Shedding A Bit of Information on Light:

(measurement & manipulation of quantum states)

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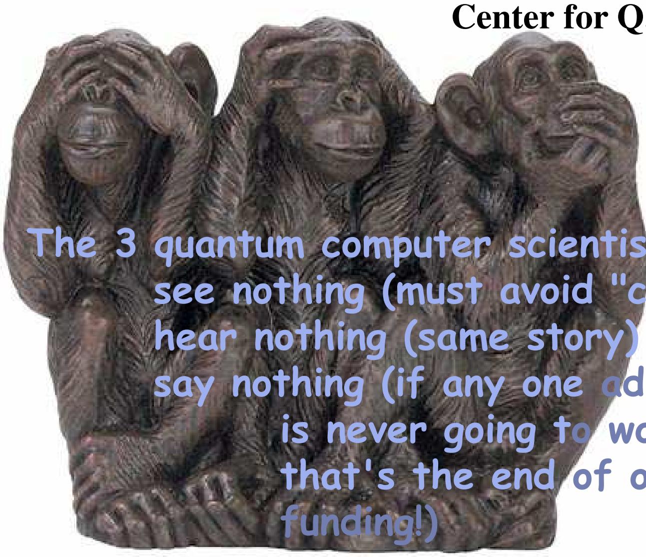

CQIQC, Fields Institute, Toronto, July 2004

DRAMATIS PERSONAE

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Some friendly theorists:
Daniel Lidar, János Bergou, Mark Hillery, John Sipe, Paul Brumer, Howard Wiseman,...

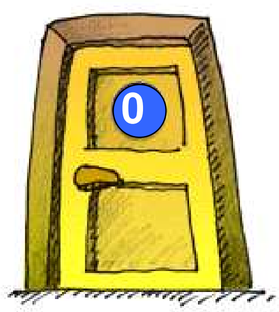
OUTLINE

0. Motivation for & introduction to quantum state & process tomography

1. Quantum state & process tomography (entangled photons and lattice-trapped atoms)

2. Experimental quantum state discrimination

3. Post-selective generation of a 3-photon path-entangled state



Quantum tomography: why?

The Serious Problem For QI

- The danger of errors grows exponentially with the size of the quantum system.
- Without error-correction techniques, quantum computation would be a pipe dream.
- To reach the thresholds for fault-tolerant computation, it is likely that error-protection techniques will first need to be tailored to individual *devices* (not just to individual *designs*); first, we must learn to measure & characterize these devices accurately and efficiently.
- The tools are "quantum state tomography" and "quantum process tomography": full characterisation of the density matrix or Wigner function, and of the "**$**uperoperator" which describes its time-evolution.

Density matrices and superoperators

One photon: H or V.
State: two coefficients

$$\begin{pmatrix} C_H \\ C_V \end{pmatrix}$$

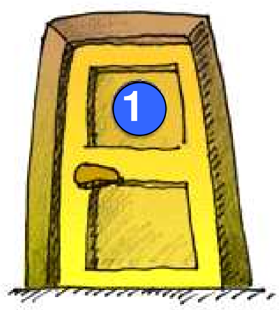
Density matrix: 2x2=4 coefficients

$$\begin{pmatrix} C_{HH} & C_{VH} \\ C_{HV} & C_{VV} \end{pmatrix}$$

Measure
- intensity of horizontal
- intensity of vertical
- intensity of 45^o
- intensity of RH circular.

Propagator (superoperator): 4x4 = 16 coefficients.

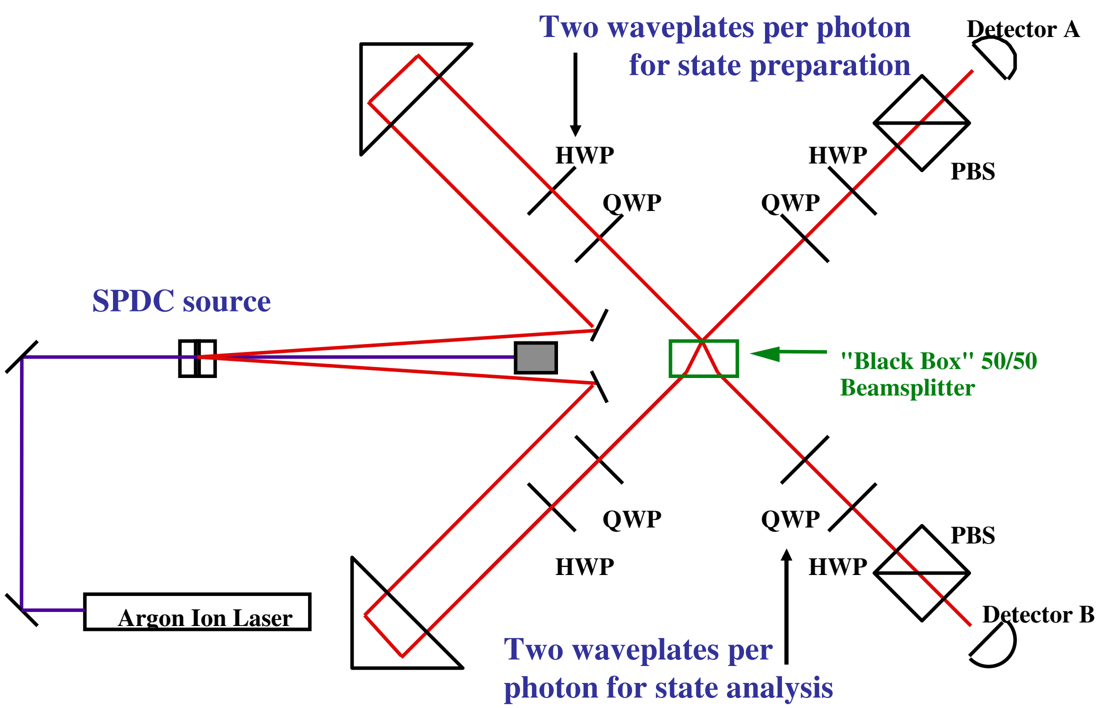
Two photons: HH, HV, VH, VV, or any superpositions.
State has four coefficients.
Density matrix has 4x4 = 16 coefficients.
Superoperator has 16x16 = 256 coefficients.



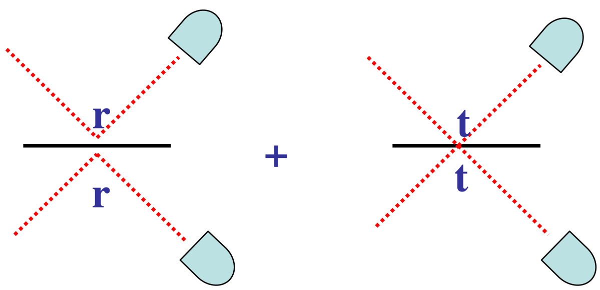
Quantum process tomography experiments
(a: entangled photons
b: trapped atoms)

Two-photon Process Tomography

[Mitchell et al., PRL 91, 120402 (2003)]



Hong-Ou-Mandel Interference



How often will both detectors fire together?

$r^2+t^2 = 0$; total destructive interference.

...iff the processes (& thus photons) *indistinguishable*.

