MOTIVIC INTEGRATION AND ITS APPLICATIONS TO P-ADIC GROUPS

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Motivic integration was first introduced by M. Kontsevich in the context of algebraic geometry. Later J. Denef and F. Loeser developed a theory of arithmetic motivic integration, which specializes to integration over the p-adics. Denef and Loeser stated the general principle that "all natural p-adic integrals are motivic". The value of a motivic integral is a geometric object defined over the field of rational numbers. The value of the initial p-adic integral can be recovered from it by a procedure that generalizes counting points on the reduction of this object over the residue field. Replacing p-adic integrals with motivic integrals has three main advantages: it clarifies exactly in what way the p-adic integral depends on p; motivic integration is an algorithmic procedure (based on elimination of quantifiers), so motivic integrals are computable; finally, motivic calculations allow to switch freely between function fields and extensions of $\mathbb{Q}_p$.

Naturally, p-adic integrals appear prominently in the representation theory of p-adic groups (e.g. orbital integrals, and Harish-Chandra distribution characters). It is, however, a largely open question whether they are "natural" in the sense of Denef and Loeser.

Recently, R. Cluckers and F. Loeser introduced a new theory of motivic integration, which encompasses all the previous ones, has a large class of integrable functions, and is complete with all the tools expected of a theory of integration, such as Fubini theorem, and Fourier transform. The goal of these talks is to give an exposition of the theory from the perspective of applying it to the problems that arise in the context of representation theory of p-adic groups.