Plenary Talks

LUC DEVROYE
McGill University

Tries and tribulations

In 1960, Fredkin proposed the "trie" data structure for storing strings. Since then, random tries have been studied at length, starting with important work by Pittel in the mid eighties. There is renewed interest in various trie parameters thanks to the resurgence of "stringology".

We will explain the structure of random tries for the simple model in which each string consists of an i.i.d. sequence of symbols drawn from a fixed discrete distribution. In particular, we will look at the profile of a trie, the core of a trie, its height and fill-up level, and generalizations to weighted tries.

HADI KHARAGHANI
University of Lethbridge

The Hadamard determinant conjecture

Currently all evidence strongly suggests the validity of the *Hadamard determinant conjecture*, which states that for each odd integer n there is a ± 1-matrix of order $4n$ with orthogonal rows. The status of the Hadamard conjecture with emphasis on two construction methods, one by Turyn and another by Williamson, will be discussed.

R. BRUCE RICHTER
University of Waterloo

On the cycle spaces of infinite graphs

If G is a locally finite infinite graph and K is any compactification of G, then there is a cycle space associated with K that involves all the cycles – including infinite ones arising, for example, from a 2-way infinite path in G having both ends converging to the same point of K. It is natural to allow restricted infinite sums in this context, so the cycle space is more than just a vector space. If K and K' are two compactifications of the same graph G, and f : K -¿ K' is a continuous function, then the cycle space associated with K' is larger than the one for K. We discuss how to minimally extend a generating set of the K-cycle space to a generating set for the K'-cycle space.

CATHERINE YAN
Texas A and M University

Crossings and Nestings in Combinatorics

In this talk we survey the recent progress on crossings and nestings in combinatorial structures. By a variant of the RSK algorithm from algebraic combinatorics, set partitions are in one-to-one correspondence with vacillating tableaux, which are certain random walks in the Hasse diagram of the lattice of integer partitions. It follows that the crossing numbers and the nesting numbers are distributed symmetrically over all partitions of [n]. In particular, the number of k-noncrossing partitions is equal to the number of k-nonnesting partitions. This technique has been extended to other geometric configurations including perfect matchings, fillings of Ferrers shapes, linked cycles, and multi-graphs, and has led to similar results between noncrossing and nonnesting structures.

WENAN ZANG
University of Hong Kong

Recent Progress on Polyhedral Combinatorics

Combinatorial optimization searches for an optimal object in a finite collection; typically the collection has a concise representation while the number of objects is huge. Polyhedral and linear programming techniques have proved to be very powerful and successful in tackling various problems in this area, and the end products of these methods are often integral polyhedra or min-max relations. In this talk I shall give a brief survey of our recent results on polydedral combinatorics, including min-max relations on packing cycles, min-max relations on packing feedback vertex sets, a characterization of all matroids with the box max-flow min-cut property, and the complexity of recognizing totally dual integral (TDI) systems.

Contributed Talks

PAUL ELLIOT-MAGWOOD
University of Ottawa

Generating Extreme Points of the Subtour Elimination Polytope

The Travelling Salesman Problem (TSP) is the problem of finding a minimum weight Hamilton cycle in a weighted complete graph on n vertices. Since this problem is well-known to be NP-hard, one direction which seems promising for finding good solutions is to study the structure of the extreme point solutions associated with the problem's linear programming (LP) relaxation. In this talk we will discuss the structure of the extrme points of the Subtour Elimination Polytope (SEP) which is the set of all feasible solutions to the LP relaxation of the TSP. We give some new results on both the underlying graph structure of these extreme points, as well as the structure of the cuts which define such solutions. We demonstrate the usefulness of these results by showing how this new theory can facilitate the generation of all extreme points of the SEP for some values of n that were previously unattainable. This allows us, for the first time with these values of n, to verify the well-known conjecture that the integrality gap is 4/3 for the metric TSP. Authors: Sylvia Boyd and Paul Elliott-Magwood

TONY EVANS
Wright State University

Representations of Graphs modulo n: Some Problems

A graph G has a representation modulo n if there exists an injective map $f : V(G) \to \{0, 1, ..., n\}$ such that vertices u and v are adjacent if and only if $f(u) - f(v)$ is relatively prime to n. The representation number $rep(G)$ is the smallest n such that G has a representation modulo n. We pose a collection of related open problems. Authors: A. B. Evans, D. A. Narayan, and J. Urick

PU GAO
University of Waterloo

Short cycle distribution in some recursively generated random regular graphs

We introduce a new method of generating random d-regular graphs by repeatedly applying an operation called pegging. The pegging operation is abstracted from a type of basic operation applied in SWAN network. We prove that for the resulting graphs, the limiting joint distribution of the numbers of short cycles is independent Poisson. We also estimate the rate at which the distribution approaches its limit.

CARLOS HOPPEN
University of Waterloo

Independent Sets in Graphs with Large Girth

This talk is devoted to the analysis of a family of randomised algorithms that find an independent set in a regular graph G. The expected size of the set they output provides a lower bound on the maximum cardinality of an independent set in G. The bounds obtained for large girth can be shown to hold asymptotically almost surely for random regular graphs.

SASCHA KURZ
University of Bayreuth

Integral point sets over finite fields

An integral point set $\mathcal{P}$ over a finite field $\mathbb{F}_q$ is a set of n points in $\mathbb{F}_q^2$ with pairwise integral distances. A pair (x_1, y_1), $(x_2, y_2) \in \mathbb{F}_q^2$ is at integral distance if there exists $d \in \mathbb{F}_q$ with $(x_1-x_2)^2+(y_1-y_2)^2 = d^2$. By $\mathcal{I}(\mathbb{F}_q, 2)$ we denote the maximum cardinality of an integral point set in $\mathbb{F}_q^2$. We prove $\mathcal{I}(q, 2) = q$ for $2 \nmid q$ and classify the extremal examples. By $\overline{\mathcal{I}}(\mathbb{F}_q, 2)$ we denote the maximum cardinality if additionally no three points are collinear. We give a conjecture and solve the case for $q \equiv 3 \mod 4$. Similar to integral point sets over $\mathbb{R}^2$ the extremal examples seem to consist of point sets on a circle. Forbiding four points on a circle leads to the numbers $\dot{\mathcal{I}}(\mathbb{F}_q, 2)$. The discovery of big examples i.e. leading to $\dot{\mathcal{I}}(\mathbb{F}_{71}, 2) = 11$ are a motivation! to search with a bit more effort for seven points in the plane $\mathbb{R}^2$ no three on a line, no four on a circle with integral coordinates and pairwise integral distances. We give an up-to-date overview on the latest computational result of this Erdős question.

PIERRE LEROUX
Universite du Quebec a Montreal

2-connected graphs with prescribed three-connected components

We will first review the classical, but not so well known, 3-factorization of any 2-connected graph, adapted to simple (no loops or multiple edges) graphs. By analogy with the block-cutpoint tree of a connected graph, we deduce from this decomposition a bicolored tree tc(g) associated to any 2-connected graph g , whose white vertices are either polygons or 3-connected components of g and whose black vertices are edge-bonds linking together these polygons and 3- connected graphs, arising, as in two-sums, from separating pairs of vertices of g. Two fundamental relationships on graphs and networks follow from this construction. They are expressed in the algebra of species (or classes) of structures, involving operations of sum, product, substitution and a special substitution, of networks for edges.

The first relationship is a functional equation which characterizes the class R=R(F) of networks all of whose irreducible (3-connected) components belong to a given class F of 3-connected graphs. It can be seen as a structural reformulation of Trakhtenbrot's theorem. The second one is a dissymmetry theorem which permits the expression of the corresponding class B=B(F) of 2-connected graphs, all of whose 3- connected components are in F, in terms of F and R(F). Some enumerative consequences will also be discussed.

This is joint work with Andrei Gagarin, Gilbert Labelle and Timothy Walsh.