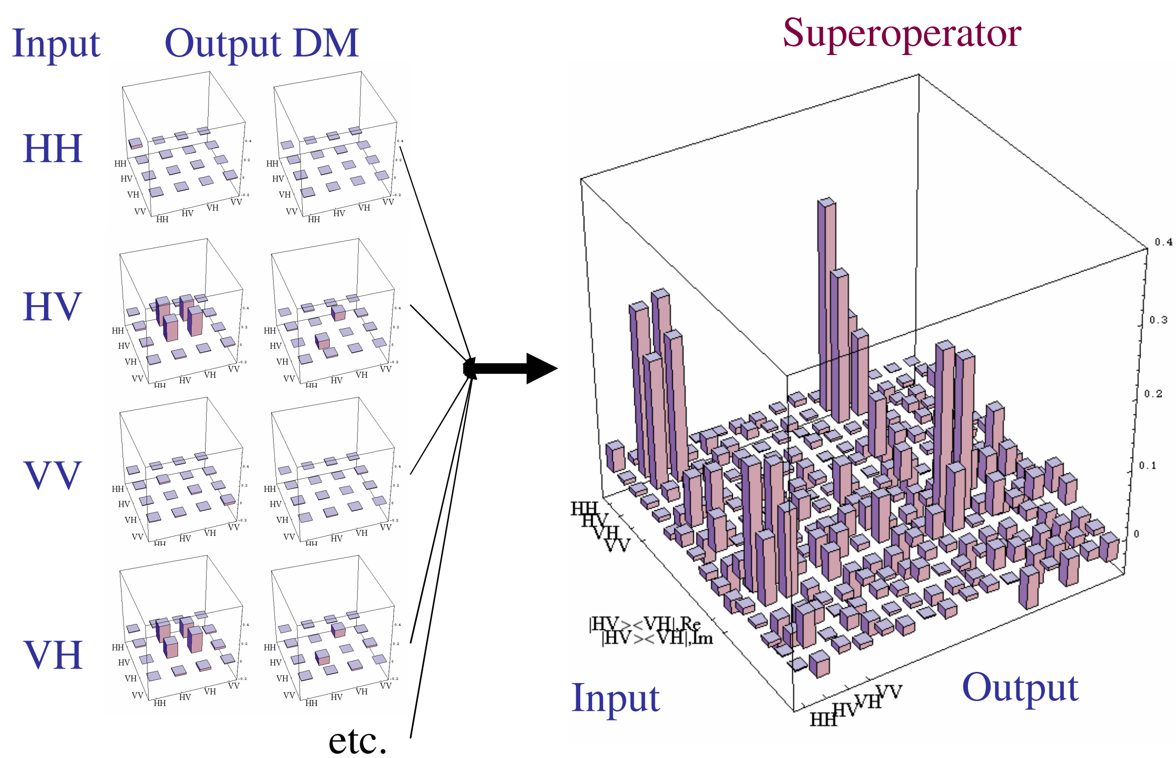
If the photons have same polarisation, no coincidences.

Only in the singlet state |HV> – |VH> are the two photons guaranteed to be orthogonal.

This interferometer is a "Bell-state filter," needed for quantum teleportation and other applications.

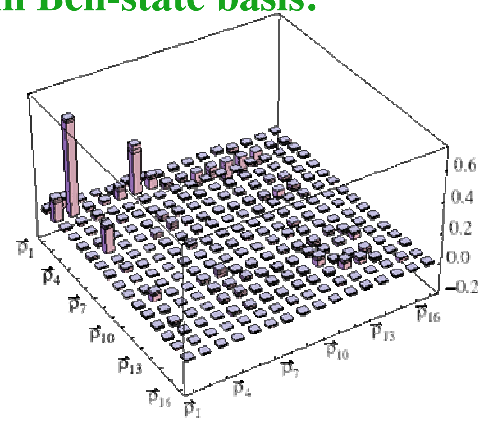
Our Goal: use process tomography to test (& fix) this filter.

“Measuring” the superoperator



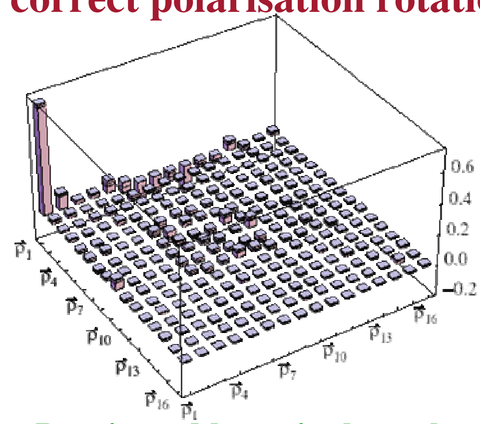
Superoperator provides information needed to correct & diagnose operation

Measured superoperator, in Bell-state basis:



The ideal filter would have a single peak.
Leading Kraus operator allows us to determine unitary error.

Superoperator after transformation to correct polarisation rotations:



Dominated by a single peak; residuals allow us to estimate degree of decoherence and other errors.

(Experimental demonstration delayed for technical reasons; now, after improved rebuild of system, first addressing some other questions...)

A sample error model:
the "Sometimes-Swap" gate

Consider an optical system with stray reflections – occasionally a photon-swap occurs accidentally:

$$\mathcal{E}(\rho) = \frac{1}{2}\left(I\rho I + S\rho S\right)$$

Two subspaces are decoherence-free:

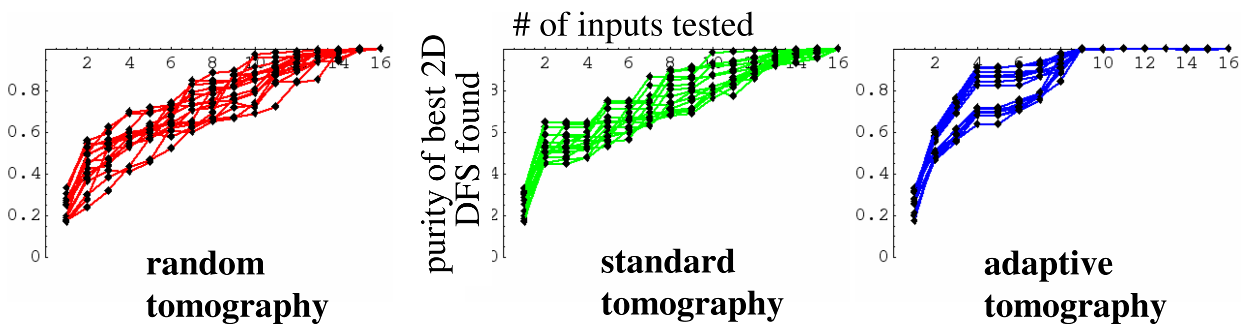
1D: $|\psi^-\rangle \equiv |01\rangle - |10\rangle$

3D: $\begin{cases} |\psi^+\rangle \equiv |01\rangle + |10\rangle \\ |\phi^-\rangle \equiv |00\rangle - |11\rangle \\ |\phi^+\rangle \equiv |00\rangle + |11\rangle \end{cases}$

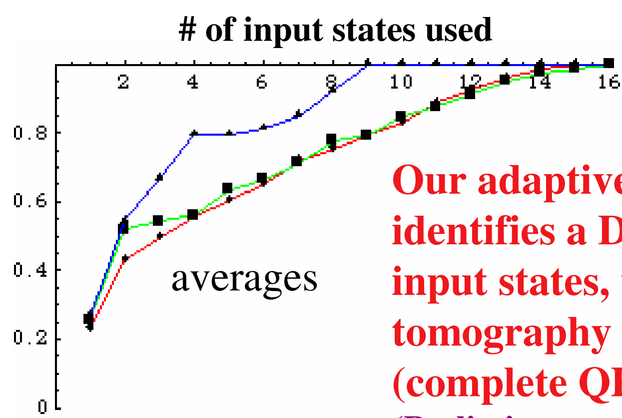
Experimental implementation: a slightly misaligned beam-splitter (coupling to transverse modes which act as environment)

TQEC goal: let the machine identify an optimal subspace in which to compute, with no prior knowledge of the error model.

Some strategies for a DFS search (simulation; experiment underway)



Best known 2-D DFS (average purity).

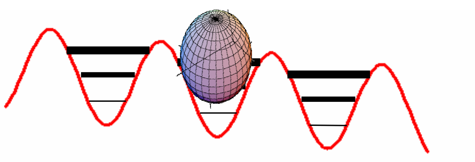


Our adaptive algorithm *always* identifies a DFS after testing 9 input states, while standard tomography routinely requires 16 (complete QPT).

