Medical Image Processing
Using Transforms

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Fields, 08, HmZhu

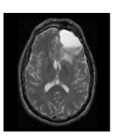
What do we want when we see a doctor?

Purpose of medical image processing

patients

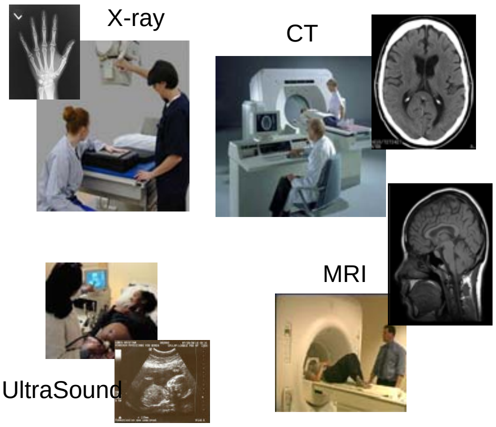
us

doctors



Medical image processing

Increase the chance of making right decisions on diagnosis, treatment, prediction, prevention, …



General Questions Concerned

- Are the images good enough to make diagnosis?
- If not, how can we improve image quality?
- What information can we draw from images?
- Can the information aid disease diagnosis?

Outlines

- Image Quality
- Gray value transforms
- Histogram processing
- Transforms in image space
- Transforms in Fourier space
- Transforms in Time-frequency space

Standard pipeline for medical image processing

Filtering

Registration

Correction

Segmentation

Analysis

Visualization

Validation

References

1. Gonzalez, R. C., Woods, R. E. and Eddins S. L. (2004). *Digital Image processing*. Pearson Prentice Hall; www.ImageProcessingPlace.com
2. www.sprawls.org
2. http://docs.gimp.org/en/
3. http://www.gnu.org/software/octave/doc/interpreter/
4. http://www.mathworks.com/access/helpdesk/help/helpdesk.html
5. http://www.math.ufl.edu/help/matlab-tutorial/matlab-tutorial.html

1. Digital Image Quality

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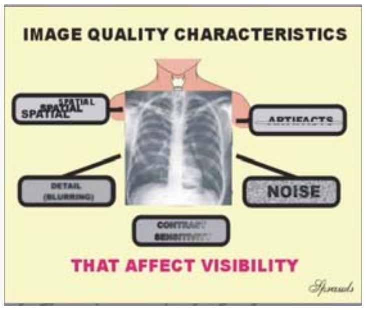
Image Quality



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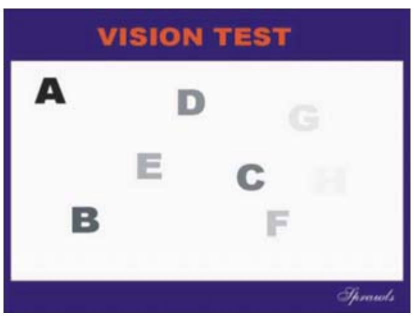
Contrast Sensitivity



