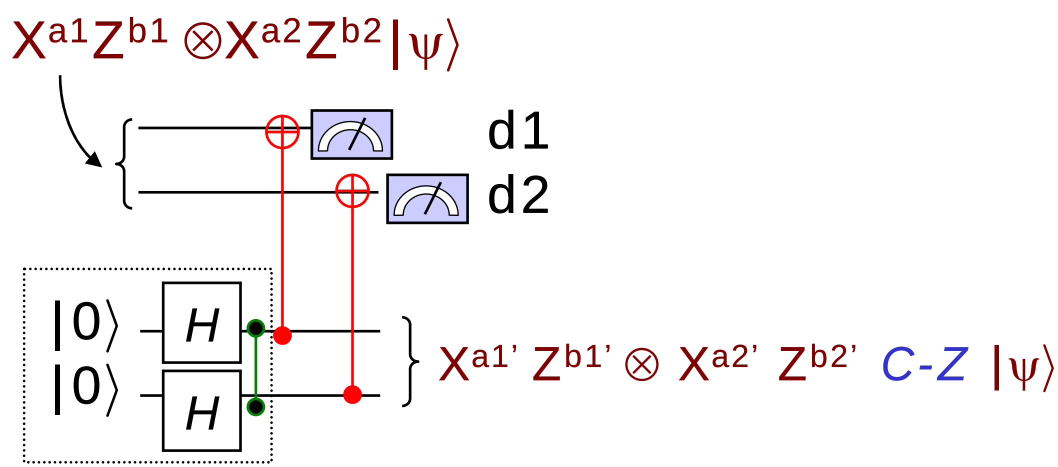
Simulating a Z rotation $e^{i\theta Z}$



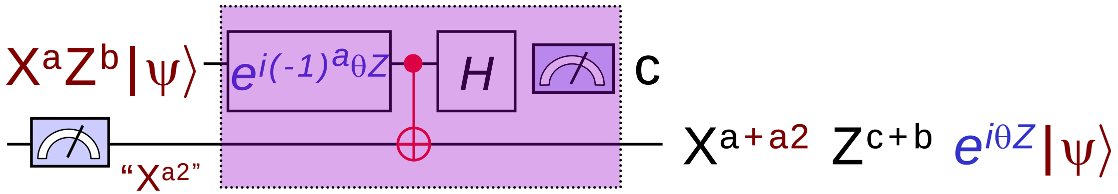
Simulating an X rotation $e^{i\theta X}$



Simulating a C-Z

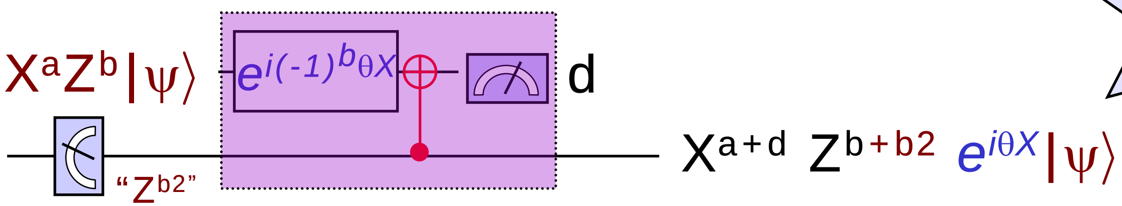


Simulating a Z rotation $e^{i\theta Z}$

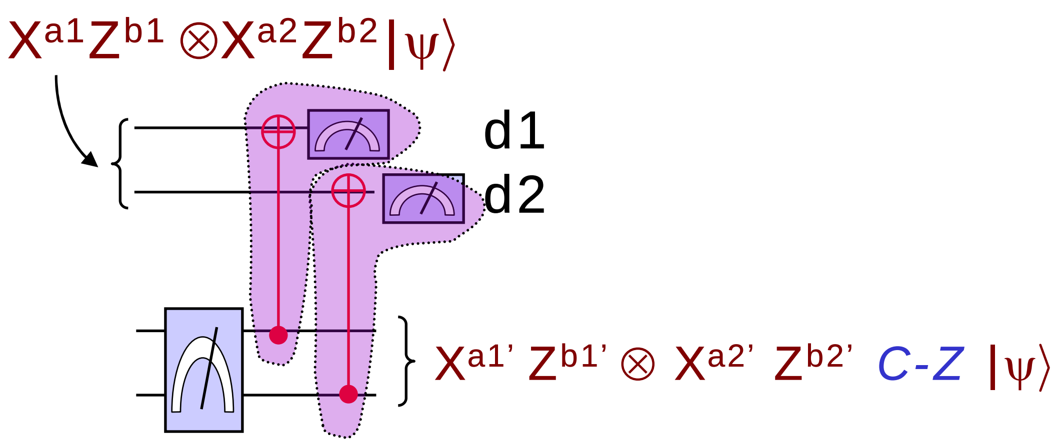


Complete recipe for TQC based on 1-bit teleportation

Simulating an X rotation $e^{i\theta X}$



Simulating a C-Z



See more improvements in quant-ph/0404132

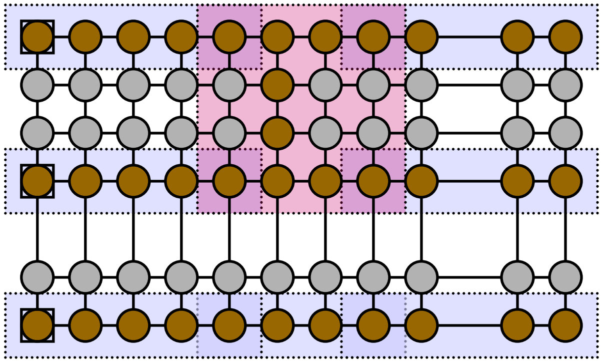
Punchline :

2m 2-qubit & 2m+n 1-qubit measurements
for a circuit of n qubits with
m C-Zs & arbitrary 1-qubit gates

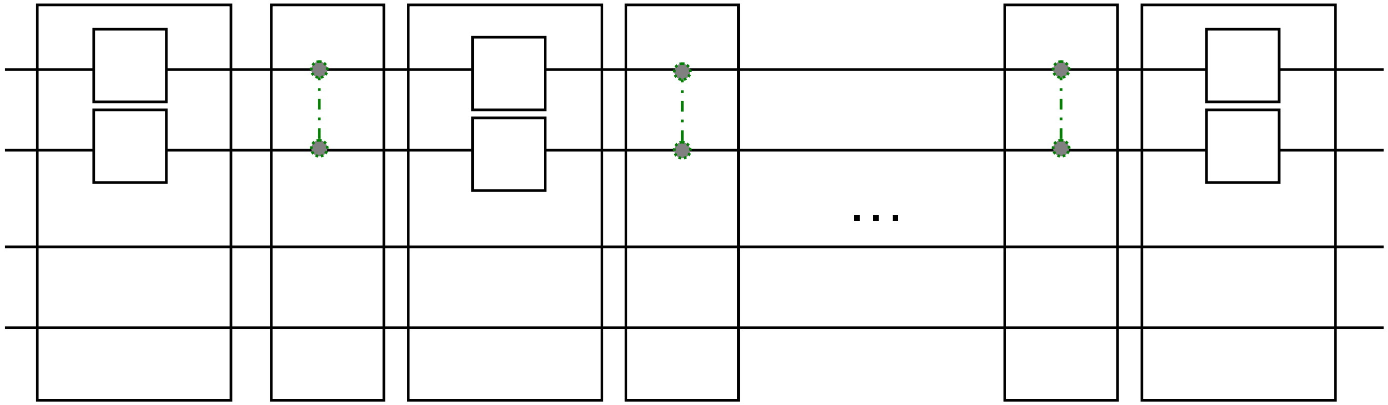
Deriving 1WQC-like schemes using gate simulations obtained from 1-bit teleportation