(Preliminary work on scaling promising)

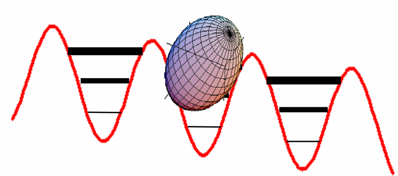
Tomography in Optical Lattices

[Myrkog *et al.*, quant-ph/0312210]



Rb atom trapped in one of the quantum levels of a periodic potential formed by standing light field (30GHz detuning, 10s of μK depth)

Complete characterisation of process on arbitrary inputs?

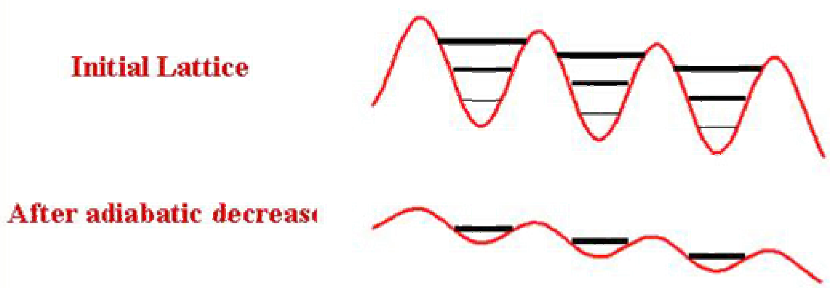


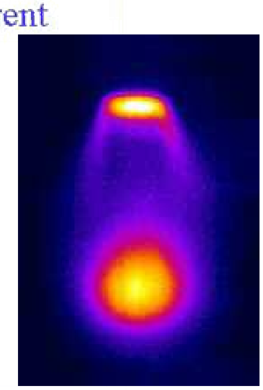
First task: measuring state populations

Adiabatically lower the depth of the wells in the presence of gravity. Highest states become classically unbound and are lost. Measure ground state occupation.

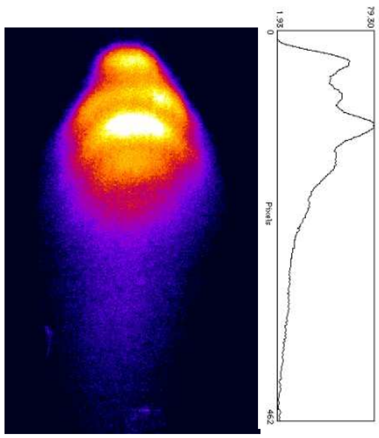
Two Methods : - Ramp down and hold. Observe population as a function of depth.

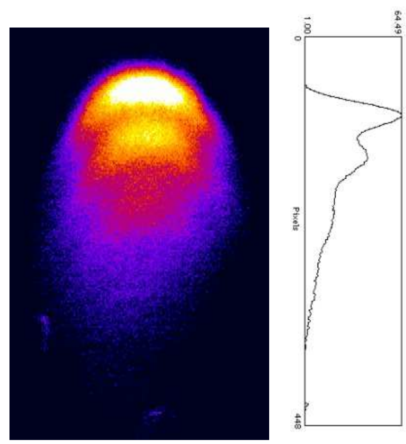
OR - Ramp down very slowly and observe different states leave at distinct times.