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Contrast Sensitivity

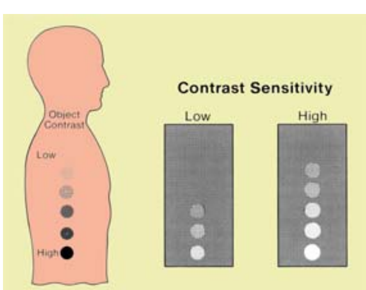

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Image Blurring

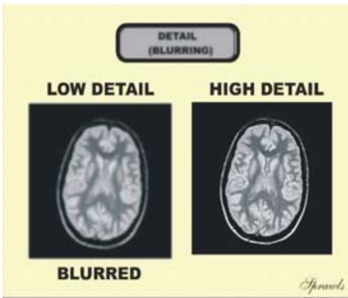

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Image Blurring

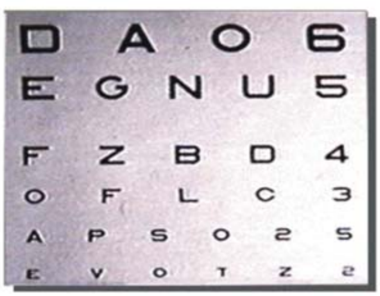

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Image Blurring

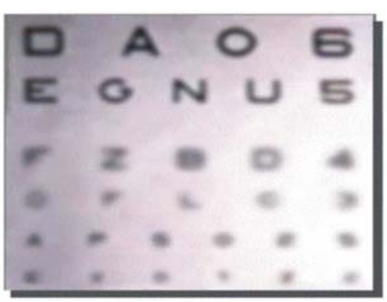

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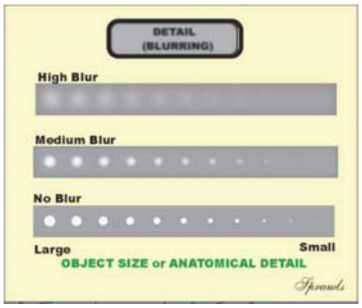
Image Blurring



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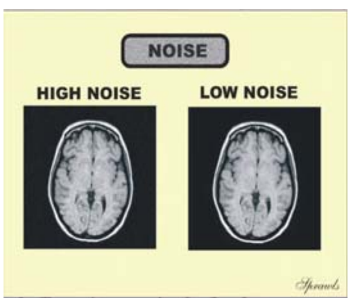
Images with different noise levels



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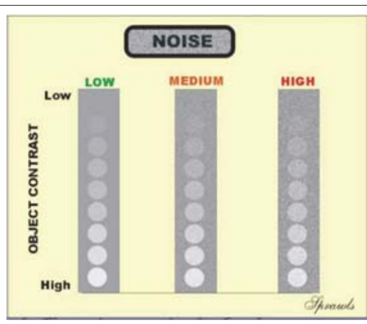
Effects of noise



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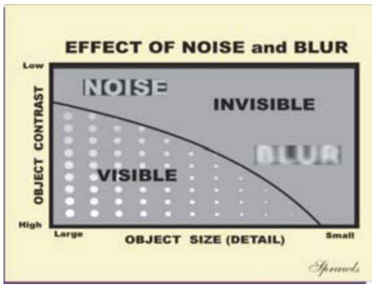
Effects of Noise and Blur



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2. Gray Value Transforms

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Histogram

Histogram shows the distribution of image intensity, often displayed as a bar graph.

The histogram of a digital image with gray levels in the range [0, L-1] is defined as

$$h(g_k) = n_k$$

where

g_k: the kth gray level, $g_k \in [0, L\text{-}1]$

n_k: the number of pixels having gray levels g_k

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Normalized Histogram

Normalized histogram estimates the probability distribution of occurrence of gray levels

Normalized histogram of a digital image with gray levels in the range [0, L-1] is defined as

$$p(g_k) = n_k / n$$

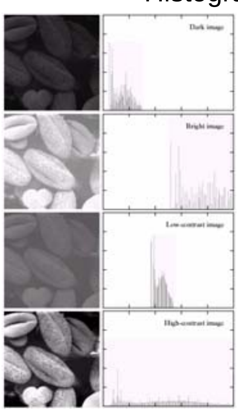
where

g_k: the kth gray level, $g_k \in [0, L-1]$

n_k: the number of pixels having gray levels g_k

n : the total number of pixels in the image

Histogram Processing



a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Dark: focuses on low values of the gray scales

Bright: biased towards the high side of the gray scales

Low contrast: has a narrow histogram

High contrast: covers a broad range of the gray scales

Gray Value Transformations

A gray level transformation of a digital image with gray levels in the range [0, L-1] is defined as

$$s = T(r)$$

where

r: the original/input gray levels

s: the transformed/output gray levels

T: gray level transformation

Gray Value Transformations

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.

