NANTEL BERGERON
York

Schubert polynomials

I will review some of the combinatorics associated with the schubert polynomials and discuss the multiplicative structure problem.

MEGUMI HARADA
Toronto

Schubert basics

In this introductory lecture I'll introduce several of the main characters in the Schubert calculus story. First we meet the Grassmannian and more general flag varieties, thought of as homogeneous spaces of $GL(n,\mathbb{C})$ or $U(n,\mathbb{C})$. Then we construct a natural torus action on these flag varieties and the Bruhat cell decomposition. I'll discuss the Bruhat ordering and Schubert varieties as closures of Bruhat cells. Time permitting, I'll say something about how this works in the general case (not just $GL(n,\mathbb{C})$).

MEGUMI HARADA
Toronto

Equivariant cohomology and GKM theory for flag varieties

In 1998, Goresky, Kottwitz, and MacPherson showed that for certain projective varieties X with a torus action T, the equivariant cohomology ring $H^*_{T(X)}$ can be described by combinatorial data obtained from its orbit decomposition. I will explain in some detail how this description works, concentrating on the case of the flag varieties discussed so far. Then, time permitting, I will give some indications of recent work by myself, Henriques, and Holm, generalizing this to other equivariant cohomology theories, and to (possibly infinite-dimensional) flag varieties G/P of a Kac-Moody group G.

JOEL KAMNITZER
UC Berkeley

Mirkovic-Vilonen cycles

The purpose of this talk is to introduce a family of varieties, called Mirkovic-Vilonen (MV) cycles, which generalize the Schubert varieties and which play an important role in geometric representation theory. We will start by discussing the affine Schubert varieties which generalize the Grassmannians. The study of the attracting cells for the torus action on these affine Schubert varieties leads to the MV cycles. These MV cycles are connected to representation theory by the geometric Satake correspondence. This will generally be an expository talk (following Mirkovic-Vilonen) but if time permits, we discuss some new results concerning a combinatorial description of MV cycles.

AUGUSTIN-LIVIU MARE
Regina

Quantum cohomology of full flag manifolds I. Generators and relations

The (small) quantum cohomology ring $QH^*(G/B)$ of the full flag manifold G/B is by now a well understood object. The main object I will discuss in my two presentations is the combinatorial quantum cohomology ring $QH_c^*(G/B)$. As the name suggests, this is a purely algebraic object, associated to the root system of G. In the first part of my presentation I will define this ring and I will show how can one describe it in terms of generators and relations. More precisely, I will show that Bumsig Kim's presentation of the ring $QH^*(G/B)$ can be deduced from a limited number of properties of the latter ring, call them (P). The key fact is that $QH_c^*(G/B)$ satisfies the properties (P).

AUGUSTIN-LIVIU MARE
Regina

Quantum cohomology of full flag manifolds II. The quantum Chevalley formula

I will sketch a proof of the fact that the rings $QH^*(G/B)$ and $QH_c^*(G/B)$ are identical (in the first part I have proved that they are isomorphic). The idea is that the properties (P) mentioned in the previous talk characterize the ring uniquely. This will enable us to deduce two important results concerning $QH^*(G/B)$: formulas for quantum Giambelli (or rather Berstein-Gelfand-Gelfand) polynomials, and the quantum Chevalley formula (this had been proved by Peterson and Fulton-Woodward). If the time permits, at the end I will also sketch the definition of $QH^*(G/B)$ and proofs of the properties (P) for this ring.

LEONARD MIHALCEA
Michigan

Equivariant quantum Schubert calculus

The (small) equivariant quantum cohomology (eq.q.coh.) of a homogeneous variety $X = G/P$ is an algebra which is a deformation of both equivariant and quantum cohomology algebras of X. It was introduced by A. Givental and B. Kim primarily to study the quantum cohomology of X.

The aim of this talk is to present several properties of the eq.q.coh. algebra when X is a Grassmannian. One is a eq. q. Pieri-Chevalley formula, i.e. a multiplication by a complex degree 1 cohomology class. Remarkably, this formula implies an algorithm to compute the eq. q. structure constants - which in this case are some polynomials. If time permits I will also mention some open problems in the field.