1WQC:

- Universal entangled initial state
- Feedforward 1-qubit measurement

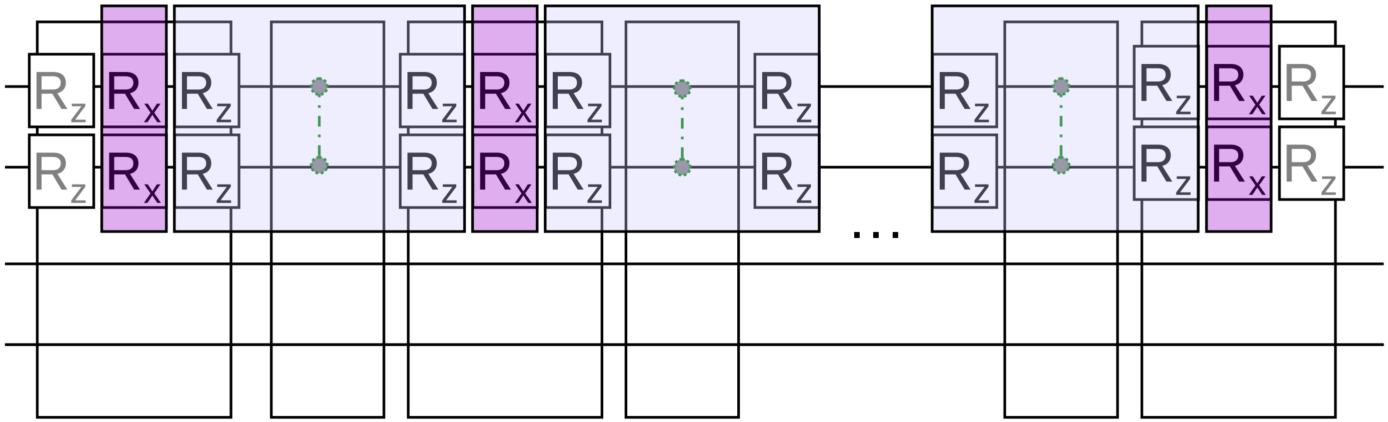


General circuit:



Alternating: (1) 1-qubit gates (2) nearest neighbor optional C-Z

General circuit:



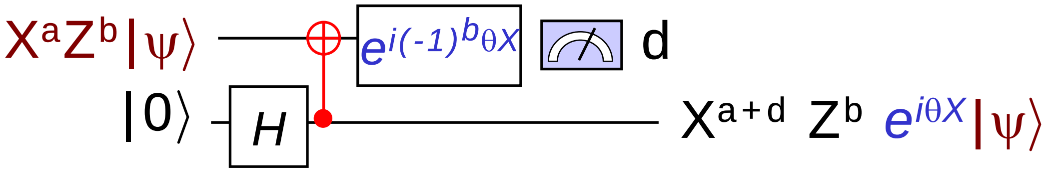
Alternating: (1) 1-qubit gates (2) nearest neighbor optional C-Z

Euler-angle
decomposition

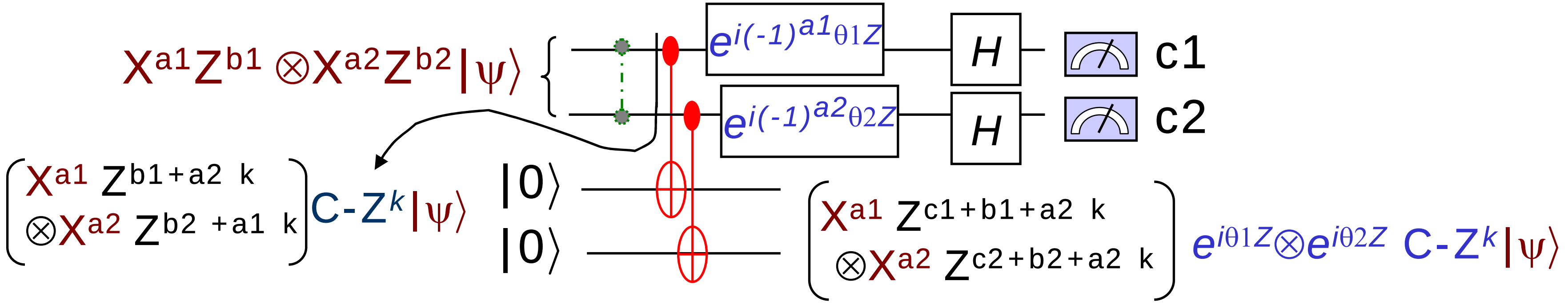
Z rotations + optional C-Z – X rotations –
Z rotations + optional C-Z – X rotations – ....