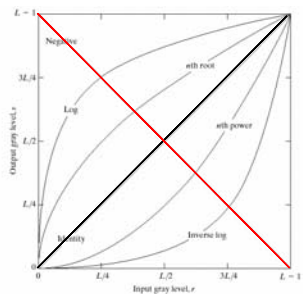
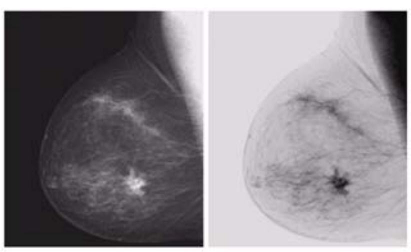


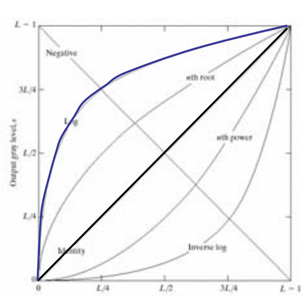
Image Negatives: s = L-1-r



a b
FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Gray Value Transformations

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.



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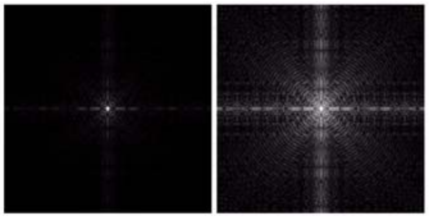
Log Transformations:
$s = c\,log(1+r)$ $(r \geq 0)$

a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



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Power-Law Transformations

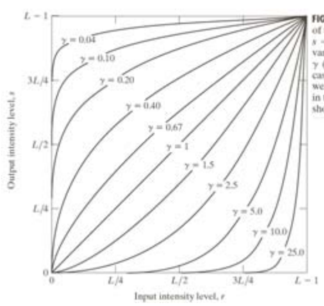

FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

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Power-Law Transformations:
$s = c\,r^\gamma$

a b
c d

FIGURE 3.8 (a) Magnetic resonance image (MRI) of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

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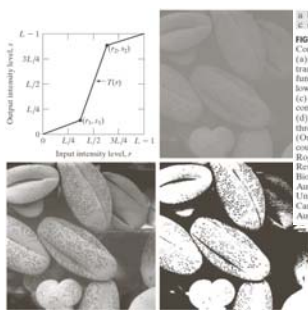
Power-Law Transformations: $s = c\,r^{\gamma}$

a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

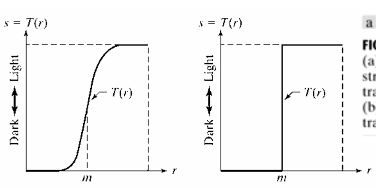
Contrast stretching transforms



a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Contrast stretching transforms

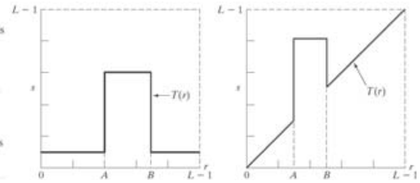


a b

FIGURE 3.4 (a) Contrast-stretching transformation. (b) Thresholding transformation.

$$s = T(r) = \frac{1}{1 + (m/r)^E}$$

Slicing transforms



a b

FIGURE 3.11 (a) This transformation highlights intensity range $[A, B]$ and reduces all other intensities to a lower level. (b) This transformation highlights range $[A, B]$ and preserves all other intensity levels.

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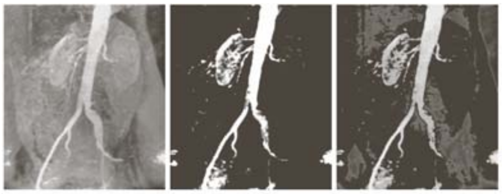
Slicing transforms



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

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2. Histogram Processing

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Histogram Equalization

$$s = T(r) = (L-1)\int_0^r p_r(\omega)\,d\omega$$

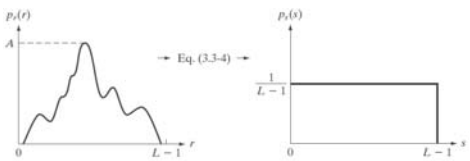
The probability density function of the output levels s is uniform.

For digital images, the equalization transform becomes

$$s_k = T(r_k) = (L-1)\sum_{j=1}^{k} p_r(r_j) = \sum_{j=1}^{k} \frac{r_j}{n}$$

where n is the total number of the pixels in the image.