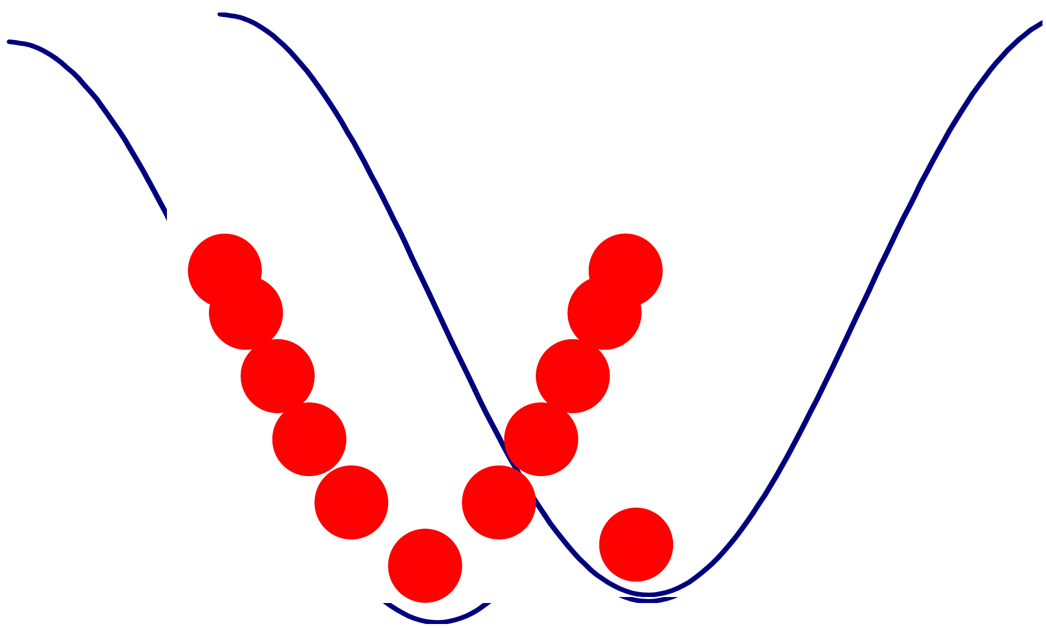


Time-resolved quantum states

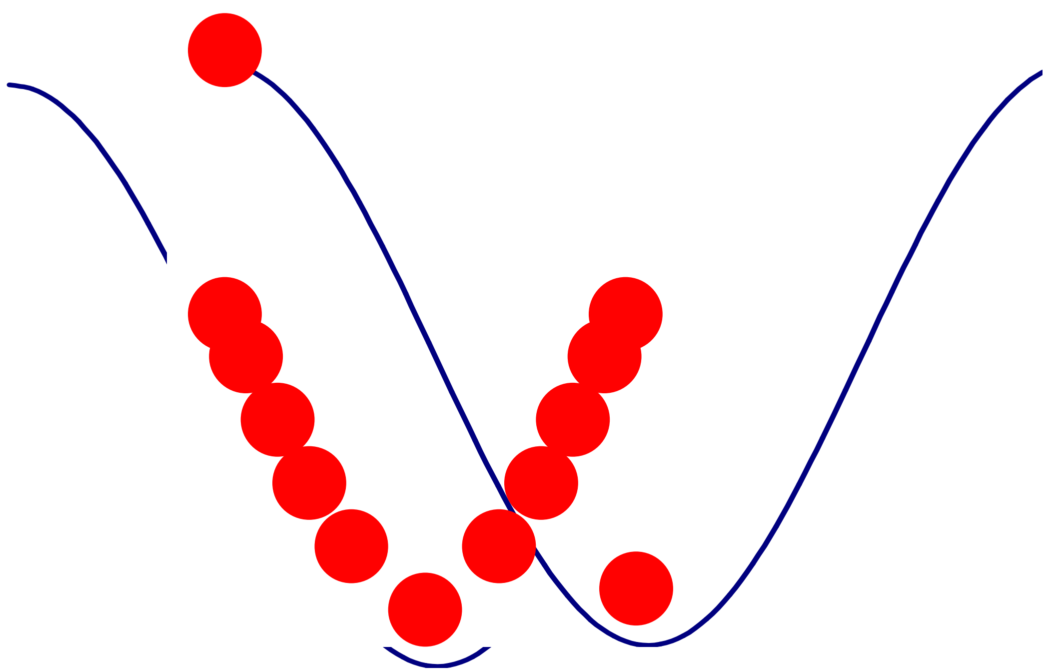




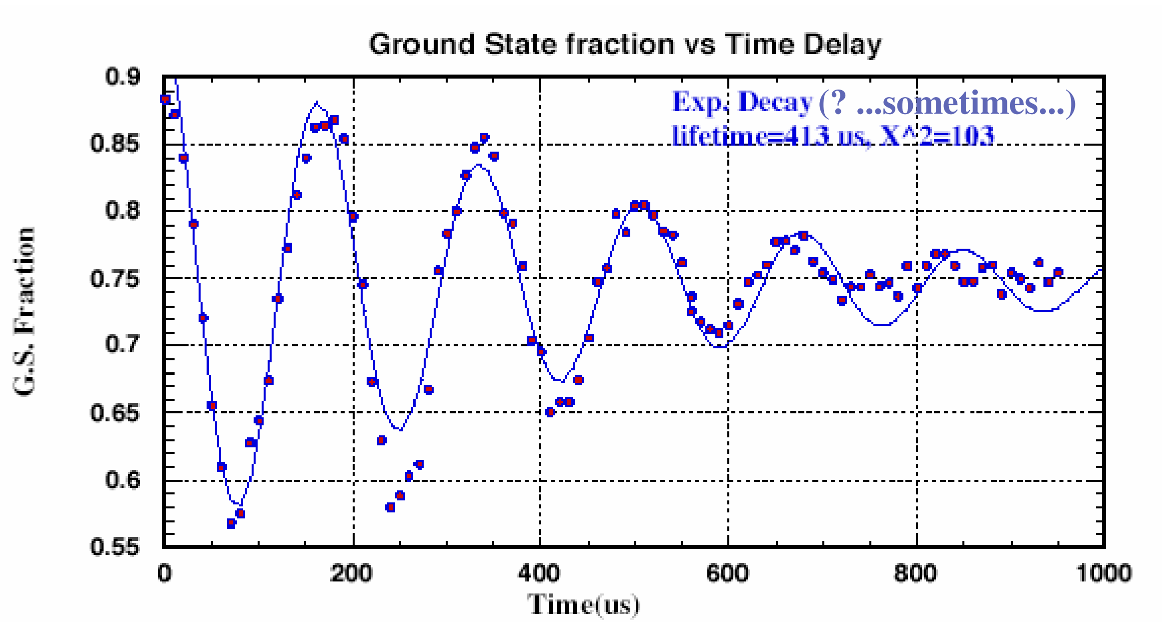
Recapturing atoms after setting them into oscillation...



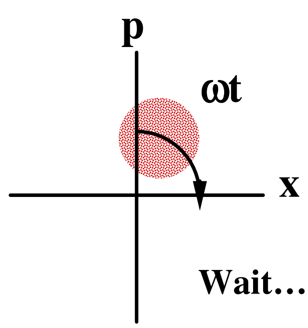
...or failing to recapture them if you're too impatient



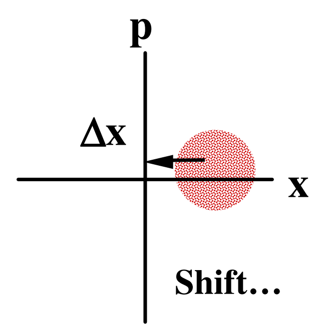
Oscillations in lattice wells



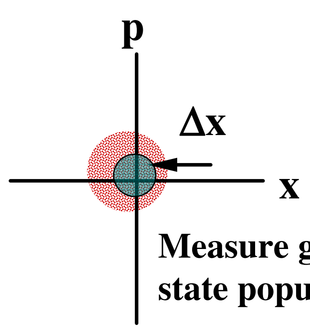
Quantum state reconstruction



Wait…



Shift…



Measure ground state population

$Q(0,0) = P_g$

$W(0,0) = \Sigma\,(-1)^n\, P_n$

(former for HO only; latter requires only symmetry)

Cf. Poyatos,Walser,Cirac,Zoller,Blatt, PRA 53, 1966 ('96)
& Liebfried,Meekhot,King,Monroe,Itano,Wineland, PRL77, 4281 ('96)

Data:"W-like" $[P_g\text{-}P_e](x,p)$ for a mostly-excited incoherent mixture

QuickTime™ and a
Photo - JPEG decompressor
are needed to see this picture.

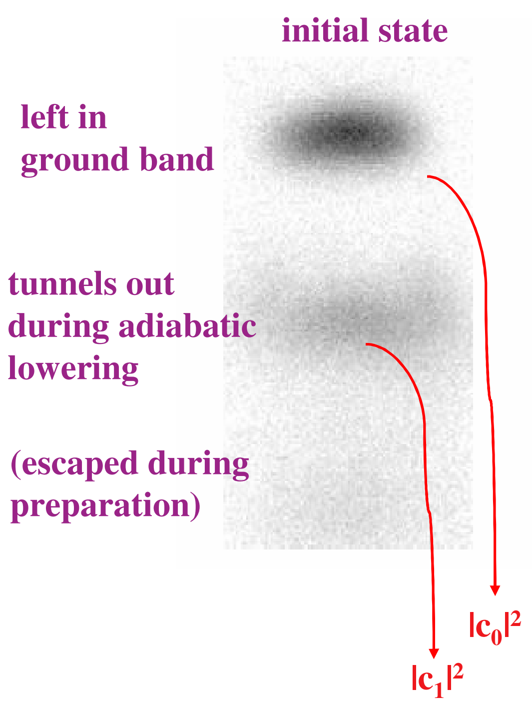
Towards QPT:
Some definitions / remarks

- "Qbit" = two vibrational states of atom in a well of a 1D lattice
- Control parameter = spatial shifts of lattice (coherently couple states), achieved by phase-shifting optical beams (via AO)
- Initialisation: prepare |0> by letting all higher states escape
- Ensemble: 1D lattice contains 1000 "pancakes", each with thousands of (essentially) non-interacting atoms.
  No coherence between wells; tunneling is a decoherence mech.
- Measurement in logical basis: direct, by preferential tunneling under gravity
- Measurement of coherence/oscillations: shift and then measure.

- Typical experiment:
  - Initialise |0>
  - Prepare some other superposition or mixture (use shifts, shakes, and delays)
  - Allow atoms to oscillate in well
  - Let something happen on its own, or try to do something
  - Reconstruct state by probing oscillations (delay + shift +measure)

Atomic state measurement

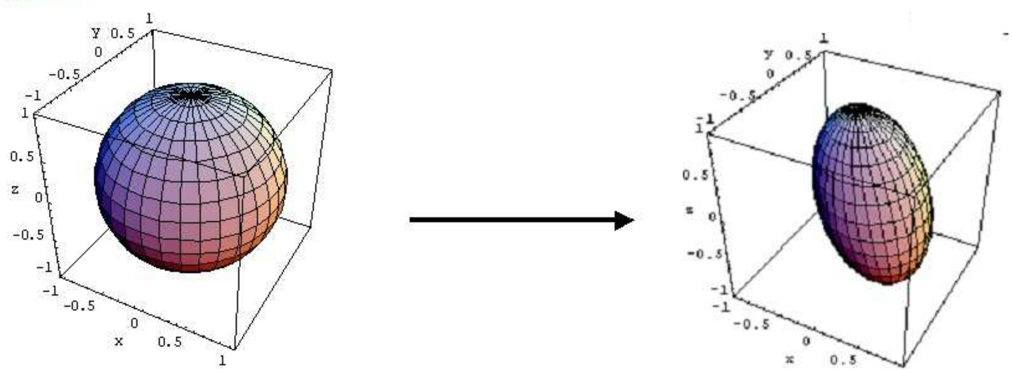
(for a 2-state lattice, with $c_0|0> + c_1|1>$)



Extracting a superoperator:

prepare a complete set of input states and measure each output

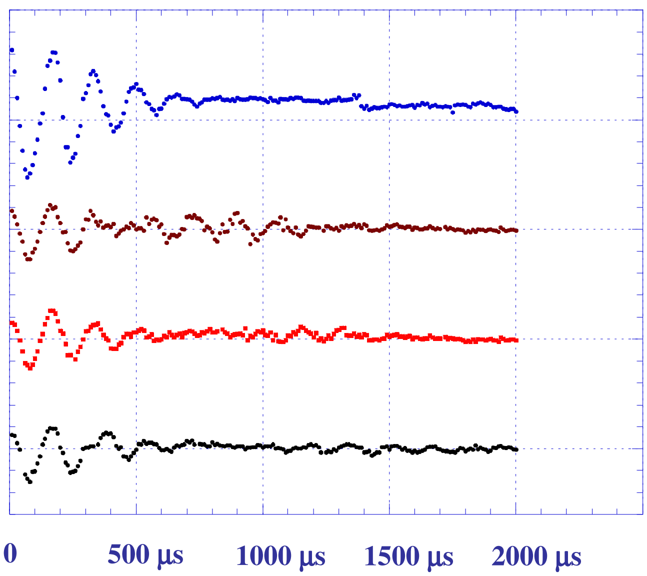
Operation:
Sitting in the lattice
for 1 period.



