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Decoherence and Noise Control in Strongly Driven Superconducting Quantum Bits

Anatoly Yu. Smirnov

D-Wave Systems Inc., Vancouver, British Columbia

Collaboration:

M. Grajcar
E. Il'ichev
A. Izmalkov
(IPHT, Jena, Germany)

M.H.S. Amin
A. Maassen van den Brink
A. Zagoskin
(D-Wave Systems Inc.)

Outline:

1. Quantum Brownian motion of a driven two-state system.
2. Rabi oscillations and decoherence suppression in a superconducting flux qubit.
3. Rabi spectroscopy and noise manipulation.
4. Conclusions.

Qubit { $\sigma_x,\sigma_y,\sigma_z$ }+ Heat bath Q + Driving force F(t)=$F_0 \cos \omega_0 t$

Hamiltonian and Heisenberg equations:

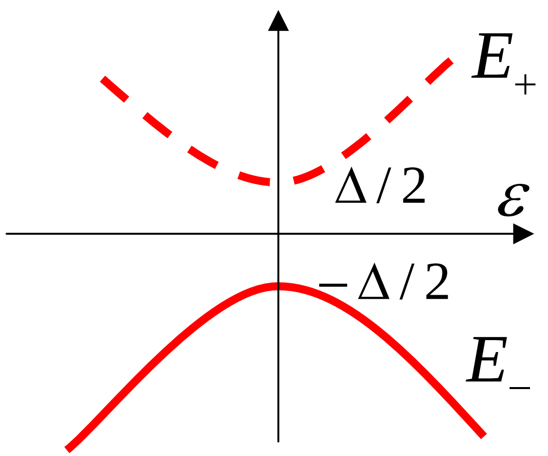
$$E_{\pm} = \pm \frac{\omega_0}{2}, \omega_0 = \sqrt{\Delta^2 + \varepsilon^2}.$$

$$H = \frac{\Delta}{2}\sigma_x + \frac{\varepsilon}{2}\sigma_z - \sigma_z Q - \sigma_z F_0 \cos \omega_0 t + H_B.$$

$$\dot{\sigma}_x = -\varepsilon\sigma_y + 2(Q + F_0 \cos \omega_0 t)\sigma_y,$$

$$\dot{\sigma}_y = -\Delta\sigma_z + \varepsilon\sigma_x - 2(Q + F_0 \cos \omega_0 t)\sigma_x,$$

$$\dot{\sigma}_z = \Delta\sigma_y.$$



$$X = (\Delta/\omega_0)\sigma_x + (\varepsilon/\omega_0)\sigma_z.$$

Heat bath:

$$\varphi(t,t_1) = \left\langle i\left[Q^{(0)}(t), Q^{(0)}(t_1)\right]_- \right\rangle \theta(t - t_1) \Leftrightarrow \chi(\omega), \chi''(\omega) = A\omega^s e^{-|\omega|/\omega_c},$$

$$M(t,t_1) = \left\langle \frac{1}{2}\left[Q^{(0)}(t), Q^{(0)}(t_1)\right]_+ \right\rangle \Leftrightarrow S(\omega) = \chi''(\omega)\coth\left(\frac{\omega}{2T}\right).$$

Non-Markovian Heisenberg-Langevin equations

$$\dot{\sigma}_x + \varepsilon\sigma_y - 2F(t)\sigma_y = \xi_x + 2\int dt_1 \left\{ M(t,t_1)\frac{\delta\sigma_y(t)}{\delta Q(t_1)} + \varphi(t,t_1)\frac{1}{2}\left[\sigma_y(t),\sigma_z(t_1)\right]_+ \right\},$$

$$\dot{\sigma}_y + \Delta\sigma_z - \varepsilon\sigma_x + 2F(t)\sigma_x = \xi_y - 2\int dt_1 \left\{ M(t,t_1)\frac{\delta\sigma_x(t)}{\delta Q(t_1)} + \varphi(t,t_1)\frac{1}{2}\left[\sigma_x(t),\sigma_z(t_1)\right]_+ \right\},$$

$$\dot{\sigma}_z = \Delta\sigma_y.$$

Fluctuation forces: $\langle \xi_x \rangle = \langle \xi_y \rangle = 0$

$$\xi_x = \left[Q^{(0)}(t),\sigma_y(t)\right]_+ - 2\int dt_1 M(t,t_1)\frac{\delta\sigma_y(t)}{\delta Q(t_1)},$$

$$\xi_y = -\left[Q^{(0)}(t),\sigma_x(t)\right]_+ + 2\int dt_1 M(t,t_1)\frac{\delta\sigma_x(t)}{\delta Q(t_1)}.$$