simulate these 2 things

Simulating an X rotation $e^{i\theta X}$

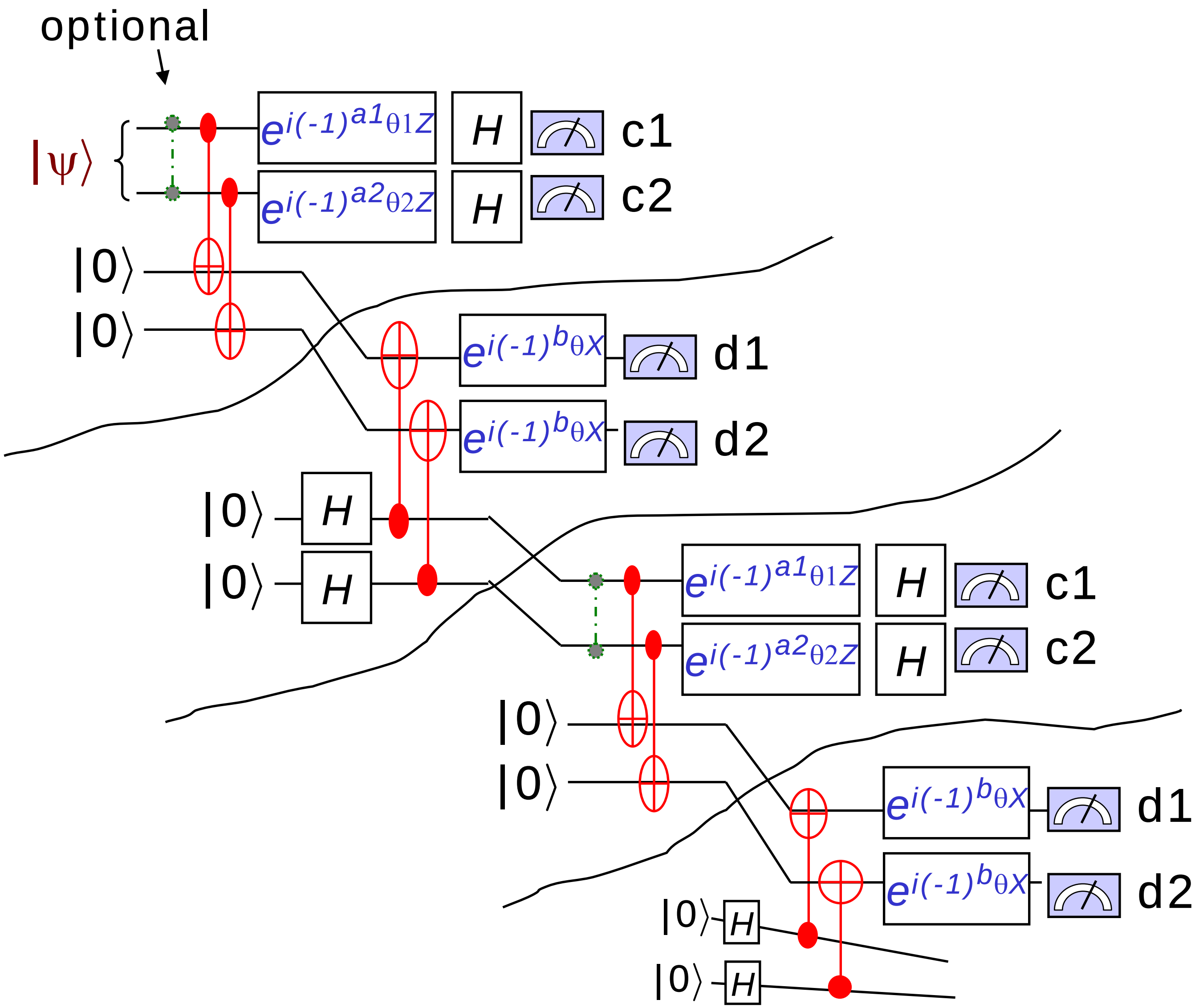


Adding an *optional* C-Z right before Z rotations



Chaining up

C-Z+Z rotations --- X rotations --- C-Z+Z rotations --- X rotations ...



Chaining up

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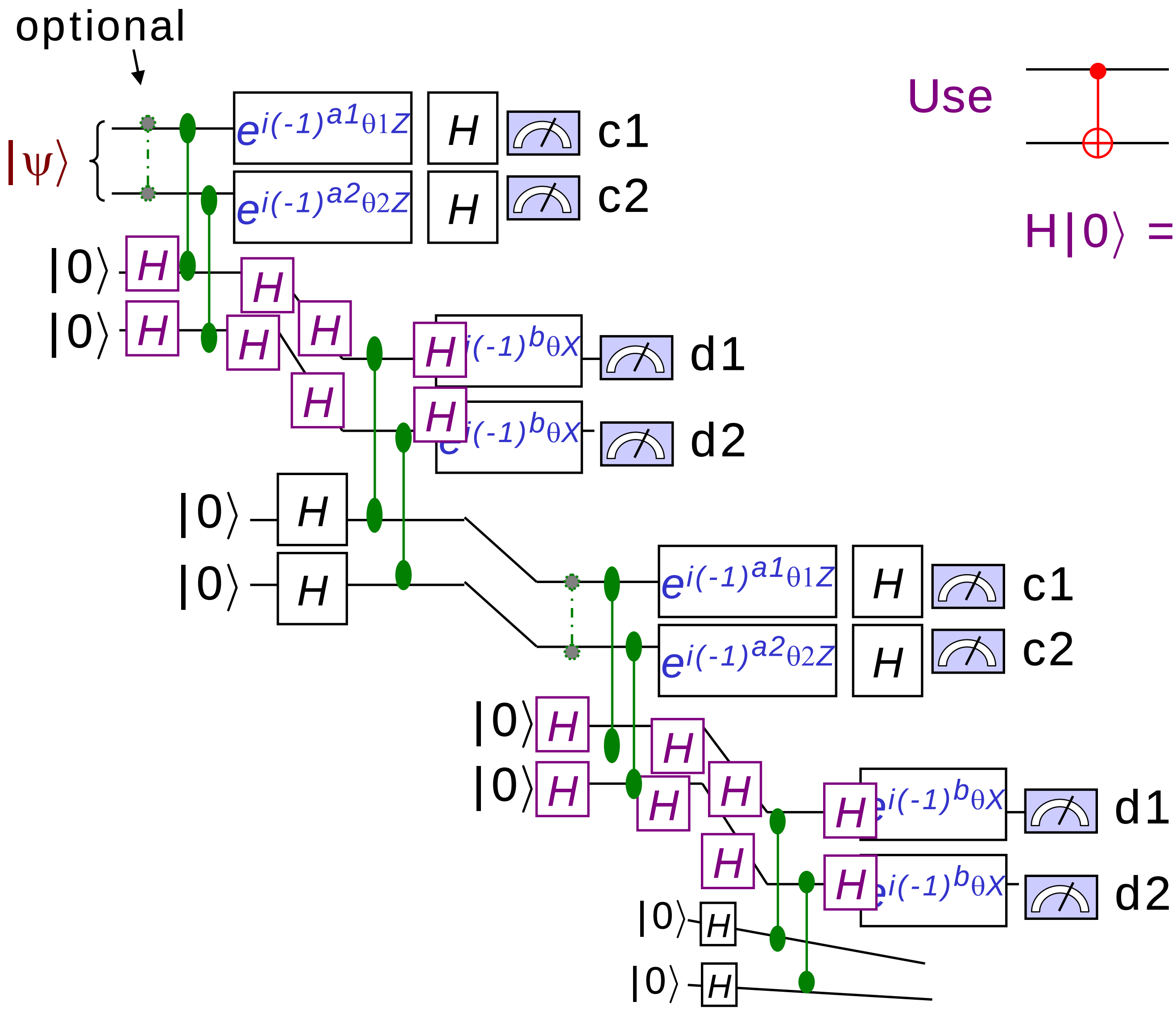
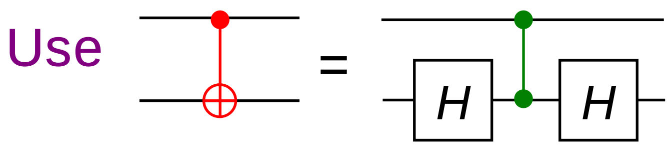
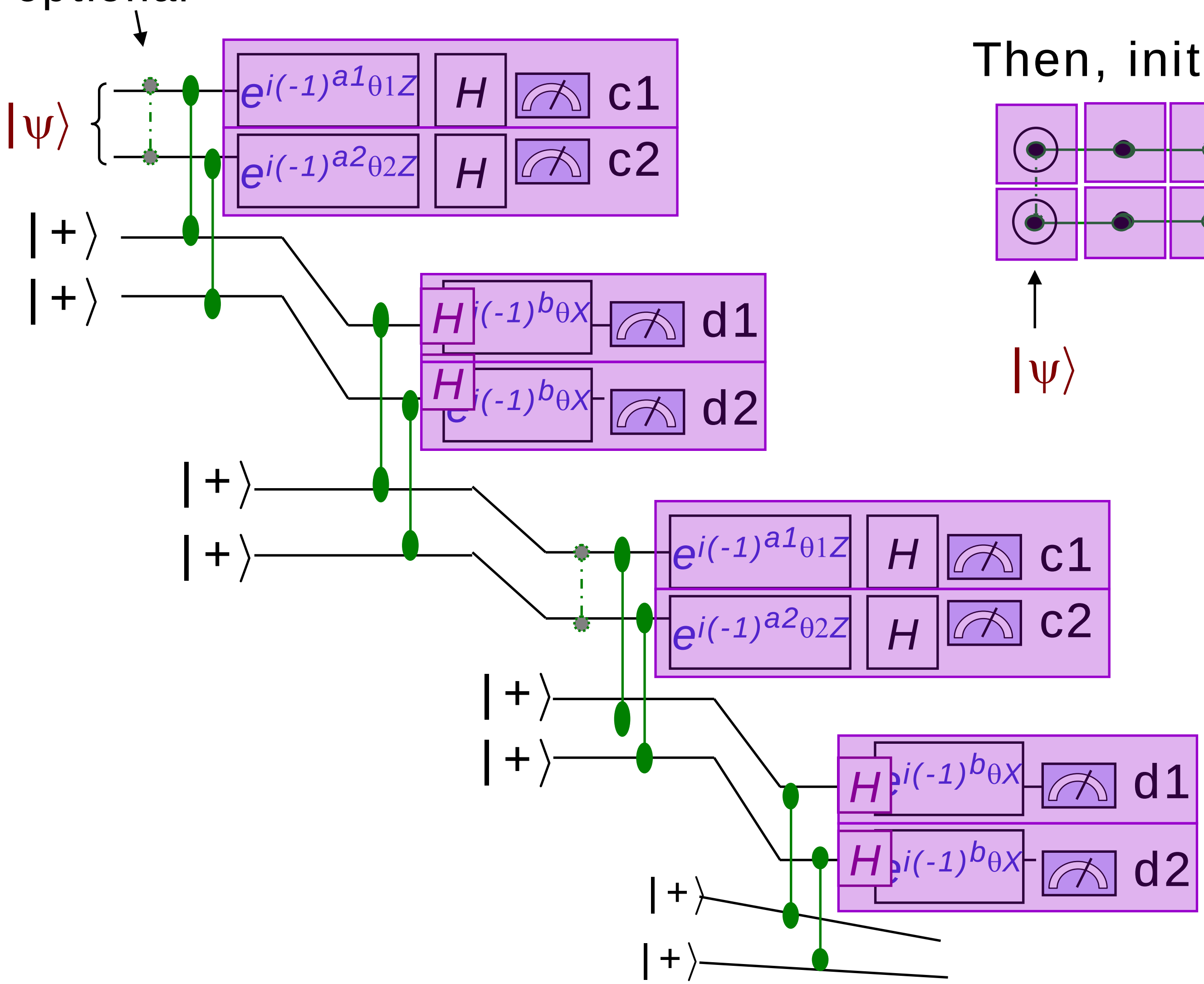




