Ottawa – Carleton Discrete Mathematics Day 2008
Abstracts of invited speakers

- David Bremner, Faculty of Computer Science, University of New Brunswick
  `Title: The duality between maximal separation and minimal distance`

  Abstract: The use of (Euclidean) distance minimization to find "best" separating hyperplanes runs from proofs of the Separating Hyperplane Theorem for convex sets through current software for machine learning. Although separation is not a metric property, a metric approach seems to follow naturally from trying to find a best separator. In this talk I discuss generalizing this duality relationship to an arbitrary Minkowski metric and to the inseparable case, where it is still useful to find a "best classifying" hyperplane. It turns out separability of the input implies a convex (and hence efficiently solvable) relaxation of the distance minimization yields the correct answer, while the general non-convex minimization problem is able to solve polytope width, and hence is NP-hard.

- Ian Goulden, Department of Combinatorics & Optimization, University of Waterloo
  `Title: Maps and Branched Covers - Combinatorics, Geometry and Physics`

  Abstract: This is an expository account of recent work on the enumeration of maps (graphs embedded on a surface of arbitrary genus) and branched covers of the sphere. These combinatorial and geometric objects can both be represented by permutation factorizations, in the which the subgroup generated by the factors acts transitively on the underlying symbols (these are called "transitive factorizations"). Various results and methods are discussed, including a number of methods from mathematical physics, such as matrix integrals and the KP hierarchy of integrable systems. A notable example of the results is a recent recurrence for triangulations of a surface of arbitrary genus obtained from the simplest partial differential equation in the KP hierarchy. The recurrence is very simple, but we do not know a combinatorial interpretation of it, yet it leads to precise asymptotics for the number of triangulations with n edges, of a surface of genus g.

- ORTRUD R. OELLERMANN, Department of Mathematics, University of Winnipeg
  `Title: Graph Classes Characterized by Local Convexities`

  Abstract: All known graph convexities can be defined in terms of intervals. The most well-known graph intervals are the geodesic and monophonic intervals. The geodesic (monophonic) interval between a pair u, v of vertices in a connected graph G is the collection of all vertices that belong to some shortest (respectively, induced) $u - v$ path in G. A set U of vertices in G is g-convex (m-convex) if for every pair u, v of vertices in U the geodesic (respectively, monophonic) interval between every pair u, v of vertices in U belongs to U. For a set S of vertices in a graph G a Steiner tree for S is a connected subgraph of smallest size that contains S. The Steiner interval for S is the collection of all vertices that belong to some Steiner tree for S. A set U of vertices of G is g_3-convex if it contains the Steiner interval of every 3-set of vertices in U. We describe several local convexities and survey known characterizations of graph classes t hat possess these local convexity properties. We conclude with some new results and open problems. (Recent results are joint work with M.A. Henning and M. Nielsen.)

- BRUCE SHEPHERD, Department of Mathematics and Statistics, McGill University
  `Title: Robust Optimization of Networks`

  Abstract: We discuss several models for designing a network when the traffic demand is either unknown in advance or will be rapidly changing. The models vary especially according to how fast the network may change its routing strategy when traffic patterns change. In certain cases, extra flexibility to change the routing leads to a more difficult design problem.

- QING XIANG, Department of Mathematical Sciences, University of Delaware
  `Title: Symplectic analogues of Hamada's Formula`

  Abstract: Let V be an $(n+1)$-dimensional vector space over GF(q), where $q = p^t$, p is a prime. For $1 < r \leq n$, let $A^n_{1,r}(q)$ be the (0,1)-incidence matrix with rows and columns respectively indexed by the r- and 1-dimensional subspaces of V, and with (X, Y)-entry equal to one if and only if the 1-dimensional subspace Y is contained in the r-dimensional subspace X. In 1969, Hamada derived an exact formula for the p-rank of $A^n_{1,r}(q)$, which is now known as Hamada's formula. We remark that the Smith normal form of $A^n_{1,r}(q)$ was determined only recently by Chandler, Sin and this speaker.

  Next assume that $n + 1 = 2m \geq 4$ and equip V with a nonsingular alternating bilinear form $\langle -, - \rangle$. Let $\mathcal{I}_r$ denote the set of totally isotropic r-dimensional subspaces of V with respect to $\langle -, - \rangle$, where $1 \leq r \leq m$. The *symplectic polar space* Sp$(2m, q)$ is the geometry with flats $\mathcal{I}_r$, $1 \leq r \leq m$. We derive symplectic analogues of Hamada's formula for the p-rank of the incidence matrix between 1-flats and m-flats of Sp$(2m, q)$. It turns out that there is no uniform formula for the p-rank of these "symplectic" incidence matrices for the p odd case and the $p = 2$ case except when $m = 2$. In the

case where $m = 2$, we have the the following attractive closed formula for the p-rank of the incidence matrix between the points and lines of $\mathrm{Sp}(4, q)$.

Theorem. Let $q = p^t$, where p is a prime and $t \geq 1$. Then the p-rank of the incidence matrix between 1-flats and 2-flats of $\mathrm{Sp}(4, q)$ is equal to

$$1 + \alpha_1^t + \alpha_2^t,$$

where

$$\alpha_1, \alpha_2 = \frac{p(p+1)^2}{4} \pm \frac{p(p+1)(p-1)}{12}\sqrt{17}.$$

(Joint work with David B. Chandler and Peter Sin)