KONSTANZE RIETSCH
King's College London

Geometric realizations of quantum cohomology of SL_n/P

By a beautiful theory of Dale Peterson's, the quantum cohomology ring of G/P is the coordinate ring of some (not-necessarily reduced, and usually not irreducible) affine variety. We will describe this theory explicitly in type A and make some connections with total positivity.

BRIAN ROTHBACH
UC Berkeley

Some comments about varieties defined by rank conditions

A (type A) Schubert variety can be defined as the set of all matrices in SL_n/B satisfying certain rank conditions. One can then pull back the conditions to $M_n(K)$ to get matrix Schubert varieties. One can further generalize by looking at certain unions of matrix Schubert varieties that are defined set-theoretically by rank conditions. I will show that the corresponding ideals for these varieties are reduced, and give an easy combinatorial formula for the dimension of the top dimensional components of any such variety.

ALISTAIR SAVAGE
Toronto

Cohomology of flag varieties

In this introductory lecture, we will cover the cohomology of the full flag varieties and its intersection pairing. We will see how this gives us a specific relationship between the cohomology class of Schubert varieties (the Schubert classes) and Schubert polynomials.

ALISTAIR SAVAGE
Toronto

Geometric representation theory on flag varieties

In this expository talk, we will describe a construction due to Ginzburg which endows the homology of flag varieties and certain other, closely related, varieties with the structure of representations of $\mathfrak{sl}_n$. The action of $\mathfrak{sl}_n$ is described in terms of natural geometric operations and the construction yields bases in the representations with remarkable properties.

JULIANNA TYMOCZKO
Michigan

Regular semisimple Hessenberg varieties

Hessenberg varieties are a family of subvarieties of the flag variety that are in a colloquial sense "transverse" to Schubert varieties. In this talk, we describe a particular kind of Hessenberg varieties: regular semisimple Hessenberg varieties. We collect what's known about these varieties, including geometric properties (like smoothness) and combinatorial properties (like how they count the number of descents of a permutation). We also describe some preliminary results about their equivariant cohomology and pose some questions about how regular semisimple Hessenberg varieties relate to Schubert varieties.

MATTHIEU WILLEMS
Toronto

K-Theory

In this introductory talk, I will give basic definitions on the ordinary and equivariant K-theory rings and I will give a description of these rings in the case of flag varieties.

MATTHIEU WILLEMS
Toronto

Equivariant K-theory of Bott-Samelson varieties and flag varieties

In this talk, I will define Bott-Samelson varieties and explain the link with flag varieties. I will give a description of the equivariant K-theory of Bott-Samelson varieties and thanks to this description I will deduce results on the equivariant K-theory of flag varieties.

ALEXANDER WOO
UC Berkeley

Introduction to the Singularities of Schubert Varieties

I will attempt to give an elementary and broad introduction to the study of singularities of Schubert varieties, which includes questions on where Schubert varieties are singular, and on various numerical invariants of the singularities such as multiplicity and Kazhdan-Luzstig polynomials. I am by no means an expert on this subject; this is an area I have read only a little about and want to think about some more.

ALEXANDER YONG
Berkeley/Fields Institute

Schubert calculus: algebraic geometry and combinatorics

In this talk, I will provide an overview of some of the classical motivations and modern themes of research that will be discussed during the week.

ALEXANDER YONG
Berkeley/Fields Institute

Degeneracy loci

Fix an variety X and a pair of vector bundles E and F or ranks e and f with a morphism Φ between them. Describe the *degeneracy locus* Y of X where this morphism gives a linear transformation Φ_x with a less than expected rank $k \leq min(e, f)$.

Reformulations of this problem (and its generalizations) arise in topology, commutative algebra, representation theory of quivers, combinatorics, as well as algebraic geometry – and with beautiful connections between one another.

I'll explain some of the problems that arise in the subject and describe some potential avenues for future work.