$H|0\rangle = |+\rangle$, $HH=I$

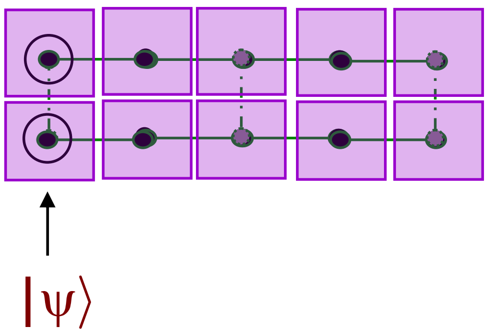
Chaining up

C-Z+Z rotations –-- X rotations –-- C-Z+Z rotations –-- X rotations ...



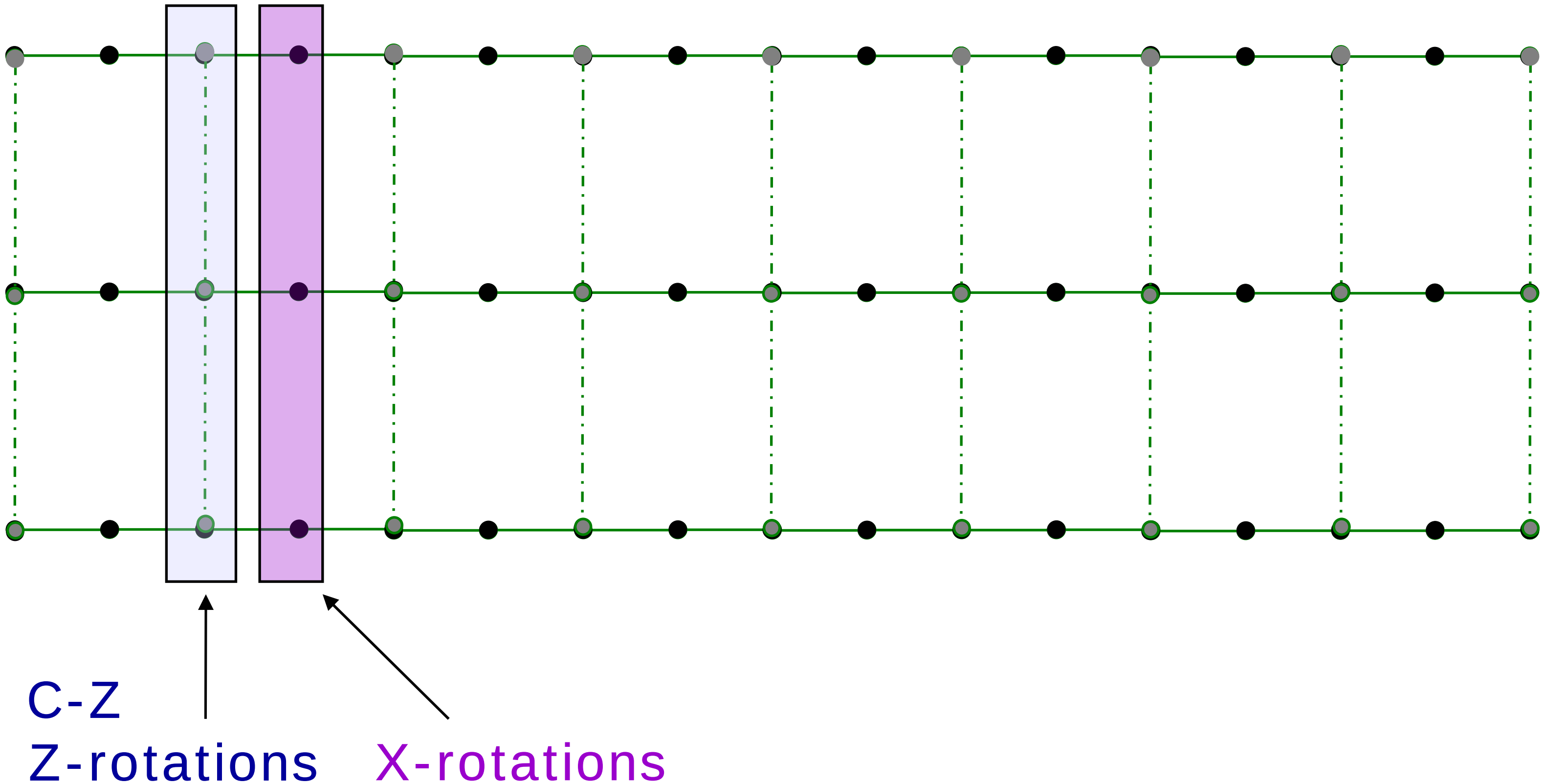
Let • = |+⟩ ,

Then, initial state =



Circuit dependent initial state:

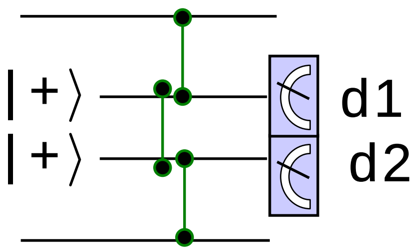
3 qubits, 8 cycles of C-Z + 1-qubit rotations

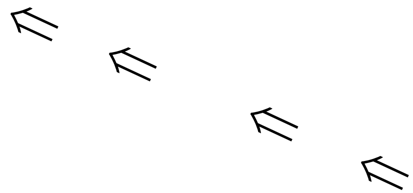


Simulating an optional C-Z: 

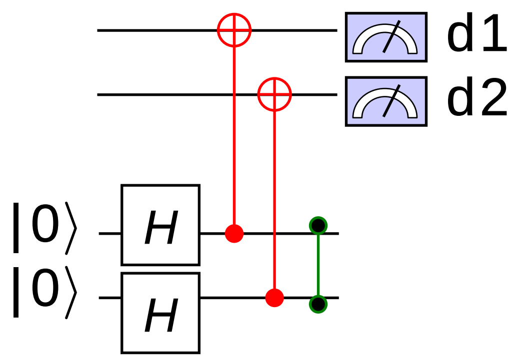
To do the C-Z:

turns out equiv to Gottesman's remote-CNOT



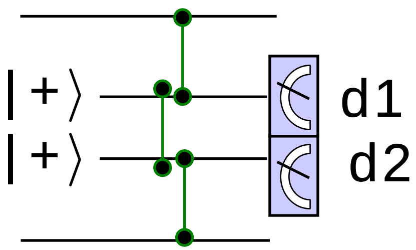


Simulating a C-Z

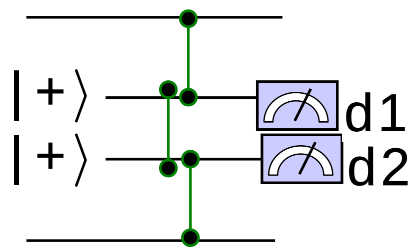


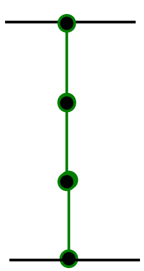
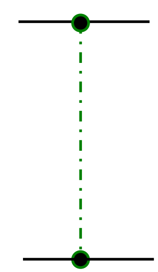
Simulating an optional C-Z: 

To do the C-Z:



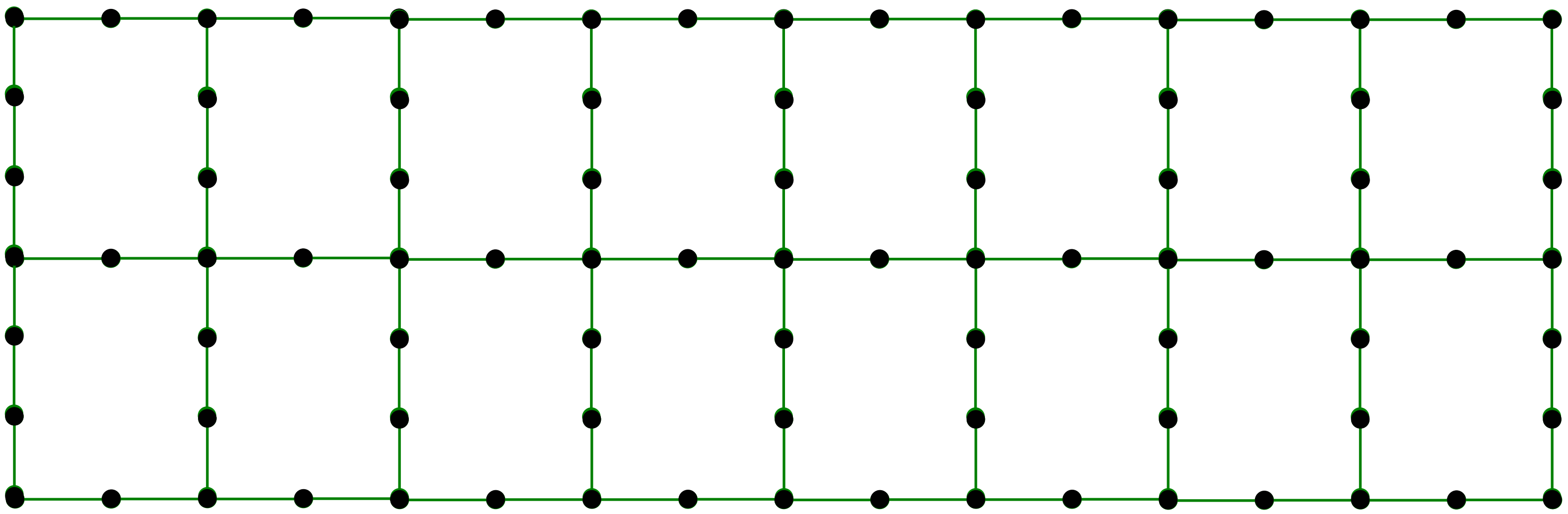
To skip the C-Z:



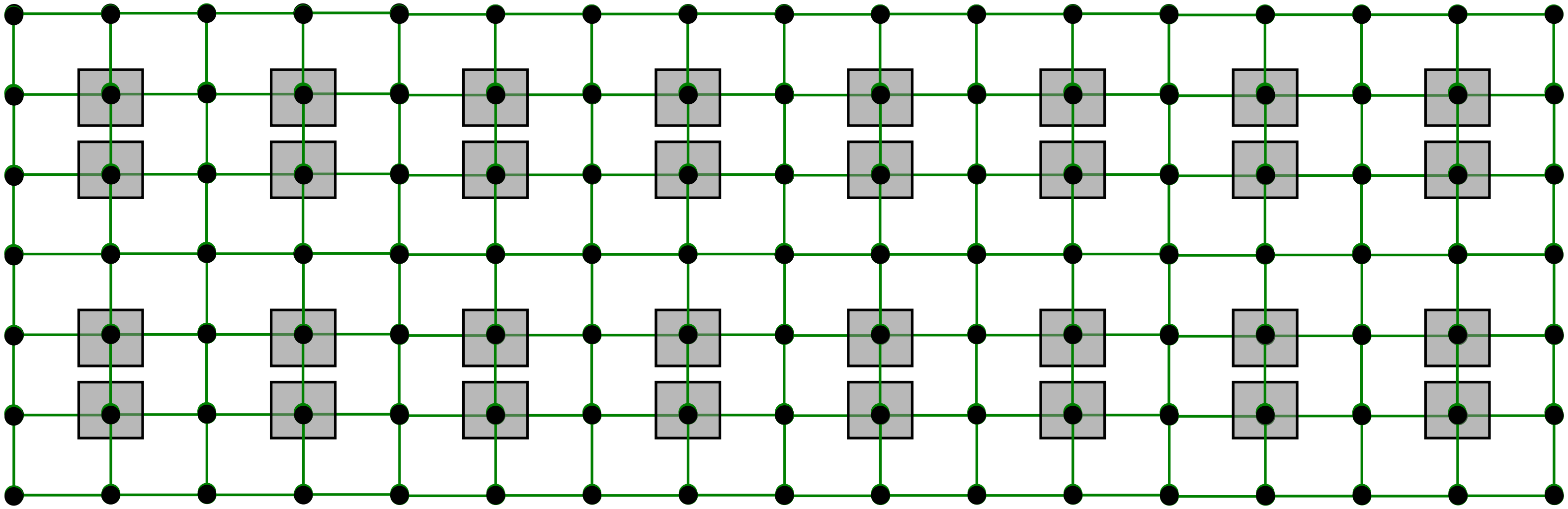
Thus:  gives the ability for simulating 

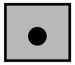
Universal Initial state

3 qubits, 8 cycles



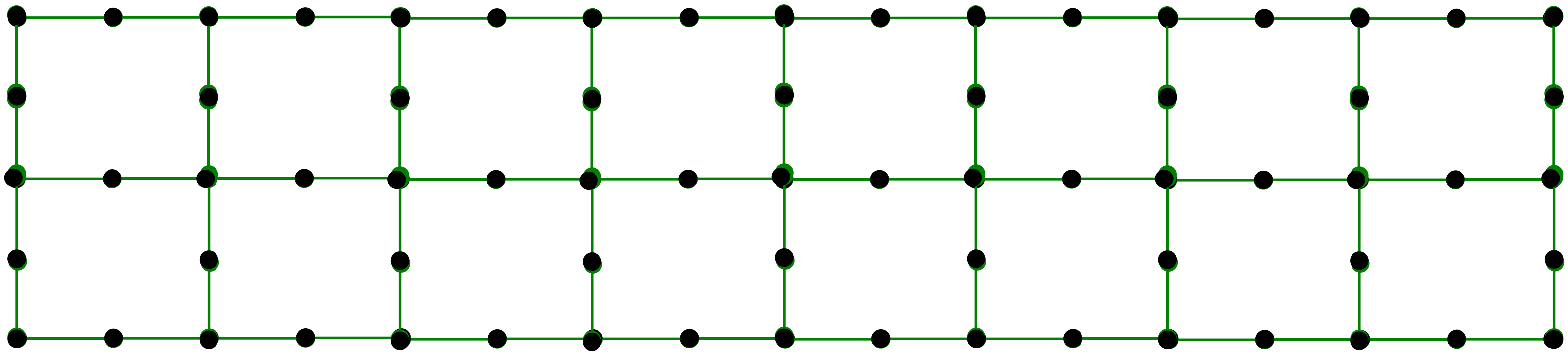
Starting from the cluster state



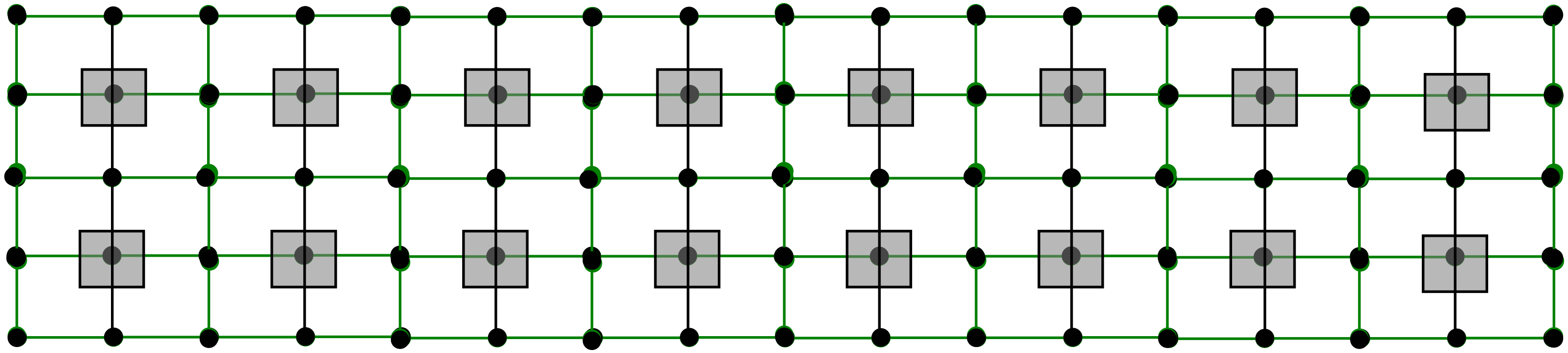


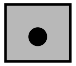
measure in Z basis

Universal Initial state 3 qubits, 8 cycles



Starting from the cluster state





measure in Z basis

Summary:

Unified derivations, using 1-bit teleportation,

for 1WQC & TQC + simplifications

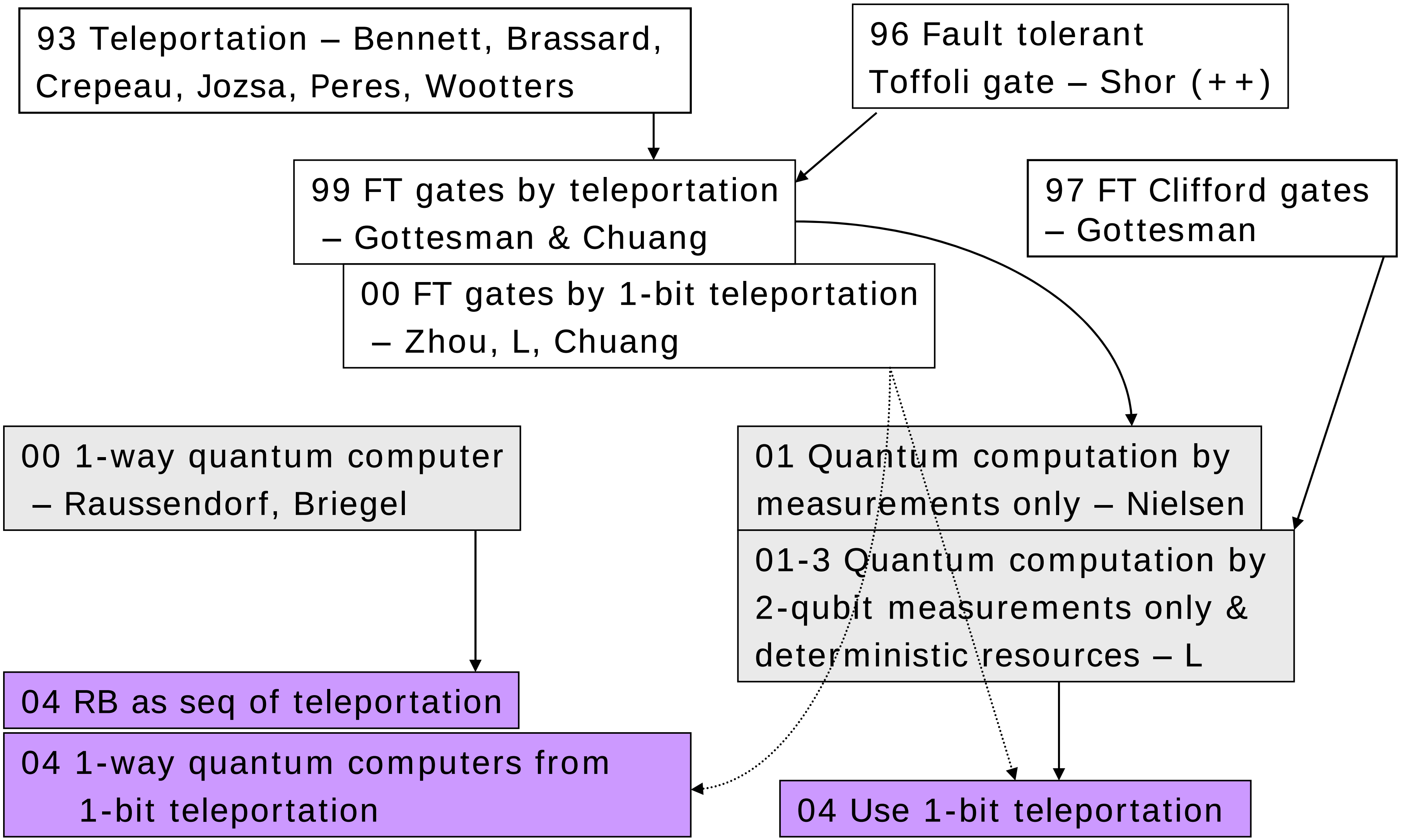
Details in quant-ph/0404082,0404132

...

