ARITHMETIC AND GEOMETRY OF ALGEBRAIC VARIETIES
WITH SPECIAL EMPHASIS ON
CALABI–YAU VARIETIES AND MIRROR SYMMETRY
OCTOBER 21–22, 2006

ABSTRACTS

OCTOBER 21, 2006

9:30am: Matthias Schuett (Harvard University)

CM-modular forms and Calabi–Yau varieties

Almost all known examples of higher-dimensional modular Calabi-Yau varieties are related to newforms with complex multiplication. Here, higher-dimensional should be understood as dimension 2 or at least 4. The aim of the talk will therefore be to give a classification of CM- newforms with rational coefficients. We will particularly emphasize the case of weight 3 which arises in connection with singular K3 surfaces.

10:30am: Cheol Hyun Cho (University of Toronto)

On the counting of J-holomorphic discs

We will explain that the count of J-holomorphic discs with Lagrangian boundary condition in a manifold with involution is well-defined under some conditions. We show examples that shows ill-definedness in general situations.

11:30am: Shuang Cai (University of Alberta)

A simple definition of algebraic connective K-theory

Two homology theories stand out prominently for a given quasiprojective variety X: the Chow group and the algebraic K-group, owing both to their interesting internal structures and relations to various other properties of the variety.

In light of Voevodsky's recent work on motivic cohomologies, one naturally seeks the unification of the Chow group and algebraic K-theory for X. One also favors to have a description of such a theory as clear and straightforward as possible.

In this work, we aim to progress toward this goal. We first need to place Chow groups and K groups as pieces in the bigger frame of doubly indexed homology theories for schemes over some field k, a.k.a., oriented Borel-Moore functors. In this theoretical setting, we claim the correct objects to be located lie in the Gerstein-Brown-Quillen spectral sequence of X. In more concrete terms, they are the image of the natural maps: $K_n(\mathcal{M}_{p-1}(X)) \longrightarrow K_n(\mathcal{M}_p(X))$, where $\mathcal{M}_p(X)$ is the category of sheaves on X with support of dimension no greater than p. We denote this image by $CK^2_{p,n-p}(X)$. It is immediately clear that $CK^2_{p,n-p}(X)$ admits natural maps to the corresponding doubly-indexified Chow group and K-group. The task now is to verify that these CK groups form oriented Borel-Moore functors.

This amounts to verifying the following properties: (i) correct functorial properties including the push-forwards and pull-backs, and essential properties of an oriented homology theory which are (ii) localization exact sequences and (iii) homotopy invariance properties (from which follows the existence of Chern classes).

Now that the basic definition has been set. We may start to investigate its actual connection to Chow groups and K-groups. Given a quasi-projective variety X, we first embed $Ch_n(X)$ as the $A_{n,-n}(X)$ term in a double indexed Borel-Moore Theory $A_{p,q}(X)$, or the K-homology groups as they came to be called. We also extend the definition of K-groups by formally stipulating $K_{p,q}(X) = K_{p+q}(X)\beta^p$, in particular, β

is identified with a generator of $K_{1,-1}(k) \cong CK_{1,-1}(k)$. We prove that $A_{*,*}(X) = CK_{*,*}/\beta CK_{*,*}(X)$ and $K_{*,*}(X) = CK_{*,*}(X)_\beta$.

2:30pm: Kentaro Hori (University of Toronto)

Phases of $N = 2$ theories in $1 + 1$ dimensions with boundary

Let $G(x_1, ..., x_n)$ be a homogeneous polynomial of degree n such that $G = 0$ defines a smooth Calabi-Yau hypersurface X_G in the projective space. We derive equivalences of the category of graded matrix factorizations of G and the derived category of coherent sheaves on X_G, which were originally constructed by D. Orlov. In physics terms, we find maps of D-branes in the Landau-Ginzburg orbifold and D-branes in the geometric regime. We find the equivalences using supersymmetric gauge theories in $1 + 1$ dimensions with boundary. This has potential applications to stability conditions and (homological) mirror symmetry. This is a joint work with Manfred Herbst and David Page.