Likely sources of decoherence/dephasing:

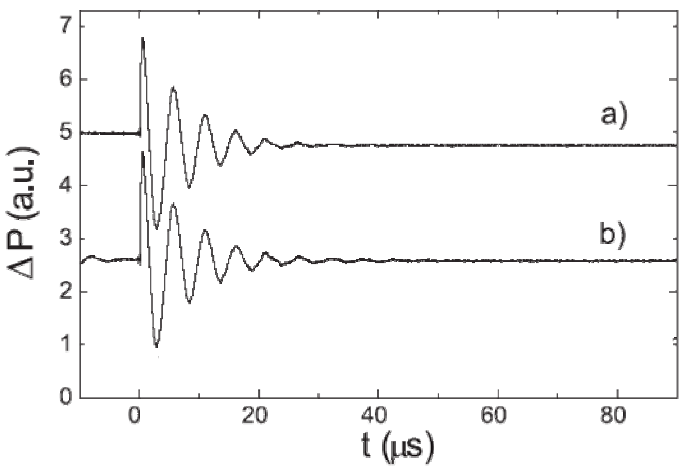
- **Real photon scattering (100 ms; shouldn't be relevant in 150 μs period)**
- **Inter-well tunneling (10s of ms; would love to see it)**
- **Beam inhomogeneities (expected several ms, but are probably wrong)**
- **Parametric heating (unlikely; no change in diagonals)**
- **Other**

Towards bang-bang error-correction: pulse echo indicates T2 ≈ 1 ms...



[Cf. Buchkremer, Dumke, Levsen, Birkl, and Ertmer, PRL **85**, 3121 (2000).]

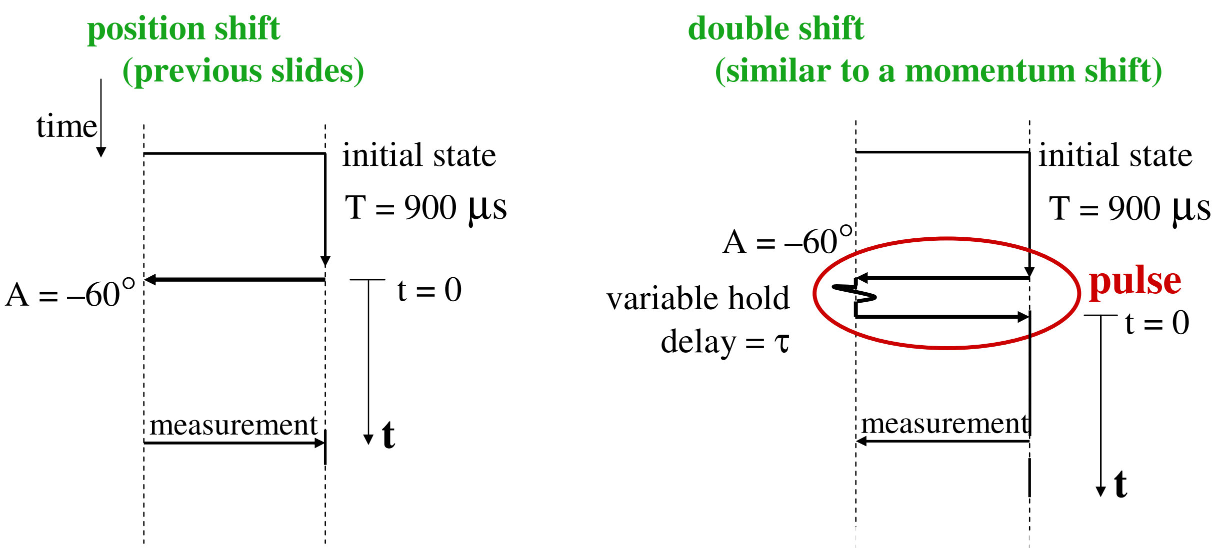
Cf. Hannover experiment



Far smaller echo, but far better signal-to-noise ("classical" measurement of <X>)
Much shorter coherence time, but roughly same number of periods
– dominated by anharmonicity, irrelevant in our case.

Buchkremer, Dumke, Levsen, Birkl, and Ertmer, PRL **85**, 3121 (2000).

A better "bang" pulse for QEC?

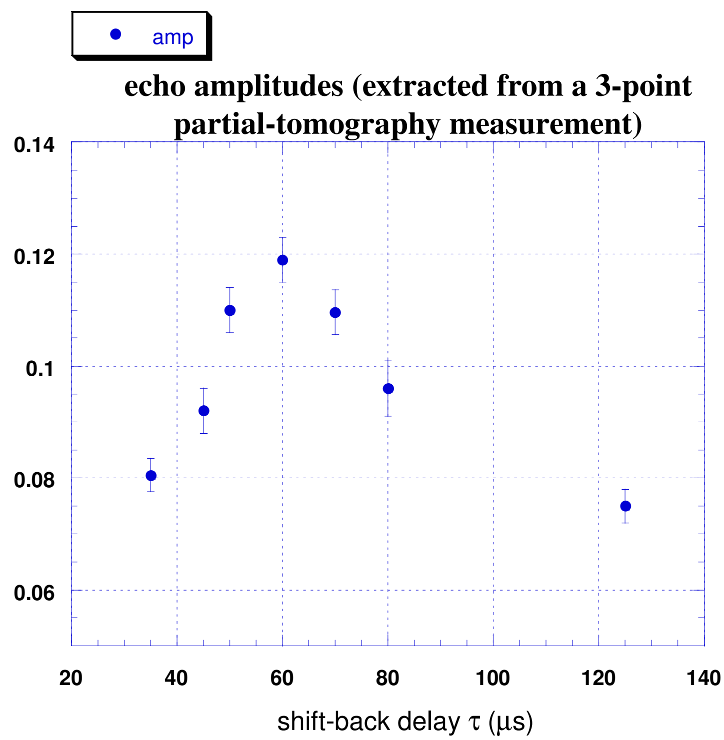


Under several (not quite valid) approximations, the double-shift is a momentum displacement.