but perhaps you don't need to see them, you only need to remember what is a simulation (milk), what 1-bit teleportation does (strawberry), and the rest (mix/freeze) comes naturally.

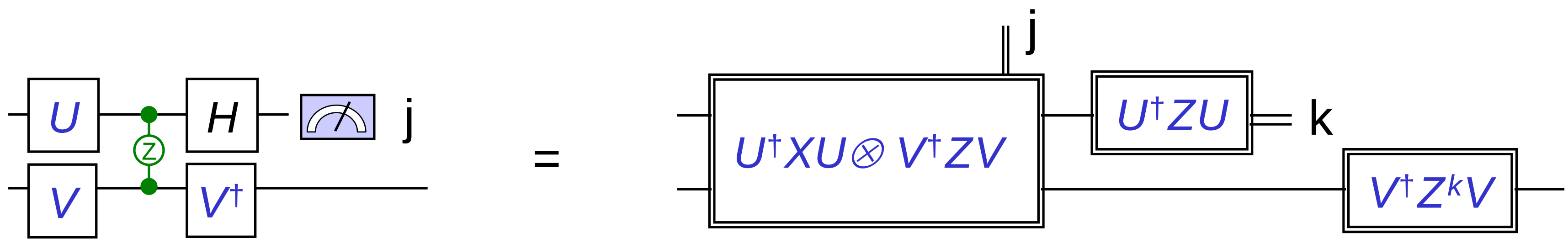
Related results by Perdrix & Jorrand, Cirac & Verstraete.

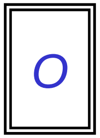
Simple measurements can be universal:



NB. Related results by Perdrix & Jorrand, Cirac & Verstraete

A little fact:



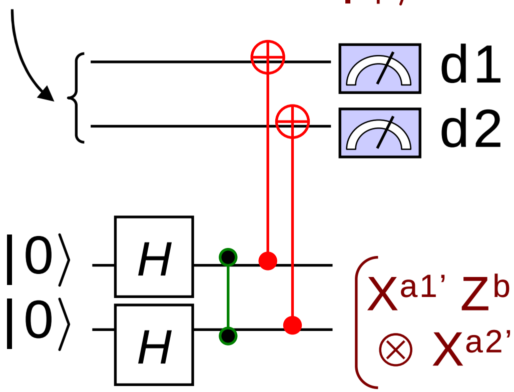


= measurement of operator O

Simulating an optional C-Z

Recall : simulating a C-Z

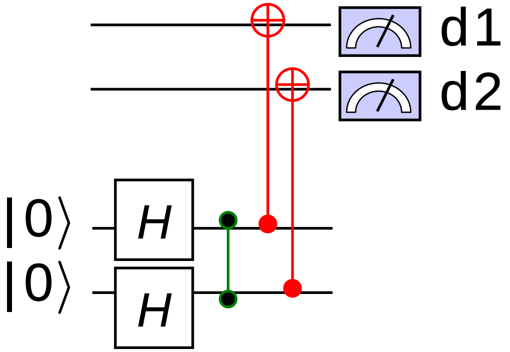
$X^{a1}Z^{b1} \otimes X^{a2}Z^{b2}|\psi\rangle$



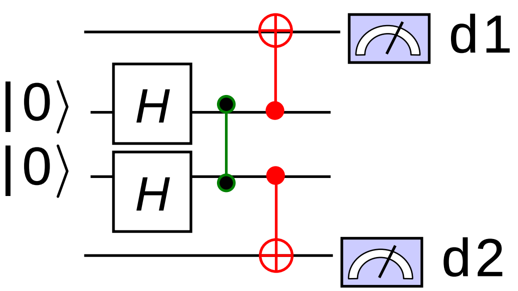
$\begin{pmatrix} X^{a1'} Z^{b1'} \\ \otimes X^{a2'} Z^{b2'} \end{pmatrix}$ *C-Z* $|\psi\rangle$

Simulating an optional C-Z

Recall : simulating a C-Z

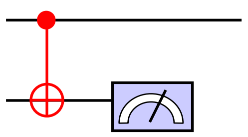


1. Moving the 2nd input to the bottom:

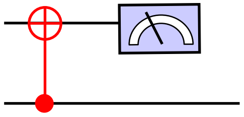


Simulating an optional C-Z

2. Use symmetry:



Just measures the parity of the 2 qubits

Up to reordering of the output bit, it is equal to 

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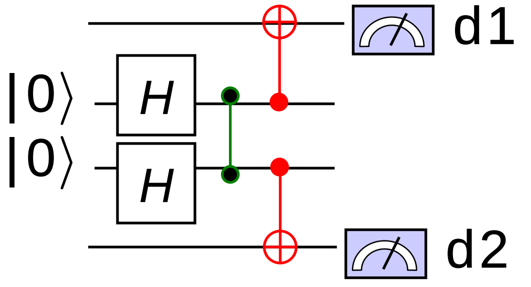

Simulating an optional C-Z

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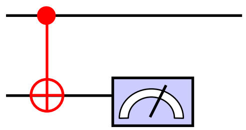
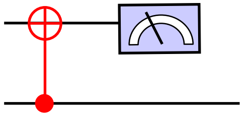
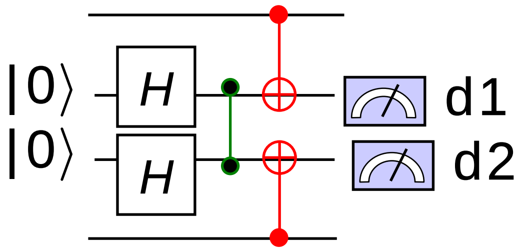


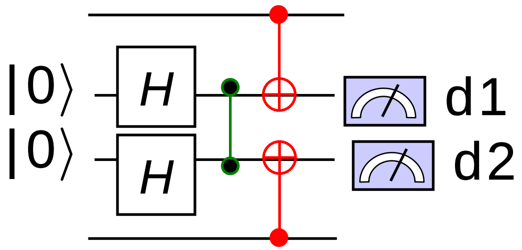
Just measures the parity of the 2 qubits

