Distillation of Qubits through Zeno-like Measurements

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References:

H. Nakazato, T. Takazawa, and K. Y., Phys. Rev. Lett. **90**, 060401 (2003);
H. Nakazato, M. Unoki, and K. Y., Phys. Rev. A **70**, 012303 (2004);
G. Compagno *et al.*, quant-ph/0405074 (2004).

L.-A. Wu, D. A. Lidar, and S. Schneider, quant-ph/0402209 (2004).

Introduction/Motivation

- How to prepare a quantum state

  with high quantum coherence (entangled state, ...)

- Purification/Distillation

  ··· Extraction of a pure state *from an arbitrary mixed state*

  ▷ in case a state preparation *via a direct projective measurement* is not possible

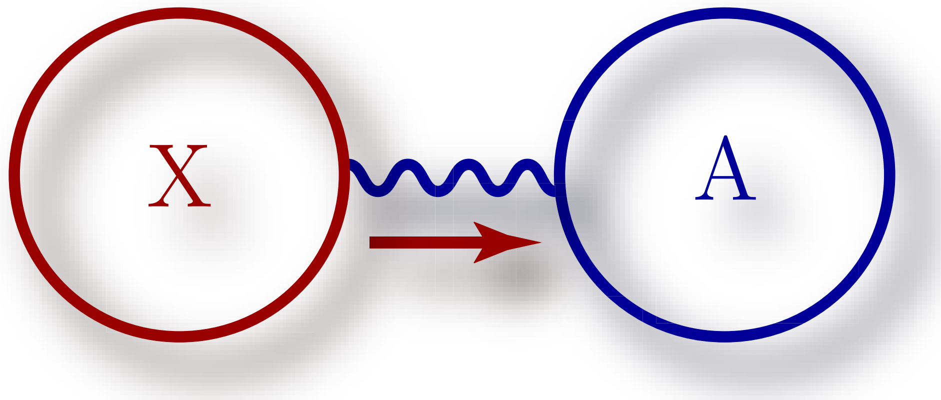
Abstract

- A novel method of purification/distillation *based on "Zeno-like measurements"*
  - ▷ general scheme ▷ mechanism ▷ optimization

- Possible Applications to Qubit-Systems
  - ▷ *Initialization of Multiple Qubits* $\varrho \to |{\downarrow\downarrow\ldots}\rangle\langle{\downarrow\downarrow\ldots}|$
  - ▷ *Entanglement Distillation* $\varrho \to |\Psi^-\rangle\langle\Psi^-|$
  - ▷ *Distillation of Entanglement between Spatially Separated Qubits*

Purification through Zeno-like Measurements

H. Nakazato, T. Takazawa, and K. Y., Phys. Rev. Lett. **90**, 060401 (2003);
H. Nakazato, M. Unoki, and K. Y., Phys. Rev. A **70**, 012303 (2004).



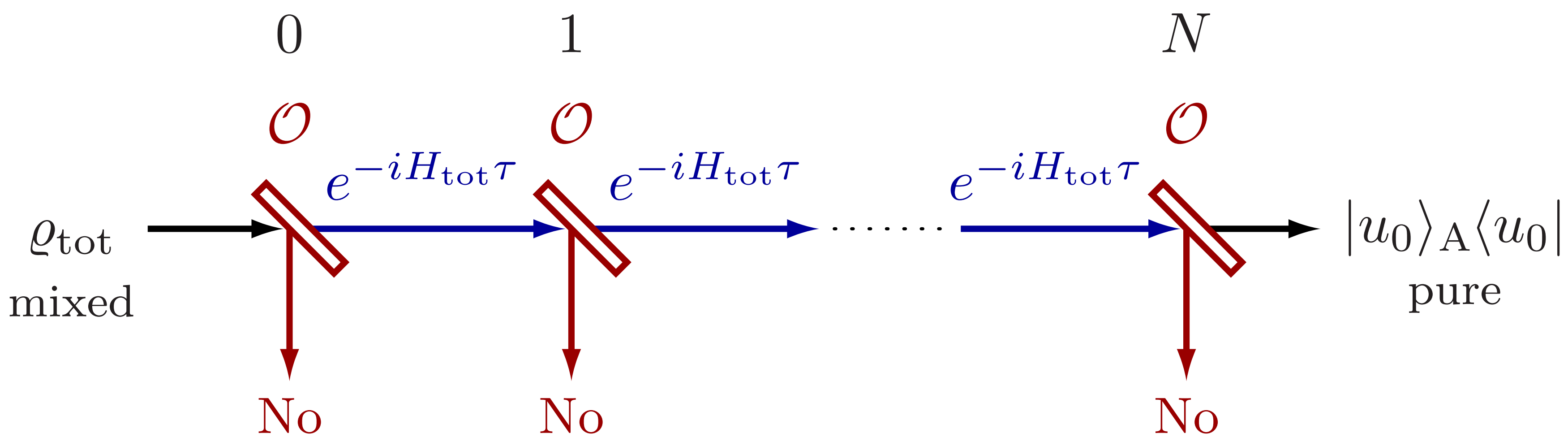
Repeated Measurements on X
(Zeno-like Measurements)