G.F. Efremov, A.Yu. Smirnov, Sov.Phys. JETP 53, 547(1981)

Qubit with heat bath (no driving force) :

Population difference: $\langle X(t) \rangle = X(0) + X^0(1 - e^{-t/T_1}), \quad X^0 = -\tanh\left(\frac{\omega_0}{2T}\right)$

Evolution of z-polarization:

$$\langle \sigma_z(t) \rangle = \sigma_z^0 + \left(\frac{\varepsilon^2}{\omega_0^2} e^{-t/T_1} + \frac{\Delta^2}{\omega_0^2} e^{-t/T_2} \cos\omega_0 t \right) \left[\sigma_z(0) - \sigma_z^0 \right] +$$

$$\frac{\varepsilon\Delta}{\omega_0^2} \left(e^{-t/T_1} - e^{-t/T_2} \cos\omega_0 t \right) \left[\sigma_x(0) - \sigma_x^0 \right], \qquad \sigma_x^0 = \frac{\Delta}{\omega_0} X^0, \sigma_z^0 = \frac{\varepsilon}{\omega_0} X^0$$

Equilibrium *relaxation* and *dephasing* rates **(**$\omega_0 = \sqrt{\Delta^2 + \varepsilon^2}$**):**

$$T_{1,eq}^{-1} = 2\frac{\Delta^2}{\omega_0^2} S(\omega_0), T_{2,eq}^{-1} = \frac{\Delta^2}{\omega_0^2} S(\omega_0) + 2\frac{\varepsilon^2}{\omega_0^2} S(\omega)_{|\omega=0},$$

Qubit with heat bath and driving force.

Exact resonance +*Weak* qubit-bath coupling + Rotating Wave Approximation :

$$\omega_0 = E_+ - E_- = \sqrt{\Delta^2 + \varepsilon^2}, \Gamma << \omega_0.$$

Rabi oscillations of the excited level population $P_{Exc}(t)$: $P_{Exc}(0) = 0,$

$$P_{Exc}(t) = \frac{1 + \langle X(t) \rangle}{2} = \frac{1}{2}\left(1 - e^{-t/T_1} \cos \Omega_R t\right) \Rightarrow \frac{1}{2},$$

with the frequency: $$\Omega_R = \frac{\Delta}{\sqrt{\Delta^2 + \varepsilon^2}} F_0 >> T_1^{-1}.$$

and the damping rate

$$T_1^{-1} = \frac{\Delta^2}{\omega_0^2} S(\omega_0) + \frac{\Delta^2}{2\omega_0^2} \frac{S(\omega_0 + \Omega_R) + S(\omega_0 - \Omega_R)}{2} + \frac{\varepsilon^2}{\omega_0^2} S(\Omega_R).$$

Rabi oscillations of the "dipole moment" (at zero bias):

$$\langle \sigma_z(t) \rangle = e^{-t/T_1} \sin \Omega_R t \sin \omega_0 t + Z_0 \left(1 - e^{-\Gamma_z t}\right) \cos \omega_0 t$$

with the steady-state z-polarization: $\langle \sigma_z(t) \rangle = Z_0 \cos \omega_0 t,$

$$Z_0 = \frac{\chi''(\omega_0 + \Omega_R) - \chi''(\omega_0 - \Omega_R)}{S(\omega_0 + \Omega_R) + S(\omega_0 - \Omega_R)},$$

and the additional decoherence rate (non-zero bias):

$$\Gamma_z = \frac{\Delta^2}{\omega_0^2} \frac{S(\omega_0 + \Omega_R) + S(\omega_0 - \Omega_R)}{2} + 2\frac{\varepsilon^2}{\omega_0^2} S(\Omega_R).$$

A.Yu. Smirnov,
Phys.Rev. B 67, 155104(2003);
Phys.Rev. B 68, 134514(2003).