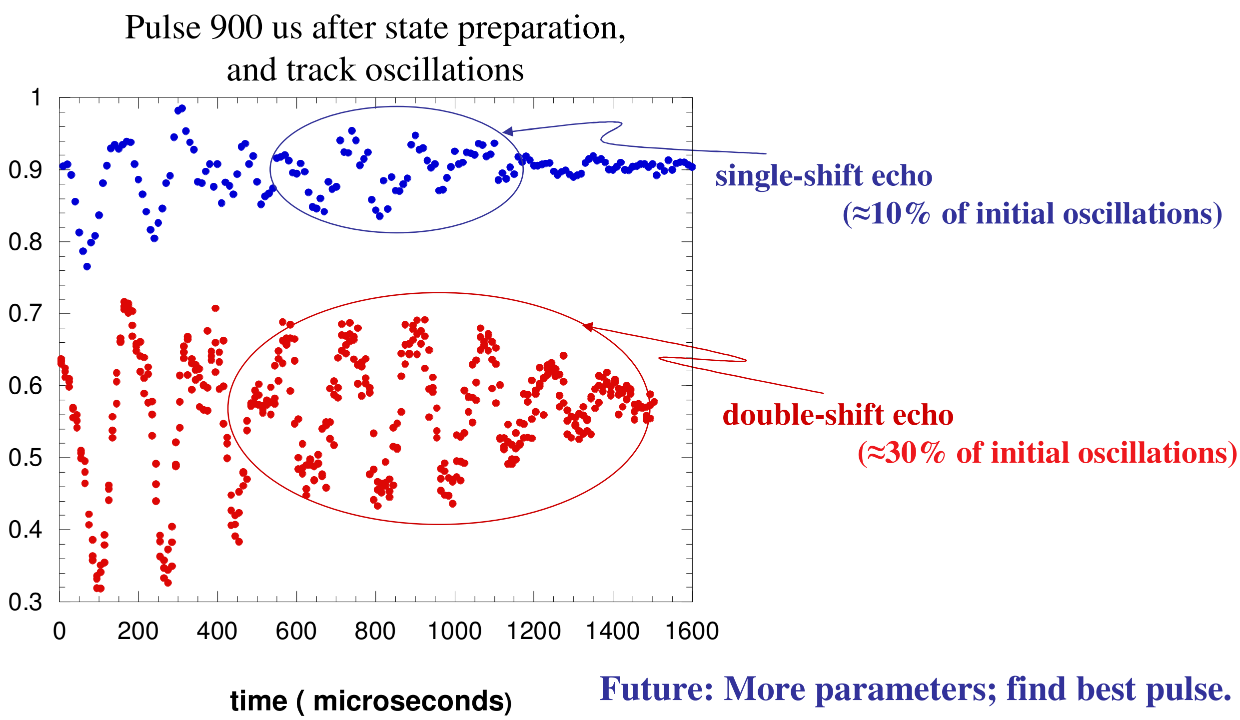
We expected a momentum shift to be *at least* as good as a position shift.

In practice: we want to test the idea of letting learning algorithms search for the best pulse shape on their own, and this is a first step.

Optimising the pulse

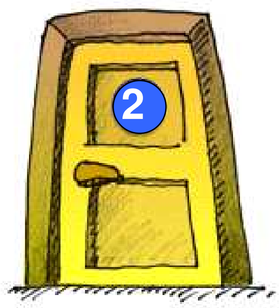


Echo from optimized pulse



Future: More parameters; find best pulse.

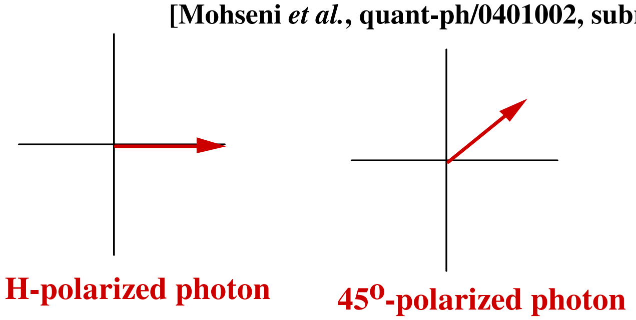
Step 2 (optional?): figure out why it works!



Distinguishing the indistinguishable...

Can one distinguish between nonorthogonal states?

[Mohseni *et al.*, quant-ph/0401002, submitted to PRL]



- Single instances of non-orthogonal quantum states cannot be distinguished with certainty. Obviously, ensembles can.

- This is one of the central features of quantum information which leads to secure (eavesdrop-proof) communications.

- Crucial element: we must learn how to distinguish quantum states as well as possible -- and we must know how well a potential eavesdropper could do.

Theory: how to distinguish non-orthogonal states optimally

Step 1:

Repeat the letters "POVM" over and over.

Step 2:

Ask some friendly theorists for help.
[or see, e.g., Y. Sun, J. Bergou, and M. Hillery, Phys. Rev. A 66, 032315 (2002).]

The view from the laboratory:

A measurement of a two-state system can only yield two possible results.

If the measurement isn't guaranteed to succeed, there are three possible results: (1), (2), and ("I don't know").

Therefore, to discriminate between two non-orth. states, we need three measurement outcomes – no 2D operator has 3 different eigenstates, though.

Into another dimension...

If we had a device which could distinguish between |a> and |b>, its action would by definition transform them into «pointer states» |"It's A!"> and |"It's B!">, which would be orthogonal (perfectly distinguishable).

Unfortunately, unitary evolution conserves the overlap:

$$\langle a|b\rangle = \langle "A"|"B"\rangle \stackrel{?}{=} 0$$

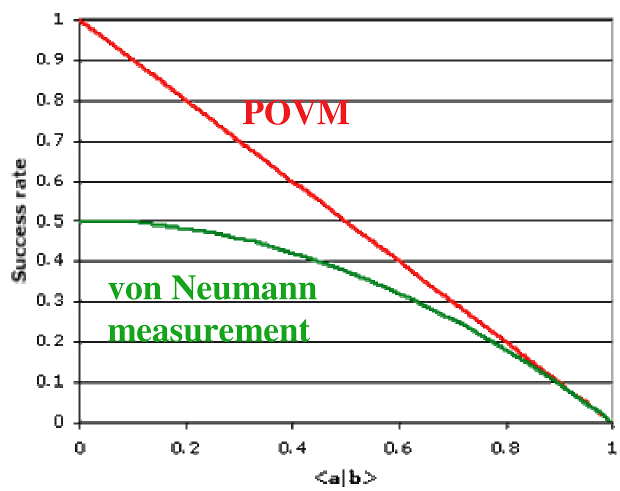
So, to get from non-orthogonal *a* and *b* to orthogonal "A" and "B", we need a *non-unitary* operation.

Quantum *measurement* leads to such non-unitary operations – put another way, we have to accept throwing out some events.

$$\begin{aligned} |a\rangle &\rightarrow u|A\rangle + v|DK\rangle \\ |b\rangle &\rightarrow w|B\rangle + x|DK\rangle \end{aligned}$$

By throwing out the "Don't Know" terms, we may keep only the orthogonal parts.

How well can standard (projective) measurements do?



At <alb> = 0.707, the von Neumann strategy succeeds 25% of the time, while the optimum is 29.3%.

The advantage is higher in higher dim.

Consider these three non-orthogonal states, prepared with equal *a priori* probabilities:

$$|\psi_1\rangle_{in} = \begin{pmatrix} \sqrt{2/3} \\ 0 \\ 1/\sqrt{3} \end{pmatrix} \; ; \; |\psi_2\rangle_{in} = \begin{pmatrix} 0 \\ 1/\sqrt{3} \\ \sqrt{2/3} \end{pmatrix} \; ; \; |\psi_3\rangle_{in} = \begin{pmatrix} 0 \\ -1/\sqrt{3} \\ \sqrt{2/3} \end{pmatrix}$$

Projective measurements can distinguish these states with *certainty* no more than 1/3 of the time.

(No more than one member of an orthonormal basis is orthogonal to *two* of the above states, so only one pair may be ruled out.)