$\Downarrow$

Extraction of a Pure State in A

$$\underset{\text{mixed}}{\varrho_{\mathrm{A}}} \longrightarrow \underset{\text{pure}}{|u_0\rangle_{\mathrm{A}}\langle u_0|}$$

Zeno-like Measurements on X



We retain only those events where $|\phi\rangle_{\rm X}$ is found at *every* measurement.

▷ projection op. $\mathcal{O} = |\phi\rangle_{\rm X}\langle\phi|$

Parameters:

τ ⋯ interval between measurements

$|\phi\rangle_{\rm X}$ ⋯ measuring state

$H_{\rm tot}$ ⋯ parameters in the Hamiltonian (coupling constants g, …)

Evolution under Zeno-like Measurements

State after N *Successful* Confirmations

$$\varrho_{\rm tot}^{(\tau)}(N) = (\mathcal{O}e^{-iH_{\rm tot}\tau})^N \mathcal{O}\varrho_{\rm tot}\mathcal{O}(e^{iH_{\rm tot}\tau}\mathcal{O})^N / P^{(\tau)}(N)$$
$$= |\phi\rangle_{\rm X}\langle\phi| \otimes \varrho_{\rm A}^{(\tau)}(N)$$

$$\varrho_{\rm A}^{(\tau)}(N) = (V_\phi(\tau))^N \tilde{\varrho}_{\rm A} (V_\phi^\dagger(\tau))^N / P^{(\tau)}(N) \xrightarrow{N \text{ increases}} ?$$

Success Probability

$$P^{(\tau)}(N) = \mathrm{Tr}_{\rm A}[(V_\phi(\tau))^N \tilde{\varrho}_{\rm A} (V_\phi^\dagger(\tau))^N]$$

$V_\phi(\tau) = {}_{\rm X}\langle\phi|e^{-iH_{\rm tot}\tau}|\phi\rangle_{\rm X}$, $\tilde{\varrho}_{\rm A} = {}_{\rm X}\langle\phi|\varrho_{\rm tot}|\phi\rangle_{\rm X}$ $\cdots$ operators in $\mathcal{H}_{\rm A}$

Purification, Conditions & Optimization

$$(V_\phi(\tau))^N = \sum_n \lambda_n^N |u_n\rangle_{\rm A}\langle v_n| \xrightarrow{N \text{ increases}} \lambda_0^N |u_0\rangle_{\rm A}\langle v_0| \qquad (0 \le |\lambda_n| \le 1)$$

- $\lambda_0 \cdots$ largest (in magnitude) eigenvalue (*unique, discrete, and nondegenerate*)

$$\varrho_{\rm A}^{(\tau)}(N) = (V_\phi(\tau))^N \tilde{\varrho}_{\rm A} (V_\phi^\dagger(\tau))^N / P^{(\tau)}(N) \xrightarrow{N \text{ increases}} |u_0\rangle_{\rm A}\langle u_0| \cdots \text{ Pure State!}$$

- $|u_0\rangle_{\rm A}$ is the right eigenvector of $V_\phi(\tau) = {}_{\rm X}\langle\phi|e^{-iH_{\rm tot}\tau}|\phi\rangle_{\rm X}$ belonging to λ_0.
- $|u_0\rangle_{\rm A}$ is irrespective of the initial mixed state $\varrho_{\rm tot}$.