3:30pm: Shengda Hu (University of Montreal)

T-duality and H-flux from generalized Kähler geometry

The construction of reduction by Poisson Lie group action in Poisson geometry can be extended to generalized complex geometry. A generalized Kähler manifold has two compatible generalized complex structures. We present a construction in generalized Käher geometry via Poisson Lie reduction on either of the generalized complex structure. The two quotient manifolds thus obtained are possible candidates of a T-dual pair with H-flux.

4:30pm: Vincent Bouchard (Perimeter Institute, Waterloo)

Orbifold Gromov–Witten invariants from mirror symmetry

The B-model topological string partition function on a Calabi-Yau threefold X is a wave function of a quantum mechanical system with phase space $H^3(X)$. As such, it transforms in a well defined way under canonical transformations of the phase space. We thus obtain a simple way of relating the B-model topological string partition function near different points in the complex structure moduli space. In this talk I will explain how this point of view can be used to give predictions for certain orbifold Gromov-Witten invariants, the key role being played by mirror symmetry.

5:30pm: Nam-Hoon Lee (KIAS and Queen's University)

Some attempts at constructing infinitely many families of Calabi–Yau manifolds

Some Calabi-Yau construction by smoothing normal crossings, that may possibly lead to construction of infinitely many families of Calabi-Yau manifolds, will be discussed. It depends on the existence of a specific type of threefolds and some K3 surfaces. As byproducts, we will also discuss some generalization of Enriques Calabi-Yau threefolds.

OCTOBER 22, 2006

9:30am: Jeng-Daw Yu (Queen's University)

Special liftings of ordinary $K3$ surfaces and applications

Given an ordinary K3 surface over a finite field, one can lift it canonically to a K3 surface, the so-called canonical lift, over characteristic zero. In this talk, we study the cohomology of this canonical lift. In particular, we investigate the Hodge structure of the Betti cohomology and apply it to the Hodge conjecture of self product of certain K3 surfaces.

10:30am: Alina Cojocaru (The Fields Institute and University of Illinois at Chicago)

Frobenius fields for elliptic curves

Let E/Q be an elliptic curve without complex multiplication and let K be an imaginary quadratic field. In 1976, Lang and Trotter conjectured that the number of primes $p < x$ for which the Frobenius fields of E at p equals K is asymptotically $\sqrt{x}/logx$. I will discuss ways of obtaining non-trivial upper bounds for this set of primes.

(Joint with Chantal David.)

11:30pm: Reinier Broker (The Fields Institute)

***p*-adic class invariants**

The theory of complex multiplication provides us with a means of computing a generating polynomial for the Hilbert class field of a given imaginary quadratic number field. The classical approach of using the modular j-function yields polynomial with huge coefficients, and as was discovered by Weber already, we can do better by using 'smaller' functions.

In this talk we focus on new p-adic algorithms to compute such generating polynomials. For the j-function this goes back to a paper of Couveignes and Henocq, and we explain how to generalize their approach to cope with smaller functions over p-adic fields by using modular curves.

12:30pm: Noriko Yui (Queen's University)

Quasimodular forms and mirror symmetry for elliptic curves

We look into the formula due to Douglas and Dijkgraaf on the generating function, $F_g(q)$, of the number of simply ramified covers of genus $g \geq 1$ over a fixed elliptic curve. Their result is that $F_g(q)$ is a quasimodular form of weight $6g - 6$ on the full modular group $PSL_2(Z)$.

There are two ways of computing $F_g(q)$: the fermionic count and the bosonic count. The fermionic counting is a mathematical treatment. However, the bosonic counting rests on physical arguments, which involves Feynman diagrams (and integrals) on trivalent graphs.

I will report on our attempt to understand this fascinating formula for the generating function $F_g(q)$ from the mathematical point of view.

This is a joint work with Mike Roth (Queen's) and Brendan McLellan (Toronto).