Histogram Equalization



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Example: 3-bit image

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

Example: 3-bit image

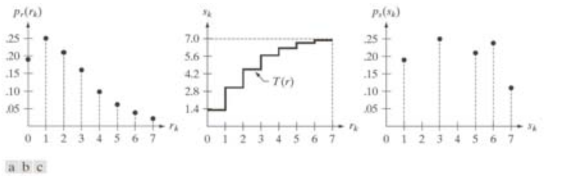
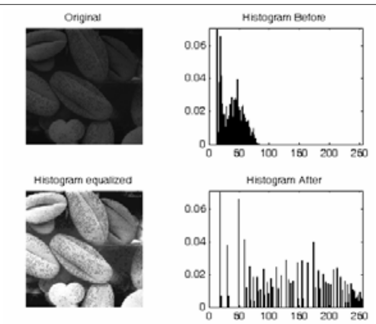
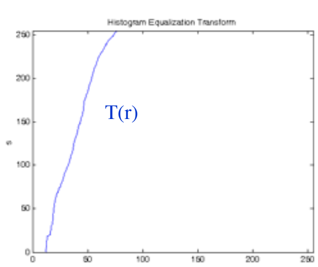


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Example: Pollen



Example: Pollen



Example: Pollen

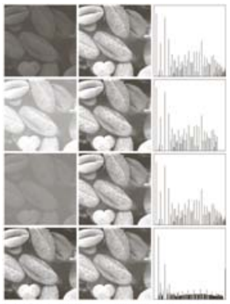
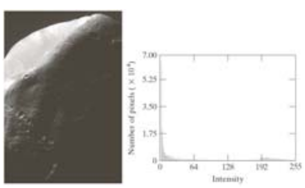


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

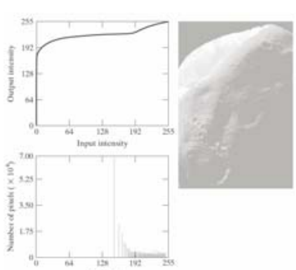
Example: Mars_Moon



a b

FIGURE 3.23 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

Example: Mars_Moon



a b
c

FIGURE 3.24 (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).

Histogram Matching

$$s = T(r) = (L-1)\int_0^r p_r(\omega)\,d\omega$$

results in intensity levels s that is uniform distributed.
Suppose we define a variable z such that

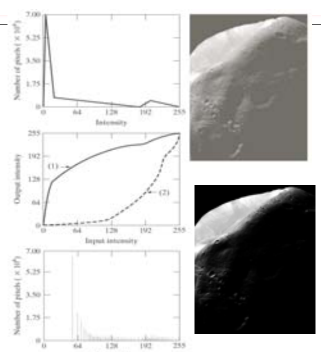
$$H(z) = (L-1)\int_0^z p_z(\omega)\,d\omega = s,$$

where intensity level z has the specific density $p_z(z)$.
Then we have

$$z = H^{-1}(s) = H^{-1}[T(r)].$$

That is, we transform intensity levels r with density function $p_r(r)$ to intensity levels z with specific density function $p_z(z)$.

Histogram Matching



a c
b
d
FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

3. Application:
MR Intensity Calibrations

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MR Image Intensity Calibration

Magnetic Resonance in Medicine 42:1072–1081 (1999)

On Standardizing the MR Image Intensity Scale

László G. Nyúl and Jayaram K. Udupa*

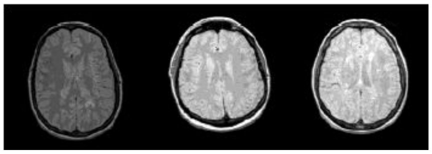


FIG. 5. Original slices from three studies acquired as per the same FSE Pd protocol before standardization displayed at a fixed window that was actually set up for the first image (first row)

MR intensity variation

- MR signal intensity doesn't have a fixed unit of measure
- Although the relative difference between tissue types will remain roughly constant from scan to scan, the absolute value of the scale is not fixed
- May pose a problem in image segmentation or quantification