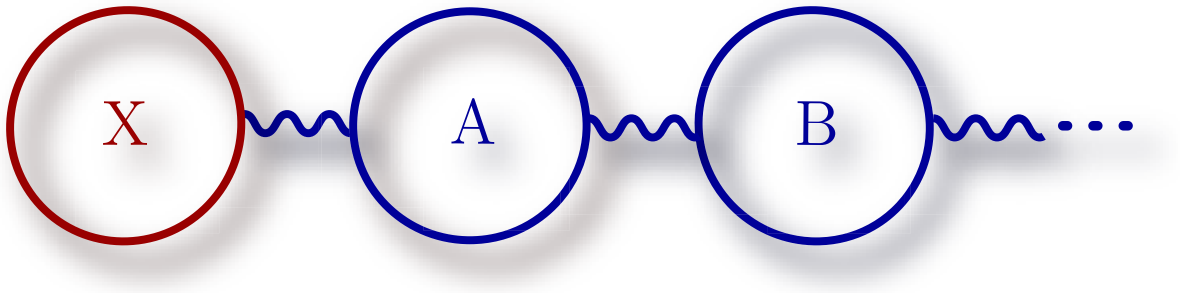
$$P^{(\tau)}(N) \sim |\lambda_0|^{2N}{}_{\rm XA}\langle\phi v_0|\varrho_{\rm tot}|\phi v_0\rangle_{\rm XA} \quad (0 \le |\lambda_n| \le 1)$$

Optimization

- $|\lambda_0| = 1 \quad \to \quad$ suppression of the decay of $P^{(\tau)}(N)$
- $|\lambda_n/\lambda_0| \ll 1 \quad \to \quad$ faster purification

- adjust τ, $|\phi\rangle_{\rm X}$, $H_{\rm tot}$

Possible Applications to Qubit-Systems



$$H_0 = \Omega \frac{1+\sigma_3^{\mathrm{X}}}{2} + \Omega \frac{1+\sigma_3^{\mathrm{A}}}{2} + \Omega \frac{1+\sigma_3^{\mathrm{B}}}{2} + \cdots$$

$$H'_{\mathrm{XA}} = g(\sigma_+^{\mathrm{X}}\sigma_-^{\mathrm{A}} + \sigma_-^{\mathrm{X}}\sigma_+^{\mathrm{A}}), \quad H'_{\mathrm{AB}} = g(\sigma_+^{\mathrm{A}}\sigma_-^{\mathrm{B}} + \sigma_-^{\mathrm{A}}\sigma_+^{\mathrm{B}}), \quad \ldots$$

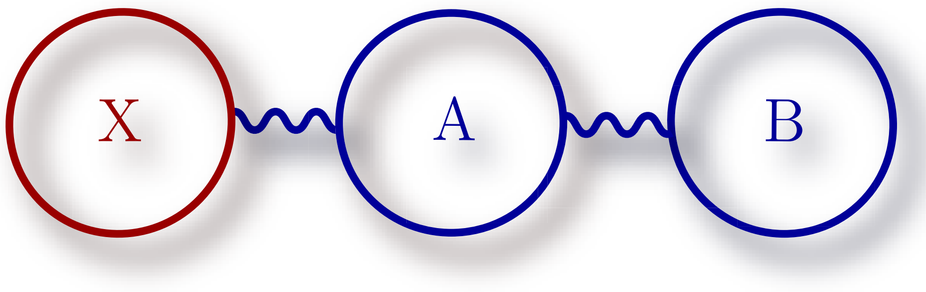
Fidelity

$$F^{(\tau)}(N) \equiv {}_{\mathrm{A}}\langle u_0|\varrho_{\mathrm{A}}^{(\tau)}(N)|u_0\rangle_{\mathrm{A}}$$