DANIEL J. MCQUILLAN
Norwich University

Vertex-magic total labelings of regular graphs

A total labeling of a graph with v vertices and e edges is a bijective map from 1,2,...,v+e into the vertices union edges of the graph. The weight of a vertex is the sum of its incident edge labels added to the label on the vertex itself. A total labeling is called vertex-magic if the weight of every vertex is the same. MacDougall has conjectured that each regular graph of degree at least 2 must have a vertex-magic total labeling, except for one exceptional graph (a union of two 3-cycles). We will discuss some progress on MacDougall's conjecture, and outline other unsolved problems.

REZA NASERASR
University of Waterloo

Cirular chormatic number of the unit distance graph

The unit distance graph is a graph with points of the plane as its vertices and with two vetices being adjacent if and only if they are at unit distance. The chromatic number of this graph is an long standing open problem. We will consider its cicular chormatic number, which is a refinement of chromatic number, and show that circular chromatic number of this graph is at least 4 (this is same known lower bound for the chromatic number).

This is a joint work with: Matt DeVos, Javad Ebrahimi, Mohammad Ghebleh, Luis Goddyn, Bojan Mohar.

SEBASTIAN RAAPHORST
University of Toronto

The existence of anti-pasch partial Steiner triple systems

A Steiner triple system of order v, denoted STS(v), exists for all v = 1 or 3 (mod 6); in the remaining cases, there are known constructions to generate maximum partial Steiner triple systems, denoted PSTS. A fairly active area of study in combinatorial design theory is the avoidance of certain substructures in designs: one of the simplest and most interesting such substructures is the pasch. In 2002, Grannell, Griggs, and Whitehead were able to resolve the remaining cases in the anti-pasch conjecture for STS(v), namely that such systems exist for v = 1, 3 (mod 6) and v not 7 or 13. This leads to direct constructions for anti-pasch PSTS of orders v = 0, 2 (mod 6). Nothing is known for v = 5 (mod 6), except that there is no maximum PSTS(11); again, through a construction, solving this case would concurrently solve the case v = 4 (mod 6), thus resolving the question of the existence of these systems completely.

I conjecture that maximum PSTS(v) that are anti-pasch exist for all v = 5 (mod 6) and v not 11. I will discuss why we're interested in such systems in the first place and some of their practical applications, and then present my work so far on this problem.

JOE SAWADA
University of Guelph

Generating Open Meandric Sequences (and meanders)

An open meandric system is a planar configuration of acyclic curves crossing an infinite horizontal line in the plane such that the curves may extend in both horizontal directions. We present a fast, recursive algorithm to exhaustively generate open meandric systems with n crossings. We then illustrate how to modify the algorithm to generate unidirectional open meandric systems (the curves extend only to the right) and non-isomorphic open meandric systems where equivalence is taken under horizontal reflection. Each algorithm can be modified to generate systems with exactly k curves. In the unidirectional case when $k = 1$, we can apply a small modification along with some additional optimization steps to generate meanders.

HU ZHANG
McMaster University

Minimizing the Number of Critical Vertices in Network Design

Given a weighted complete graph $G_K(V, E_K)$, we study a network design problem to find an edge set $E \in E_K$ such that the graph $G(V, E)$ is connected. The power of a vertex u in G is equivalent to the maximum weight of the edges in E incident with it. Minimizing the maximum vertex power is known to be polynomial time solvable. In this paper, for any fixed $\epsilon > 0$ we present a $(3/2 + \epsilon)$- approximation algorithm for minimizing the number of vertices with that maximum vertex power (critical vertices), which improves the previously best approximation ratio of $5/3$.

According to the definition of worst case analysis, we show that any solution E delivered by our approximation algorithm for an arbitrary instance is also a solution for an augmented instance, which enables us to find better lower bounds for the optimal solution of the original instance. We develop integer programs with nonlinear objectives covering all possible cases, whose optimal objective values provide upper bounds of the approximation ratio of our algorithm. Solving these integer programs yields our improved approximation ratio.

This is joint work with Tamas Terlaky and Anthony Vannelli.