ABSTRACTS 1.2

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Sharp L^p-estimates for maximal operators for $p > 2$, oscillation indices and a Fourier restriction theorem associated to hypersurfaces in $\mathbb{R}$

We study the boundedness problem for maximal operators M associated to smooth hypersurfaces S in 3-dimensional Euclidean space. For $p > 2$, we prove that if no affine tangent plane to S passes through the origin and S is analytic, then the associated maximal operator is bounded on $L^p(RR^3)$ if and only if $p > h(S)$, where $h(S)$ denotes the so-called height of the surface S. For non-analytic finite type S we obtain the same statement with the exception of the exponent $p = h(S)$. Our notion of height $h(S)$ is closely related to A. N. Varchenko's notion of height $h(\phi)$ for functions ϕ such that S can be locally represented as the graph of ϕ after a rotation of coordinates.

Several consequences of this result are discussed. In particular we verify a conjecture by E. M. Stein and its generalization by A. Iosevich and E. Sawyer on the connection between the decay rate of the Fourier transform of the surface measure on S and the L^p-boundedness of the associated maximal operator M, and a conjecture by Iosevich and Sawyer which relates the L^p-boundedness of M to an integrability condition on S for the distance function to tangential hyperplanes, in dimension three.

In particular, we also give essentially sharp uniform estimates for the Fourier transform of the surface measure on S, thus extending a result by V. N. Karpushkin from the analytic to the smooth setting and implicitly verifying a conjecture by V. I. Arnol'd in our context.

As an immediate application, we obtain an $(L^p, L^2(S))$- Fourier restriction theorem for S, which improves on a related result by A. Magyar.