Success Probability

$$P^{(\tau)}(N) \sim |\lambda_0|^{2N}\,{}_{\mathrm{XA}}\langle \phi v_0|\varrho_{\mathrm{tot}}|\phi v_0\rangle_{\mathrm{XA}}$$

Initialization of Qubits

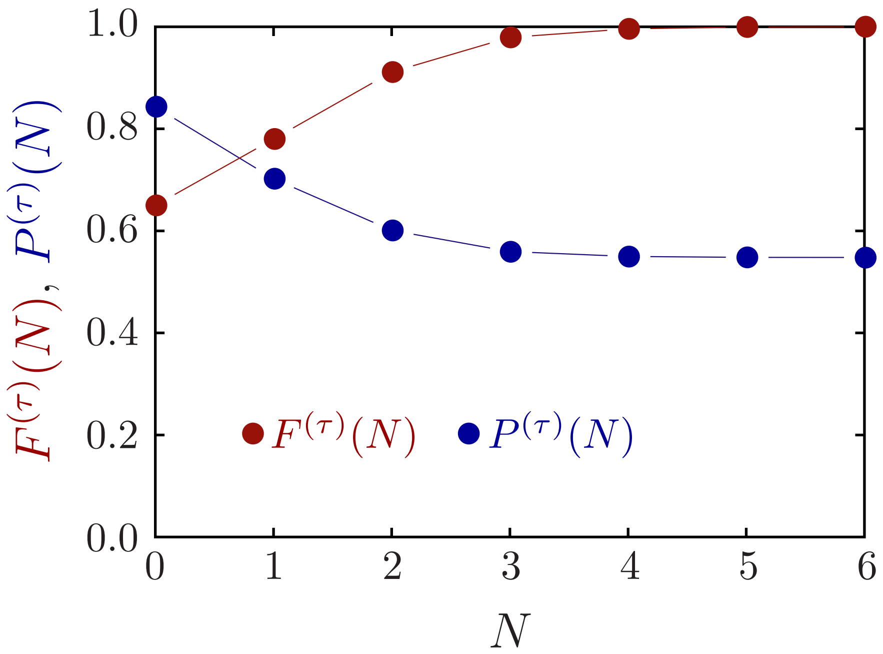


Repeated Confirmations of $|\downarrow\rangle_X$
(Zeno-like Measurements)
$\Downarrow$
Initialization of Qubits A and B

$$\underset{\text{mixed}}{\varrho_{AB}} \longrightarrow \underset{\text{initialized}}{|\downarrow\downarrow\rangle_{AB}\langle\downarrow\downarrow|}$$

Tuning

1. $|\phi\rangle_X = |\downarrow\rangle_X$
   $\Leftarrow$ for $|\lambda_0| = 1$
2. $\sqrt{2}g\tau \neq n\pi \ (n = 1, 2, \dots)$
   $\Leftarrow$ for the uniqueness of λ_0
3. $\sqrt{2}g\tau = 2n\pi \pm \zeta, \zeta = \tan^{-1}(2\sqrt{2})$
   $\Leftarrow$ for a quick initialization



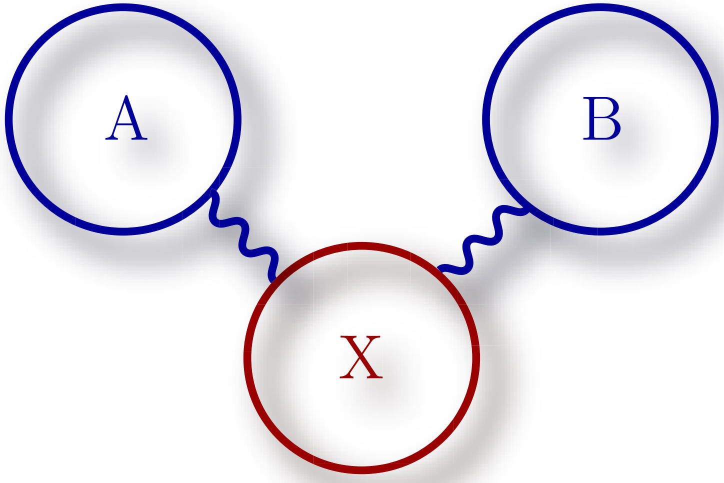
Initialization with the "optimal" prob.