For the *strongly driven* qubit ($\Omega_R >> T_1^{-1}$) Rabi oscillations of both *population* and *z-polarization* disappear for the *same* relaxation time T_1:

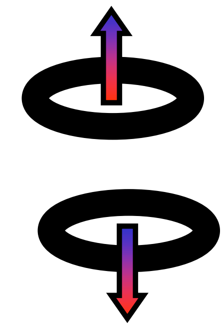
$$T_1^{-1} = \frac{\Delta^2}{\omega_0^2} S(\omega_0) + \frac{\Delta^2}{2\omega_0^2} \frac{S(\omega_0 + \Omega_R) + S(\omega_0 - \Omega_R)}{2} + \frac{\varepsilon^2}{\omega_0^2} S(\Omega_R).$$

Without the driving force:

$$T_{1,eq}^{-1} = 2\frac{\Delta^2}{\omega_0^2} S(\omega_0)$$: defines a timescale for relaxation of population

$$T_{2,eq}^{-1} = \frac{\Delta^2}{\omega_0^2} S(\omega_0) + 2\frac{\varepsilon^2}{\omega_0^2} S(0)$$: defines a dephasing rate (decay of a dipole moment)

Rabi oscillations in a flux superconducting qubit.



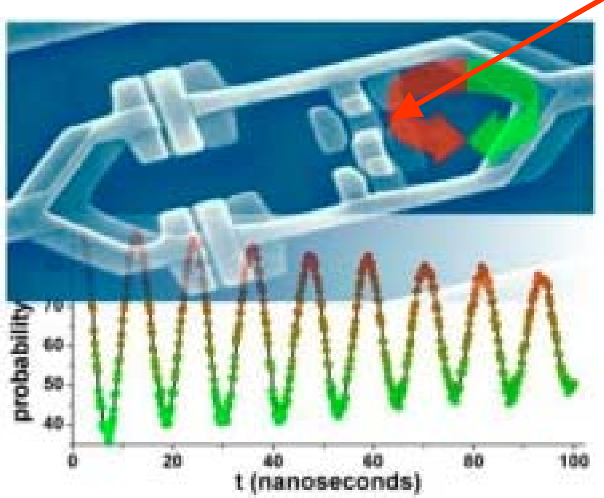
Quantum States:

$|L\rangle$ **= left rotating current**

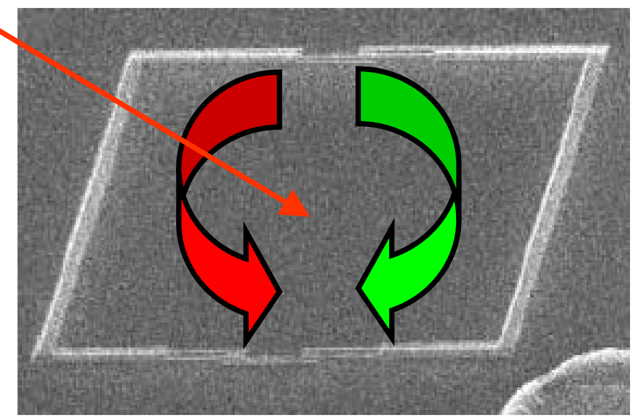
$|R\rangle$ **= right rotating current**

$$I = I_q \, \sigma_z$$

Superconducting Loop: Macroscopic two-level system



I. Chiorescu *et al*.,
Science 299, 1869 (2003);
(Time domain)



E. Il'ichev *et al*.,
Phys. Rev. Lett. 91,097906 (2003).
(Frequency domain)

Bias: $\varepsilon = (I_q / \pi)(\Phi_{ext} - \Phi_0 / 2)$

Rabi oscillations of upper level population.

$$\Omega_R \propto F_0$$

Measurements:

Decay time of Rabi oscillations:

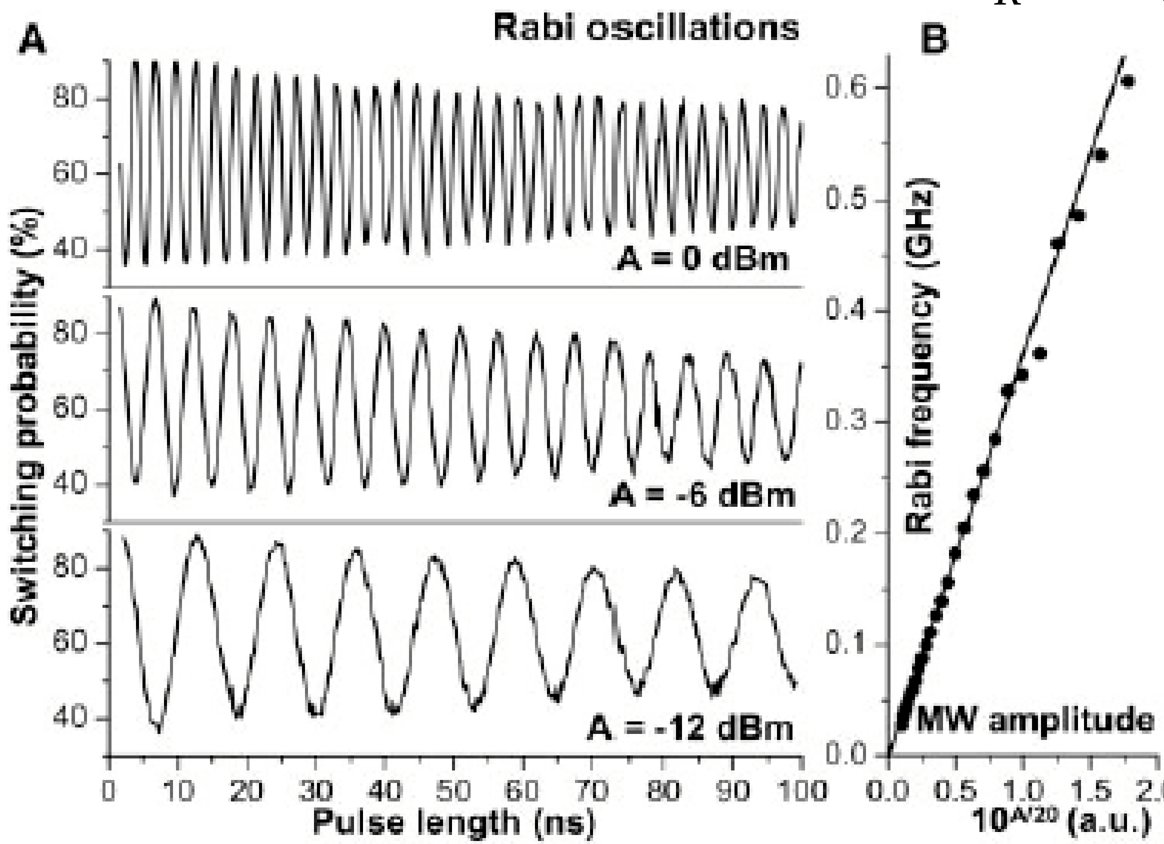
$$T_1 = \tau_{Rabi} = 150ns$$

Relaxation time of undriven qubit:

$$T_{1,eq} = \tau_{relax} = 900ns$$

Dephasing time of undriven qubit:

$$T_{2,eq} = \tau_{\varphi} = 20ns$$



$$\frac{\omega_0}{2\pi} \approx 6.6GHz >> \frac{\Omega_R}{2\pi} \approx 100MHz$$

I. Chiorescu *et al.*, Science 299, 869 (2003);

Decay rate of Rabi oscillations:

$$T_1^{-1} = \frac{3\Delta^2}{2\omega_0^2} S(\omega_0) + \frac{\varepsilon^2}{\omega_0^2} S(\Omega_R) = \frac{10^9}{150} s^{-1},$$

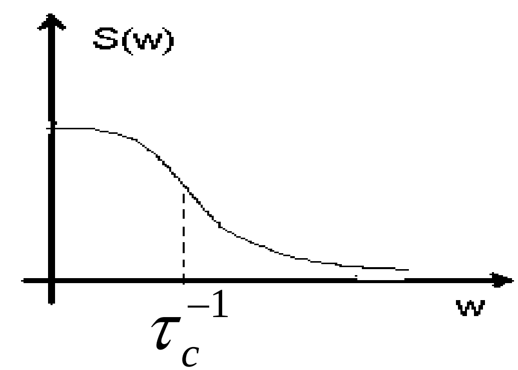
Relaxation rate of undriven qubit:

$$T_{1,eq}^{-1} = 2\frac{\Delta^2}{\omega_0^2} S(\omega_0) = \frac{10^9}{900} s^{-1},$$

Dephasing time of undriven qubit:

$$T_{2,eq}^{-1} = \frac{1}{2} T_{1,eq}^{-1} + 2\frac{\varepsilon^2}{\omega_0^2} S(\omega \cong 0) = \frac{10^9}{20} s^{-1}$$

$$\frac{S(\omega \cong 0)}{S(\Omega_R)} = \frac{24.72}{5.83} = 4.24.$$



Frequency dispersion of the heat bath spectrum

$$\tau_c^{-1} \le \Omega_R / 2\pi \cong 100MHz$$

For the flat spectrum, $S(\Omega_R) \approx S(0)$, **it should be:**

$$T_{1,\,flat} = 39.5ns = T_1 / 3.8$$

Difference between T_1 and $T_{1,flat}$ points to the *decoherence suppression* in 3.8 times by external driving field

Much higher suppression of decoherence by the high-frequency field -

Spin 1/2 irradiated by circularly polarized light (rotating magnetic field):

$$H = \frac{\Delta}{2}\left(\sigma_x \cos\omega_0 t + \sigma_y \sin\omega_0 t\right) - \vec{\sigma}\cdot\vec{Q} + H_B$$

Relaxation rate at $\Delta << \omega_0$

$$T_1^{-1} = 4\chi''\left(\frac{\Delta^2}{2\omega_0}\right)\coth\left(\frac{\Delta^2}{4\omega_0 T}\right) << 4\chi''(\Delta)\coth\left(\frac{\Delta}{2T}\right) = T_{1,eq}^{-1}$$

$$\chi''(\omega) \approx A\omega^s e^{-|\omega|/\omega_c}, s \geq 1$$

A.Yu. Smirnov, Phys.Rev.B 60, 3040 (1999)

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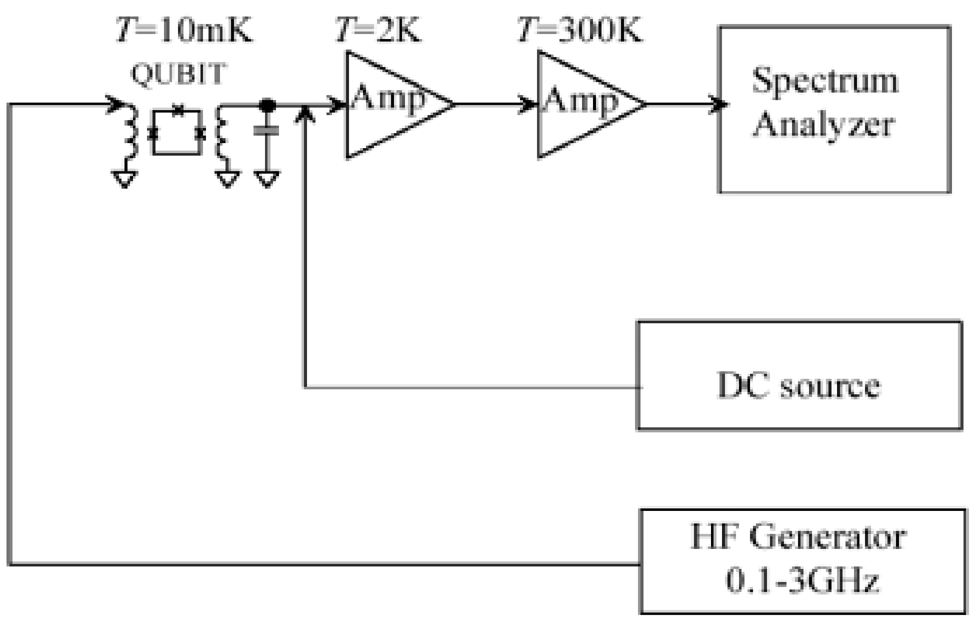
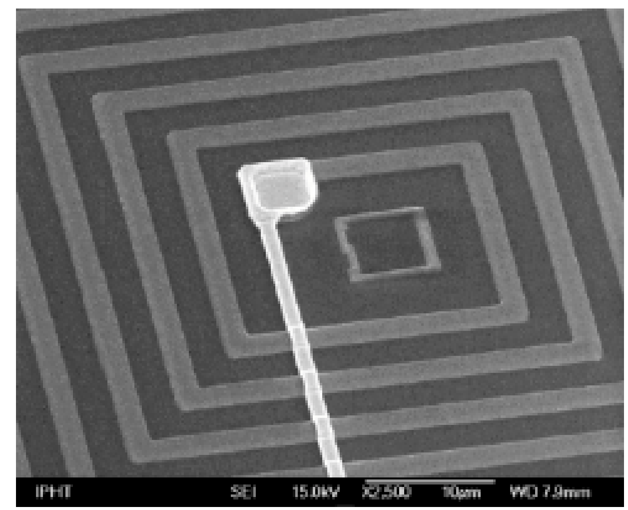
Rabi spectroscopy and noise manipulation in a flux qubit coupled to a tank circuit



Qubit: $\Delta\,,\varepsilon\,,\Gamma$



Tank: $\omega_T = \dfrac{1}{\sqrt{L_T C_T}}, \gamma_T$

Tank as
a low-frequency
linear detector
of current fluctuations
in the qubit

$$\left(\frac{d^2}{dt^2} + \gamma_T \frac{d}{dt} + \omega_T^2\right) V_T = \lambda \omega_T^2 \dot{\sigma}_z$$

$$\Delta \gg \omega_T$$

E. Il'ichev, et al.
Phys. Rev. Lett. 91, 097906 (2003).

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Spectrum of voltage fluctuations in the tank (theory)

$$S_{VQ} = 2\frac{\varepsilon^2}{\omega_0^2} k^2 \frac{L_q I_q^2}{C_T} \omega^2 \Gamma_0 \frac{\omega_T^2}{(\omega_T^2 - \omega^2)^2 + \omega^2 \gamma_T^2} \times \frac{\Omega_R^2}{(\omega - \Omega_R^2)^2 + \omega^2 \Gamma^2}$$

Peak value
of the spectrum
(at $\omega = \omega_T$)

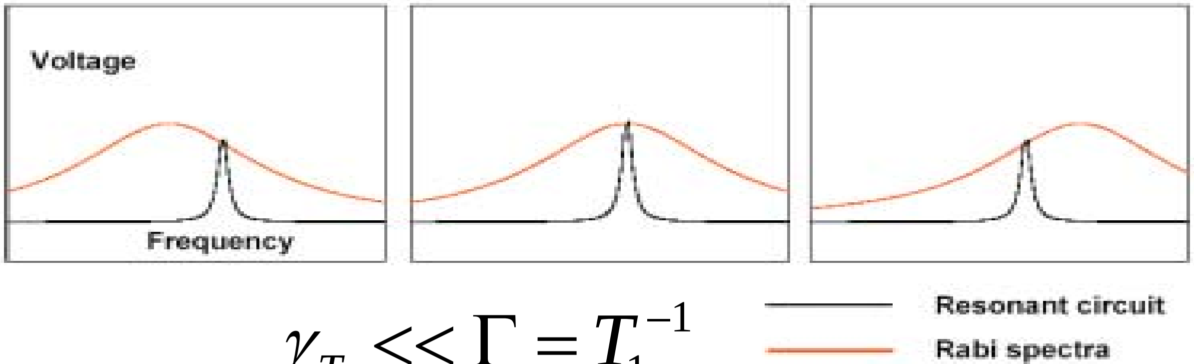
$$S_{V,\max} \sim \frac{\Omega_R^2}{(\omega_T^2 - \Omega_R^2)^2 + \omega_T^2 \Gamma^2},$$

$$\omega_0 = \sqrt{\Delta^2 + \varepsilon^2},$$

$$\omega_0 >> \omega_T >> \gamma_T$$

Direct detection
of radiation
at Rabi frequency :

$$\Omega_R = \frac{\Delta}{\sqrt{\Delta^2 + \varepsilon^2}} F_0.$$



$$\gamma_T << \Gamma = T_1^{-1}$$

A.Yu. Smirnov, Phys.Rev.B 68,134514 (2003)

Spectrum of voltage fluctuations in the tank (experiment)

$\omega_T / 2\pi$ =6.284 MHz,
$Q_T = \omega_T / \gamma_T$ = 1850,
L_q =24 pH, I_q =500 nA,
$k^2 = 10^{-3}$, T= 10 mK

Decoherence times:

$$T_1 = 2.5\mu s$$

(Jena/D-Wave)

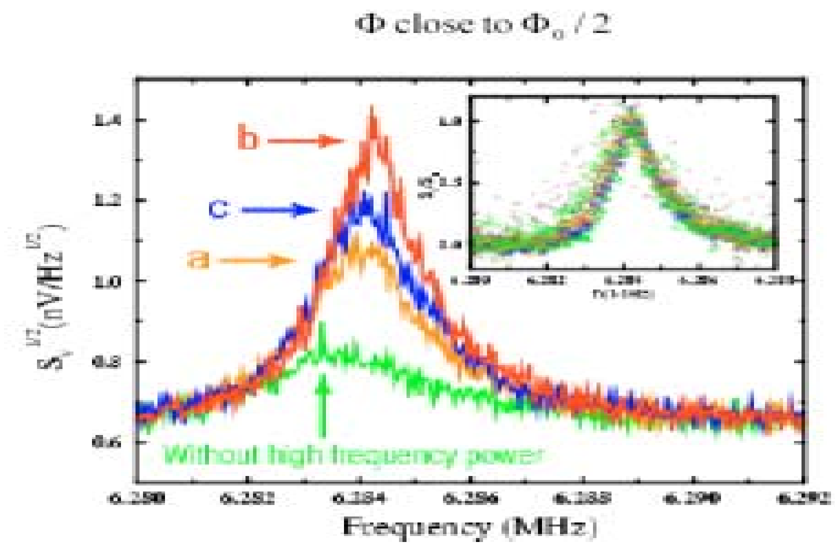
$$T_1 = 0.15\mu s$$

(Delft)

Spectrum

$$S_{VQ}^{1/2}(\omega)$$

at different HF power

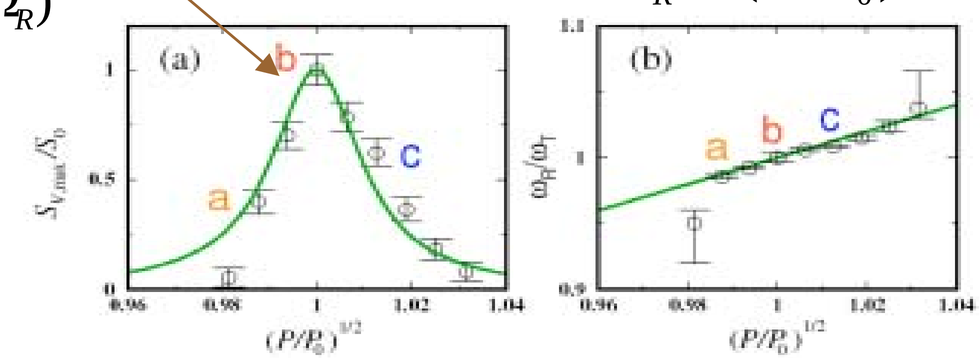


Theory

Peak value $S_{V,\max}(\Omega_R)$
of the spectrum,
as a function
of HF amplitude

$$F_0 \propto (P / P_0)^{1/2}$$

$$\Omega_R \propto (P / P_0)^{1/2}$$



Rabi spectroscopy as a weak continuous measurement

Measurement-induced decoherence (backaction of the tank on the qubit):

$$\Gamma_T = 4k^2 L_q I_q^2 \frac{\varepsilon^2}{\omega_0^2} \omega_T^2 \Gamma_0 \frac{T\gamma_T}{(\omega_T^2 - \Omega_R^2)^2 + \Omega_R^2 \gamma_T^2}$$

Internal noise of the tank circuit:

$$S_{VT} = 2\frac{\omega^2}{C_T} \frac{T\gamma_T}{(\omega_T^2 - \omega^2)^2 + \omega^2 \gamma_T^2},$$

Signal-to-noise ratio:

$$\frac{S_{VQ}(\omega)}{S_{VT}(\omega)} = k^2 \frac{\varepsilon^2}{\omega_0^2} \frac{L_q I_q^2}{T} \frac{\Gamma_0}{\gamma_T} \frac{\omega_T^2 \Omega_R^2}{(\omega_T^2 - \Omega_R^2)^2 + \omega_T^2 \Gamma^2}$$

Rabi spectroscopy is
a *weak quantum measurement*, if:

$$\gamma_T << |\Omega_R - \omega_T| < \Gamma = \Gamma_0 + \Gamma_T = T_1^{-1} << \Omega_R$$

Conclusions.

Recent measurements of Rabi oscillations in superconducting flux qubits have demonstrated a possibility to suppress decoherence and control a noise level in the flux qubits by applying a strong driving field.