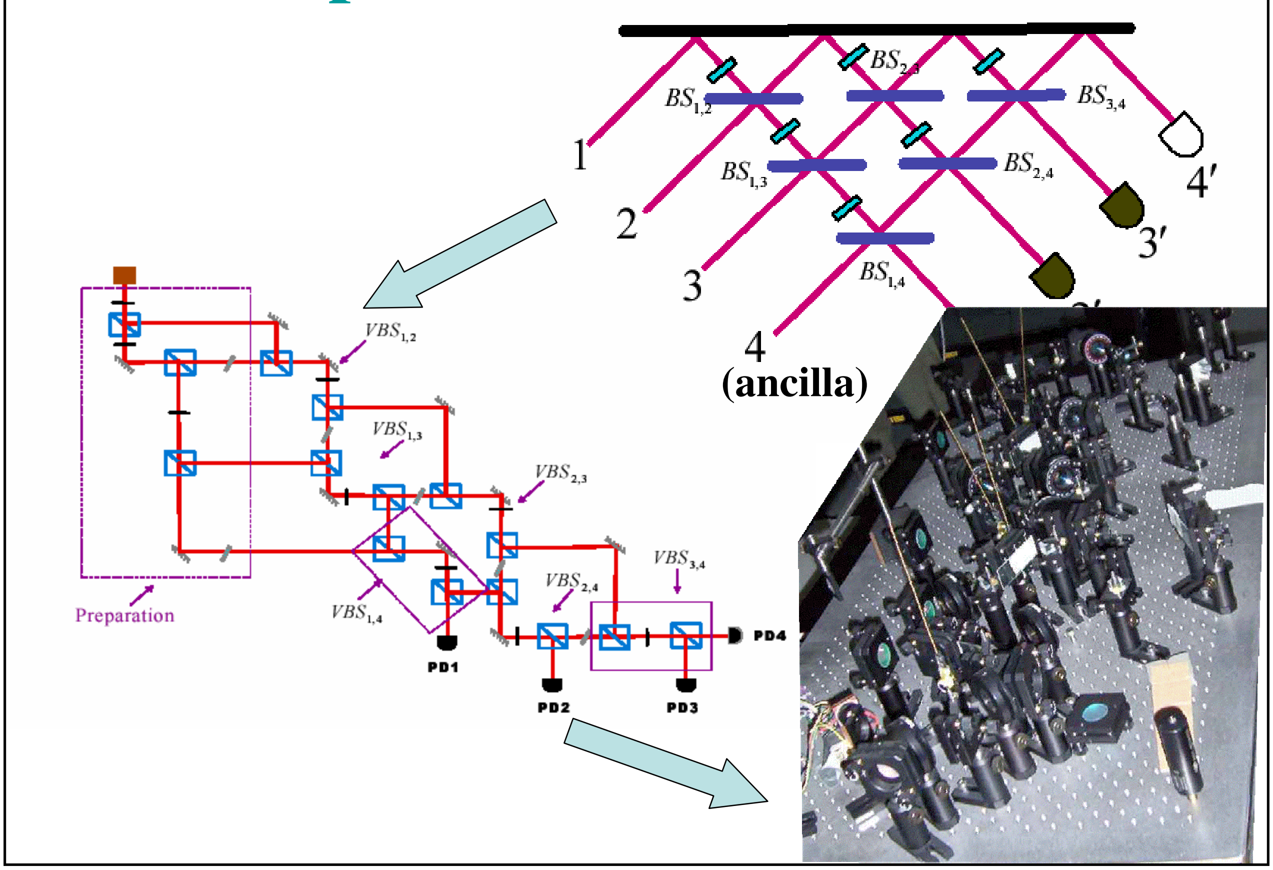
But a unitary transformation in a 4D space produces:

$$|\psi_1\rangle_{out} = \begin{pmatrix} 1/\sqrt{3} \\ 0 \\ 0 \\ \sqrt{2/3} \end{pmatrix} \qquad |\psi_2\rangle_{out} = \begin{pmatrix} 0 \\ \sqrt{2/3} \\ 0 \\ 1/\sqrt{3} \end{pmatrix} \qquad |\psi_3\rangle_{out} = \begin{pmatrix} 0 \\ 0 \\ \sqrt{2/3} \\ 1/\sqrt{3} \end{pmatrix}$$

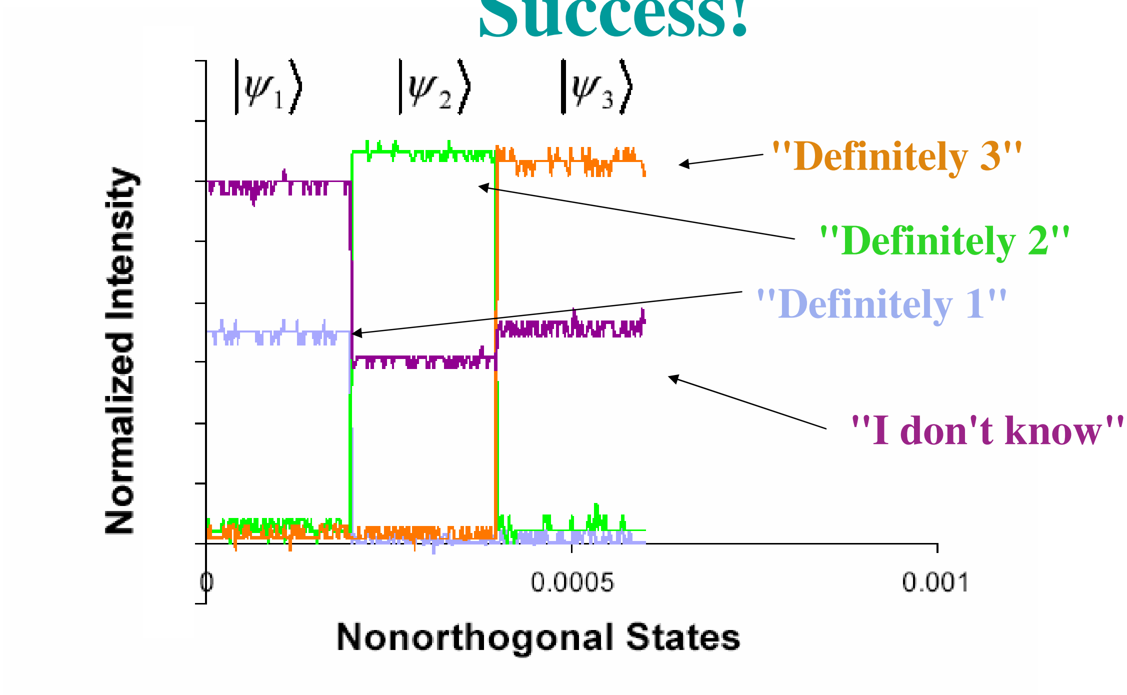
…the fourth basis state means "Don't Know," while the first indicates Ψ_1 and the 2nd and 3rd indicate Ψ_2 and Ψ_3.

These states can thus be distinguished 55% of the time (>33%).

Experimental schematic



Success!



The correct state was identified 55% of the time--
Much better than the 33% maximum for standard measurements.

Non-unitary (post-selected) operations for the construction of novel (useful?) entangled states...

Highly number-entangled states ("low-noon" experiment).

M.W. Mitchell *et al.*, Nature **429**, 161 (2004);
and cf. P. Walther *et al.*, Nature **429**, 158 (2004).

The single-photon superposition state |1,0> + |0,1>, which may be regarded as an entangled state of two fields, is the workhorse of classical interferometry.

The output of a Hong-Ou-Mandel interferometer is |2,0> + |0,2>.

States such as |n,0> + |0,n> ("high-noon" states, for n large) have been proposed for high-resolution interferometry – related to "spin-squeezed" states.

Multi-photon entangled states are the resource required for KLM-like efficient-linear-optical-quantum-computation schemes.

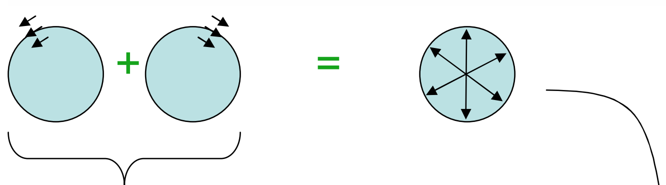
A number of proposals for producing these states have been made, but so far none has been observed for n>2.... until now!

Practical schemes?

[See for example
H. Lee *et al.*, Phys. Rev. A 65, 030101 (2002);
J. Fiurášek, Phys. Rev. A 65, 053818 (2002)]

Important factorisation:

$$\left(a^{\dagger 3} + b^{\dagger 3}\right) = \left(a^{\dagger} + b^{\dagger}\right)\left(a^{\dagger} + e^{2\pi i/3} b^{\dagger}\right)\left(a^{\dagger} + e^{-2\pi i/3} b^{\dagger}\right)$$



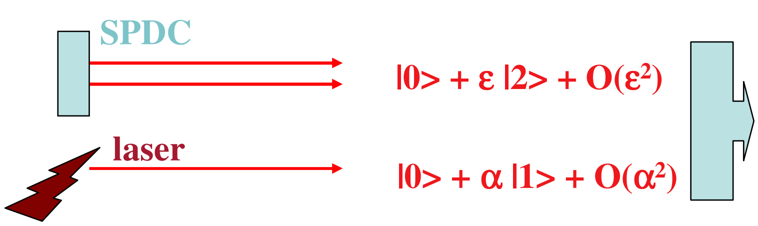
A "noon" state

A really odd beast: one 0° photon, one 120° photon, and one 240° photon... but of course, you can't tell them apart, let alone combine them into one mode!