$${}_{XAB}\langle\downarrow\downarrow\downarrow|\varrho_{tot}|\downarrow\downarrow\downarrow\rangle_{XAB}$$

$$e^{-\beta H_{tot}} \to |\downarrow\downarrow\downarrow\rangle_{XAB}\langle\downarrow\downarrow\downarrow|$$

$\Omega = 2, \quad g = 1, \quad \tau \simeq 1.73, \quad \beta = 1$

Entanglement Purification



Repeated Measurements on X
(Zeno-like Measurements)

$\Downarrow$

Entanglement between A and B

$$\varrho_{\mathrm{AB}} \longrightarrow |\Psi^-\rangle_{\mathrm{AB}}\langle\Psi^-|$$

mixed — Bell state

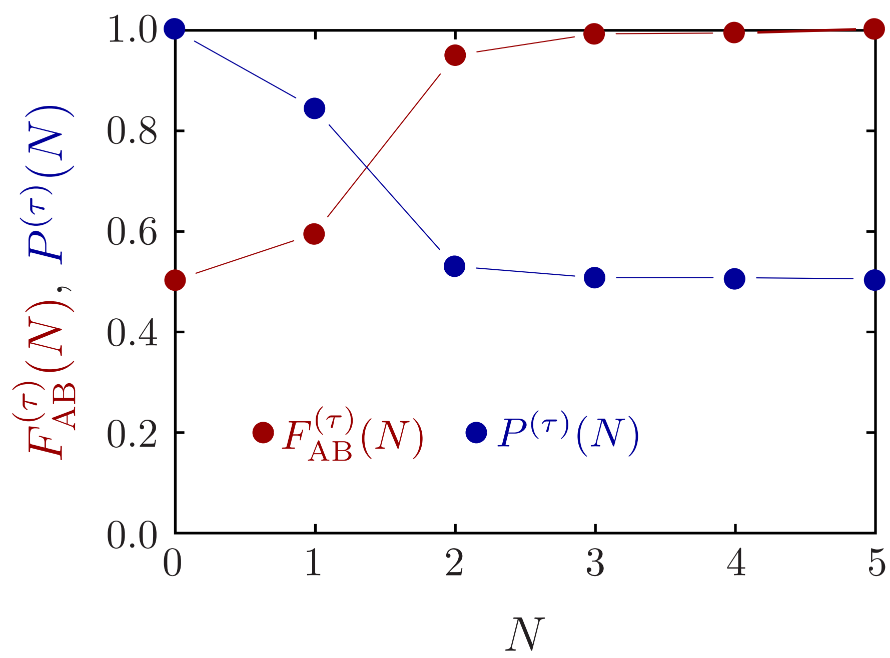
$$|\Psi^-\rangle_{\mathrm{AB}} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle_{\mathrm{AB}} - |\downarrow\uparrow\rangle_{\mathrm{AB}})$$

Tuning

1. $|\Omega|\tau = 2n\pi \;\; (n = 0, 1, \dots)$
   $\Leftarrow$ for $|\lambda_{\Psi^-}| = 1$
2. $|\phi\rangle_{\mathrm{X}} \neq |\uparrow\rangle_{\mathrm{X}}, |\downarrow\rangle_{\mathrm{X}}$ and $\sqrt{2}g\tau \neq n\pi$
   $\Leftarrow$ for the uniqueness of λ_0
3. $\sqrt{2}g\tau = 2n\pi \pm \zeta,\ \zeta = \tan^{-1}(2\sqrt{2})$
   $\Leftarrow$ for a quick purification

with the "optimal" prob.

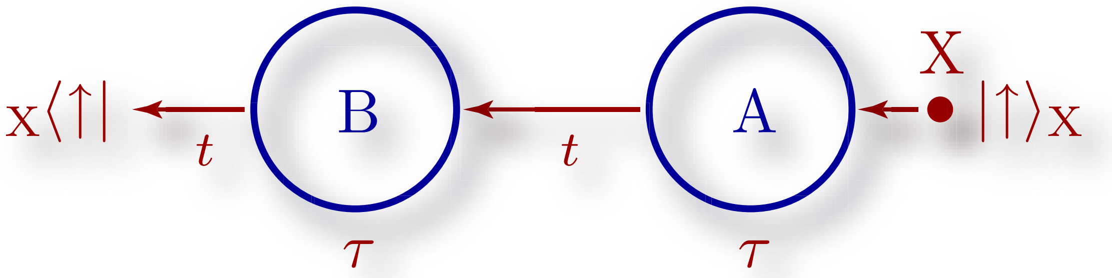
$${}_{\mathrm{XAB}}\langle\phi\Psi^-|\varrho_{\mathrm{tot}}|\phi\Psi^-\rangle_{\mathrm{XAB}}$$



$$|\rightarrow\rangle_{\mathrm{X}} \otimes |\uparrow\rangle_{\mathrm{A}} \otimes |\downarrow\rangle_{\mathrm{B}} \rightarrow |\rightarrow\rangle_{\mathrm{X}} \otimes |\Psi^-\rangle_{\mathrm{AB}}$$

$$\Omega = 0, \quad g\tau \simeq 0.5$$

Entanglement between *Separated* Qubits I



$$H'_{\mathrm{XA}} = g\sigma_1^{\mathrm{X}}\sigma_1^{\mathrm{A}}, \quad H'_{\mathrm{XB}} = g\sigma_1^{\mathrm{X}}\sigma_1^{\mathrm{B}}$$

Repeatedly Throwing X

$\Downarrow$

Entanglement between A and B

$\varrho_{\mathrm{AB}} \longrightarrow |\Psi\rangle_{\mathrm{AB}}\langle\Psi|$

mixed   entangled state

$$|\Psi\rangle_{\mathrm{AB}} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle_{\mathrm{AB}} - e^{i\chi}|\downarrow\uparrow\rangle_{\mathrm{AB}})$$

Projected Time-Evolution Op.

$$V_{\uparrow\uparrow} = {}_{\mathrm{X}}\langle\uparrow|e^{-iH_0t}e^{-i(H_0+H'_{\mathrm{XB}})\tau}e^{-iH_0t}e^{-i(H_0+H'_{\mathrm{XA}})\tau}|\uparrow\rangle_{\mathrm{X}}$$

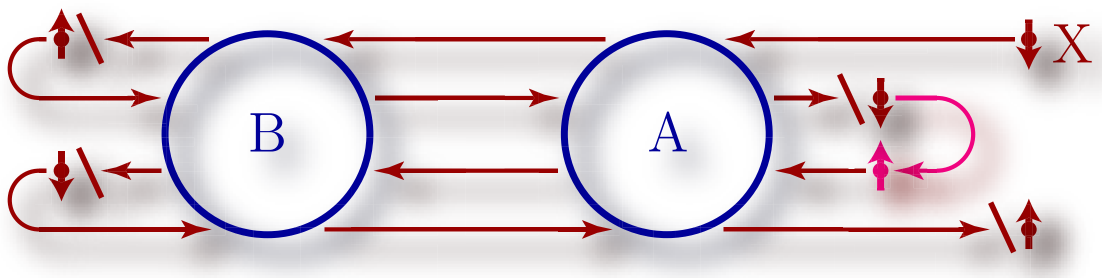
Tuning

$$\cos\sqrt{\Omega^2+g^2}\,\tau - i\frac{\Omega}{\sqrt{\Omega^2+g^2}}\sin\sqrt{\Omega^2+g^2}\,\tau = -e^{i\Omega t}\cos g\tau$$

for $|\lambda_\Psi| = 1$ and the uniqueness of λ_0

cf., A. Messina, Eur. Phys. J. D **18**, 379 (2002) and
D. E. Browne and M. B. Plenio, Phys. Rev. A **67**, 012325 (2003),
where the initial states ϱ_{tot} should be carefully prepared.