Up to reordering of the output bit, it is equal to



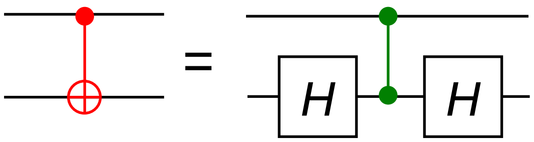


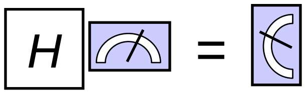
Simulating an optional C-Z

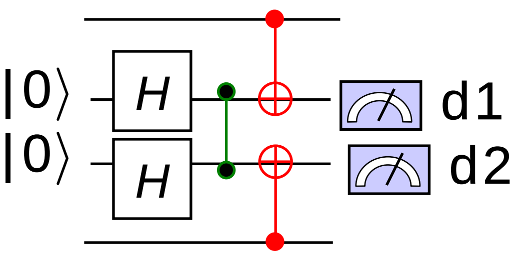


Simulating an optional C-Z

3. Use







Simulating an optional C-Z

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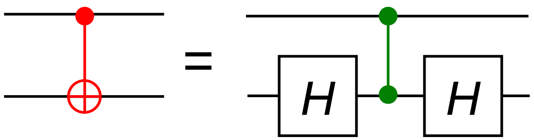
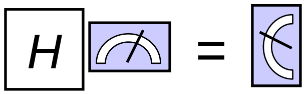
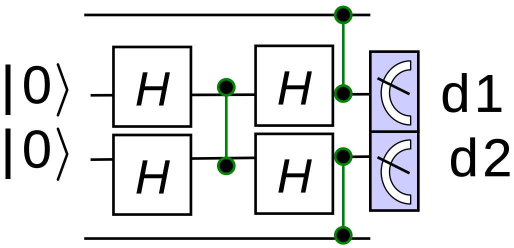


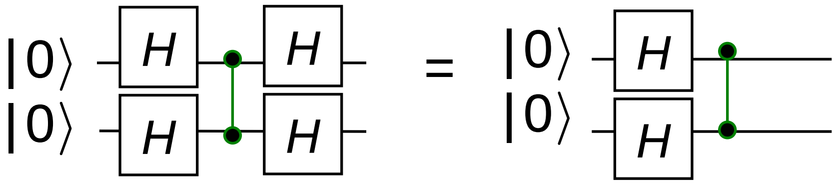


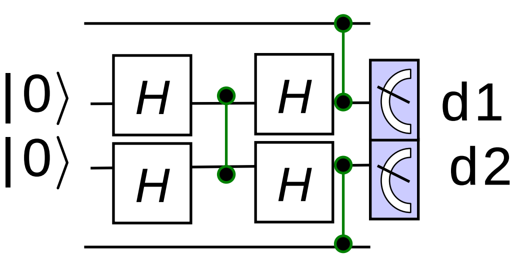


Simulating an optional C-Z

3. Use H|0⟩ =|+⟩,

and

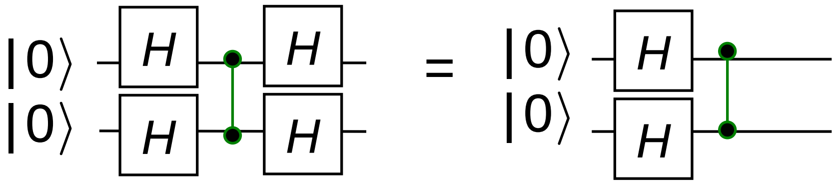


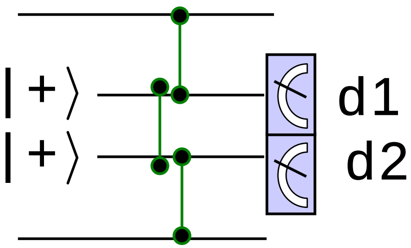


Simulating an optional C-Z

3. Use $H|0\rangle = |+\rangle$,

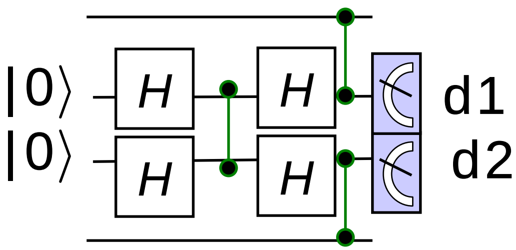
and



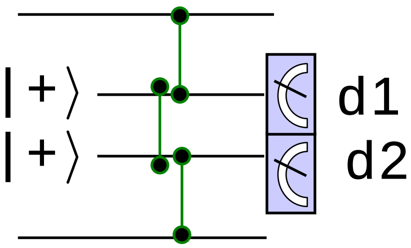


"Remote C-Z" :

Cousin of the remote CNOT by Gottesman98



Simulating an optional C-Z



"Remote C-Z" :

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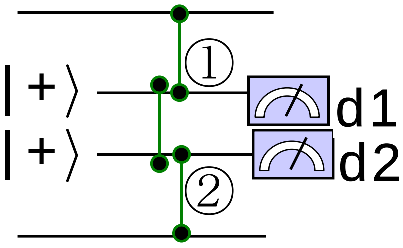
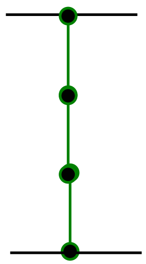


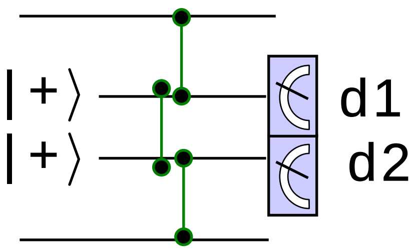
If one measures along $\{|0\rangle, |1\rangle\}$, the C-Zs labeled by ①② only acts like $Z^{d1} \otimes Z^{d2}$ – simulating identity instead!

Simulating an optional C-Z, summary:

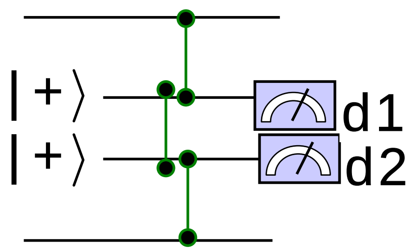


simulates

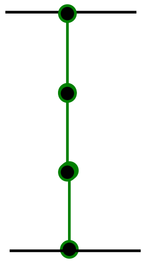
To do the C-Z:



To skip the C-Z:

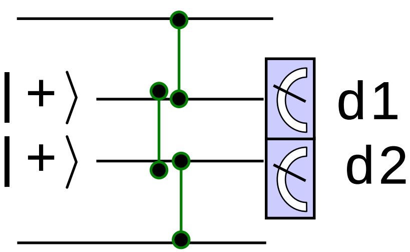


Simulating an optional C-Z, summary:

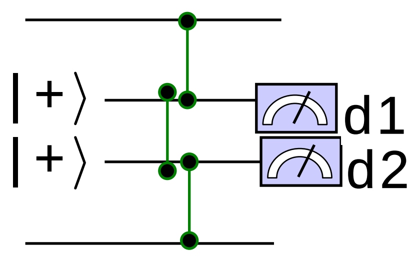


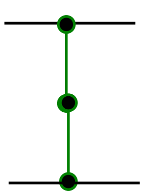
simulates

To do the C-Z:



To skip the C-Z:

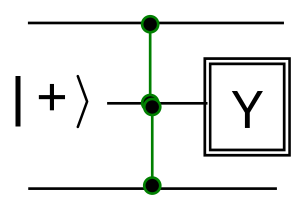




also simulates

up to Z-rotations

Do:



Skip: