On the trajectories of horizontal gradient of polynomial functions

We consider a subriemannian structure on $\mathbb{R}^n$ given by a polynomial distribution of codimension 1 (in $\mathbb{R}^n$) and a subriemannian metric on this distribution. The **horizontal gradient** vector field of a polynomial f is defined by the projection of its gradient vector field on the distribution with respect to a riemannian metric extending the above subriemannian one. We mean the set of **horizontal critical points** the one on where the horizontal gradient vanishs.

Unlike the trajectories of gradient (studies by Thom and Łojasiewicz), the length of the trajectories of horizontal gradient does not need to be bounded. Moreover these trajectories can accumulate to a "cycle". However, when f is "generic", we prove that each trajectory of the horizontal gradient of f has a limit and give a subriemannian version of Łojasiewicz's result on the uniform bound for the length.