Overview

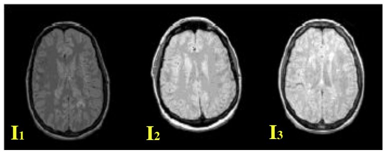
Task

- For the same tissues in all-same-settings, the resulting image intensities should be same or close
- For the same tissues in similar settings, the resulting image intensities should be similar

Methods

- Scaling or windowing (quick, intra-patients)
- Transforming to a "standard" histogram (inter-patients, various setting)

Method 1: Scaling or windowing



$$\hat{I}_i = I_i \cdot \frac{mean(I_{1,CSF})}{mean(I_{i,CSF})}$$

or

$$\hat{I}_i = (I_i - I_{i,\min}) \cdot \frac{I_{1,\max} - I_{1,\min}}{I_{i,\max} - I_{i,\min}} + I_{1,\min}$$

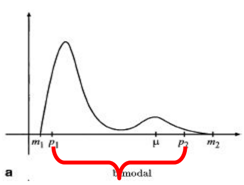
- Quick
- Can achieve display uniformity

- May not be fine enough for quantitative image analysis across different imaging protocols

Method 2: transformation

Model

Bimodal histogram



Range of intensity of interests

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Method 2: transformation

Piecewise linear mapping

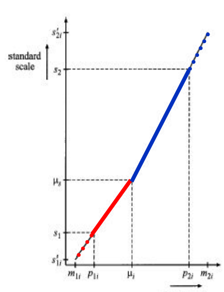

FIG. 2. The intensity mapping function for the transformation phase.

Example: histogram standardization

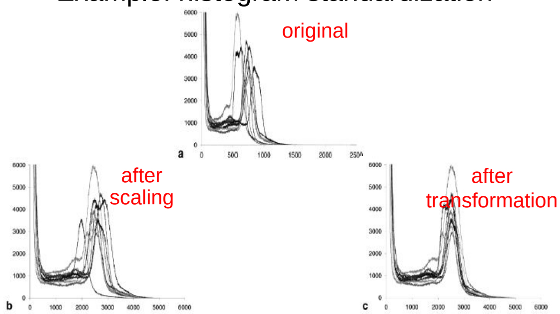

FIG. 4. Histograms at different stages of the standardization process for 10 different FSE Pd studies. Original histograms (a), histograms after intensity scaling from $[p_{1i}, p_{2i}]$ to $[s_1, s_2]$, with $s_1 = 1$, $s_2 = 4095$ (b), and after final standardization (c).

Eg: for different patients

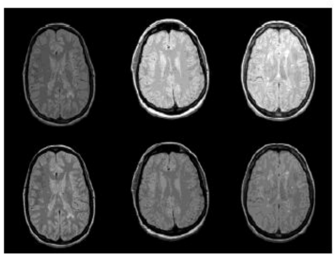

FIG. 5. Original slices from three studies acquired as per the same FSE Pd protocol before standardization displayed at a fixed window that was actually set up for the first image (first row), and after standardization displayed at a fixed "standard" window (second row).

Eg: For an non-brain region

FIG. 7. Original slices of three foot studies before (first row), and after standardization (second row). The imaging protocol for all three datasets was a T_1-weighted gradient-echo sequence with identical parameters.

Eg3: From different scanners

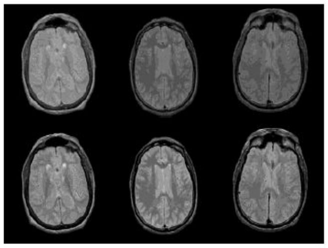

FIG. 9. Three SE Pd studies acquired as per the same protocol before standardization at default windows (first row), and after standardization displayed at a standard window (second row). The training data were acquired as per a similar protocol (with slightly different parameters) from a scanner of the same brand at a different hospital.

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Remarks

- The transform chosen for standardization has to be 1-to-1 and monotonically increasing
- The intensity calibration for patients is better done in disease-removed images or in an non-disease homogenous region.