Entanglement between *Separated* Qubits II



$$H'_{\rm XA} = g_{\rm A}(\sigma_+^{\rm X}\sigma_-^{\rm A} + \sigma_-^{\rm X}\sigma_+^{\rm A}),$$
$$H'_{\rm XB} = g_{\rm B}(\sigma_+^{\rm X}\sigma_-^{\rm B} + \sigma_-^{\rm X}\sigma_+^{\rm B})$$

Throwing X, *4 Times*

$\Downarrow$

Entanglement between A and B

$$\underset{\text{mixed}}{\varrho_{\rm AB}} \longrightarrow \underset{\text{Bell state}}{|\Psi^\pm\rangle_{\rm AB}\langle\Psi^\pm|}$$

$$|\Psi^\pm\rangle_{\rm AB} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle_{\rm AB} \pm |\downarrow\uparrow\rangle_{\rm AB})$$

Projected Time-Evolution Op.

$$V_{\rm c} = V_{\uparrow\downarrow\uparrow}V_{\downarrow\uparrow\downarrow}$$

$$V_{\downarrow\uparrow\downarrow} = {}_{\rm X}\langle\downarrow|U_{\rm I}^{(\rm A)}(\tau_{\rm A})U_{\rm I}^{(\rm B)}(\tau_{\rm B})|\uparrow\rangle_{\rm X}\langle\uparrow|U_{\rm I}^{(\rm B)}(\tau_{\rm B})U_{\rm I}^{(\rm A)}(\tau_{\rm A})|\downarrow\rangle_{\rm X}$$
$$V_{\uparrow\downarrow\uparrow} = {}_{\rm X}\langle\uparrow|U_{\rm I}^{(\rm A)}(\tau_{\rm A})U_{\rm I}^{(\rm B)}(\tau_{\rm B})|\downarrow\rangle_{\rm X}\langle\downarrow|U_{\rm I}^{(\rm B)}(\tau_{\rm B})U_{\rm I}^{(\rm A)}(\tau_{\rm A})|\uparrow\rangle_{\rm X}$$

Tuning

$$\cos g_{\rm A}\tau_{\rm A} = 0 \quad \text{and} \quad \sin g_{\rm B}\tau_{\rm B} = \pm\frac{1}{\sqrt{2}}$$

for $|\lambda_{\Psi^-}| = 1$ and the uniqueness of λ_0

- Success Prob. $P = {}_{\rm AB}\langle\Psi^\pm|\varrho_{\rm AB}|\Psi^\pm\rangle_{\rm AB}$ $\cdots$ "optimal"

Summary

- Simple (just repeat one and the same measurement).

  (⇔ combination of rotation, CNOT operation, measurement, . . .)
- High fidelity with high success probability after a few steps.
  - ▷ $|\lambda_0| = 1, \quad |\lambda_1| \ll 1$
- "Optimal" success probability $P^{(\tau)}(N) \to {}_{\mathrm{XA}}\langle\phi\Psi|\varrho_{\mathrm{tot}}|\phi\Psi\rangle_{\mathrm{XA}}$ is possible.
  - ▷ The target state $|\phi\Psi\rangle_{\mathrm{XA}}$ contained in the initial state ϱ_{tot} is fully extracted.

    (⇔ decays to zero)
- General & Flexible
  - ▷ It accepts various experimental settings.
  - ▷ Many potential generalizations.

    (⇔ special design for a specific system)

- Robustness ?
  - ▷ Precise tuning of parameters ?
  - ▷ Errors in Measurements ?
- Experimentally feasible setups ?
  - ▷ Instead of repeating measurements;

    Continuous application of an external field.
  - ▷ "One-photon" scheme ⇒ "Multi-photon" scheme, using coherent light