FIELDS

ABSTRACTS 1.2

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A critical-exponent Balian-Low theorem

Given a lattice in $\mathbb{R}^2$ and a function f in $L^2(\mathbb{R})$, the associated Gabor system is a collection of functions obtained by taking modulations and translations of f associated to points in the lattice. The classical Balian-Low theorem is a manifestation of the uncertainty principle in the setting of Gabor systems; it states that if both f and its Fourier transform are in the Sobolev space $H^1(\mathbb{R})$, then the Gabor system associated to f and the integer lattice is not an orthonormal basis (or, more generally, a frame) for $L^2(\mathbb{R})$. We generalize this result by showing that if f is in $H^{p/2}$ and its Fourier transform is in $H^{q/2}$ with p and q conjugate exponents, then the associated Gabor system is not a frame. We accomplish this by proving a variant of the endpoint Sobolev embedding into VMO.