Trick #1

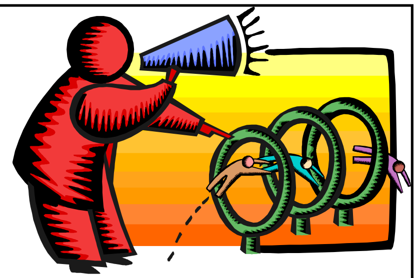
Okay, we don't even have single-photon sources.

But we can produce *pairs* of photons in down-conversion, and very *weak* coherent states from a laser, such that *if* we detect three photons, we can be pretty sure we got only one from the laser and only two from the down-conversion...

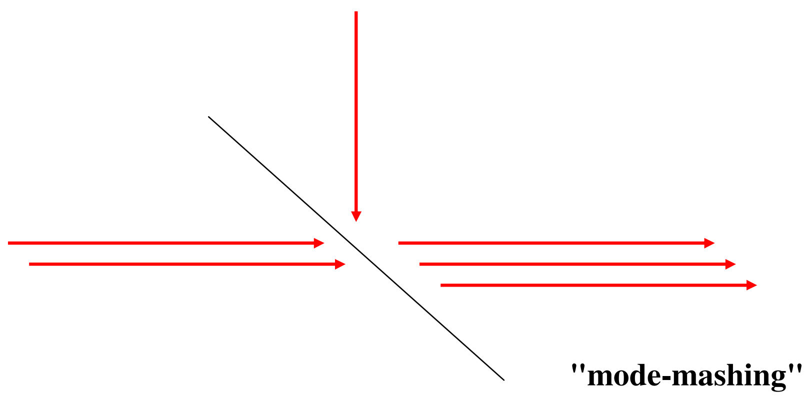


$\varepsilon\alpha\,|3\rangle + O(\alpha^3) + O(\varepsilon^2)$
+ terms with <3 photons

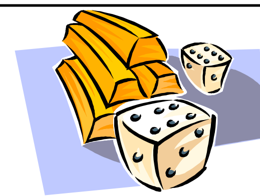
Trick #2



How to combine three non-orthogonal photons into one spatial mode?



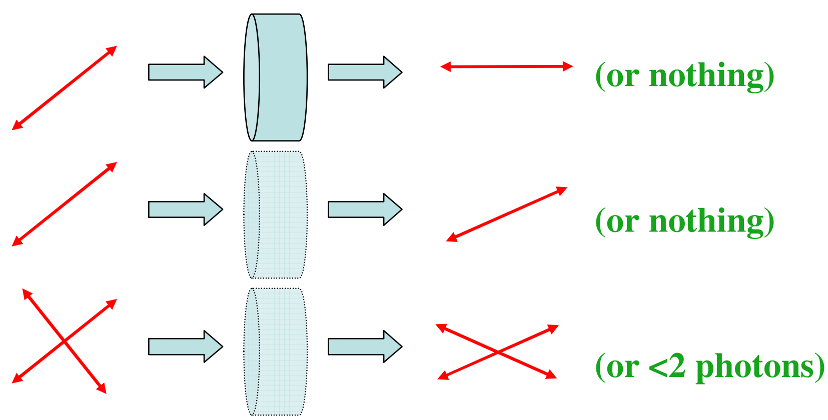
Yes, it's that easy! If you see three photons out one port, then they all went out that port.



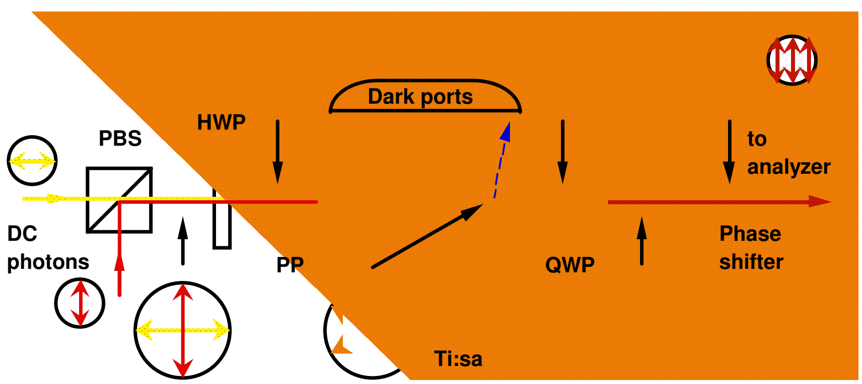
Trick #3

But how do you get the two down-converted photons to be at 120° to each other?

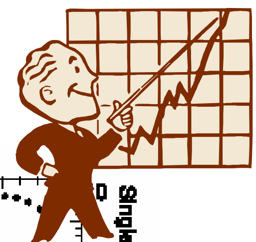
More post-selected (non-unitary) operations: if a 45° photon gets through a polarizer, it's no longer at 45°. If it gets through a *partial* polarizer, it could be anywhere...



The basic optical scheme



It works!

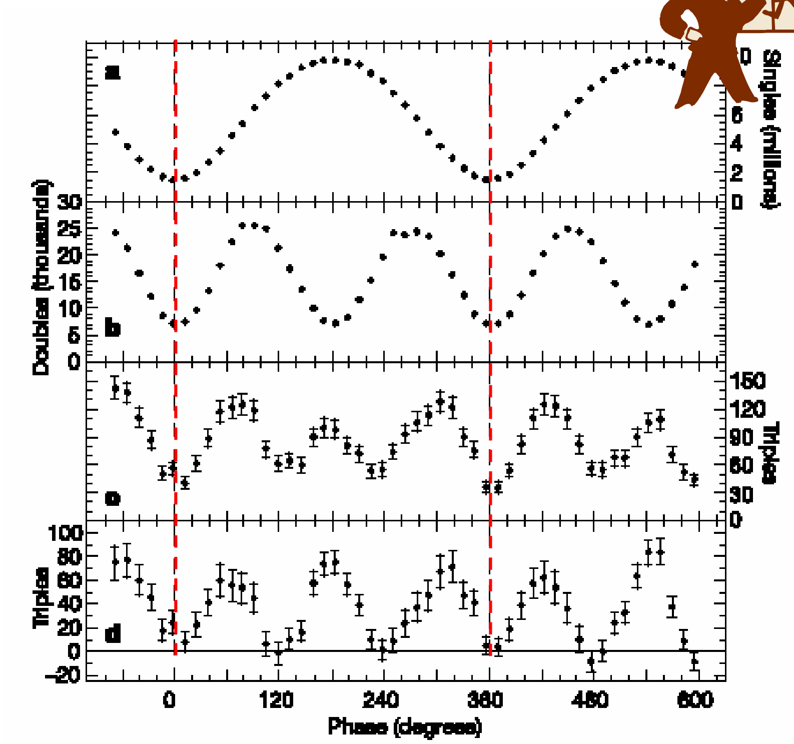


Singles:

Coincidences:

Triple coincidences:

Triples (bg subtracted):



The moral of the story

1. **Quantum process tomography can be useful for characterizing and "correcting" quantum systems (ensemble measurements). More work needed on efficient algorithms, especially for extracting only *useful* info!**

2. **Progress on optimizing pulse echo sequences in lattices; more knobs to add and start turning.**

3. **POVMs can allow certain information to be extracted efficiently even from single systems; implementation relies on post-selection.**

4. **Post-selection (à la KLM linear-optical-quantum-computation schemes) can also enable us to generate novel entangled states.**