Lindelöf spaces which are indestructible, productive, or D

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We report on recent research in collaboration with Marion Scheepers and with Leandro Aurichi. Classical combinatorial strengthenings of Lindelöfness, namely the *Menger* and *Rothberger* properties, yield new insights into longstanding open problems in topology. For example,

Theorem 1 [3]**.** *If it is consistent there is a supercompact cardinal, it is consistent with GCH that all Rothberger spaces with points G_δ have cardinality $\leq \aleph_1$, and that all uncountable Rothberger spaces of character $\leq \aleph_1$ have Rothberger subspaces of size $\aleph_1$.*

Theorem 2 [1]**.** *Menger spaces are D-spaces.*

Theorem 3 [2]**.** *Indestructibly productively Lindelöf implies Alster implies Menger.*

Theorem 4 [2]**.** *CH implies that if a T_3 space X is either separable or first countable, and is productively Lindelöf, then it is Alster and hence Menger and D.*

Theorem 5 [2]**.** *Every completely metrizable productively Lindelöf space is Menger (Alster) (σ-compact) (indestrucibly productively Lindelöf) iff there is a Lindelöf regular space M such that $M \times \mathbb{P}$ (the space of irrationals) is not Lindelöf.*

Definitions.

- A space X has the *Rothberger* (*Menger*) property if for each sequence $\{\mathcal{U}_n : n < \omega\}$ of open covers of X (each closed under finite unions), for each n there is a $U_n \in \mathcal{U}_n$ such that $\{U_n : n < \omega\}$ covers X.
- A space X is D if for each open neighborhood assignment $\{V_x : x \in X\}$ there is a closed discrete D such that $\{V_x : x \in D\}$ covers X.
- A space is *Alster* if every open cover by G_δ sets that covers each compact set finitely includes a countable subcover.
- A space X is *productively Lindelöf* if $X \times Y$ is Lindelöf for every Lindelöf space Y.
- A space is *indestructibly (productively) Lindelöf* if it remains (productively) Lindelöf in any countably closed forcing extension.

References

[1] Aurichi, L. F. D-spaces, topological games and selection principles. In preparation.

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